Chapter 14

Multi-Equation Structural Models: Simultaneous Models
Section 14.1

Modeling Approach
In some instances, econometric models might encompass elements of time-series models (for example, ARCH and GARCH models, adjustments for serial correlation).
Section 14.2

Simultaneous Systems
Simultaneous Systems

Economic models frequently involve a set of relationships designed to explain the behavior of certain variables. For instance, a simple model of the market for a given commodity may involve a supply and a demand function to determine the equilibrium price and quantity of the commodity exchange in the market. A model is said to constitute a system of simultaneous equations if all of the relationships involved are needed for determining the value of at least one of the endogenous variables included in the model. This situation implies that at least one of the relationships includes more than one endogenous variable.
Simultaneous Systems

The reduced form equations show explicitly how the endogenous variables are jointly dependent on the predetermined variables and the disturbances of the system. From the point of view of statistical inference, the single relevant characteristic of the simultaneous equation systems is the appearance of endogenous variables among the exogenous variables of at least some of the structural equations.
Section 14.3

Simultaneous Structural Models
Simultaneous Equations Model Specification

Assume $g$ endogenous (jointly dependent) variables $y_1, y_2, \ldots, y_g$ and $k$ predetermined (exogenous or lagged endogenous) variables $X_1, X_2, \ldots, X_k$.

\begin{align*}
y_1 &= \beta_{12} y_2 + \ldots + \beta_{1g} y_g + T_{11} X_1 + T_{12} X_2 + \ldots + T_{1k} X_k + \varepsilon_1, \\
y_2 &= \beta_{21} y_1 + \ldots + \beta_{2g} y_g + T_{21} X_1 + T_{22} X_2 + \ldots + T_{2k} X_k + \varepsilon_2, \\
&\vdots \quad \vdots \\
y_g &= \beta_{g1} y_1 + \beta_{g2} y_2 + \ldots + \beta_{gg-1} y_{g-1} + T_{g1} X_1 + T_{g2} X_2 + \ldots + T_{gk} X_k + \varepsilon_g.
\end{align*}

\beta's coefficients of endogenous variables  
\T's coefficients of predetermined variables  
\varepsilon's stochastic disturbance terms

Complete system $\rightarrow$ there are as many independent equations as endogenous variables.
Equations may be:

(1) Behavioral relations (demand function; consumption function)

(2) Technological relations (production function)

(3) Deterministic (definitions, identities, equilibrium conditions)
   -- Stochastic disturbance terms identically zero

Values taken by the predetermined variables, together with values of stochastic disturbance terms, determine the current values of the endogenous variables.
Vector-Matrix Notation

\[ \beta \gamma + \Gamma Z = \varepsilon \quad \text{Structural Form} \]

\[
\beta = \begin{bmatrix}
1 & \beta_{12} & \ldots & \beta_{1g} \\
\beta_{21} & 1 & \ldots & \beta_{2g} \\
\vdots & \vdots & \ddots & \vdots \\
\beta_{g1} & \beta_{g2} & \ldots & 1
\end{bmatrix};
\gamma = \begin{bmatrix}
Y_1 \\
Y_2 \\
\vdots \\
Y_g
\end{bmatrix};
\Gamma = \begin{bmatrix}
\gamma_{11} & \gamma_{12} & \ldots & \gamma_{1k} \\
\gamma_{21} & \gamma_{22} & \ldots & \gamma_{2k} \\
\vdots & \vdots & \ddots & \vdots \\
\gamma_{g1} & \gamma_{g2} & \ldots & \gamma_{gk}
\end{bmatrix};
Z = \begin{bmatrix}
X_1 \\
X_2 \\
\vdots \\
X_k
\end{bmatrix}
\]

Reduced Form (Analytically Derived)

\[
y = -\beta^{-1}\gamma Z + \beta^{-1}\varepsilon = \pi Z + V
\]
\[
\pi = -\beta^{-1}\Gamma; \quad V = \beta^{-1}\varepsilon
\]

Each endogenous variable is related to all predetermined variables and stochastic disturbance terms
Elements of the Matrix of Reduced-Form Coefficients Have a Useful Interpretation (Comparative Statics).

Change in Endogenous Variable Due to Unit Change in Predetermined Variable.

The Estimation of the Elements of the Matrix Very Important Part of Structural Analysis

Structural Form: Reduced Form

If the Predetermined Variables Include Lagged Endogenous Variables, Possible to Derive Yet Another Form of the Model - Called the Final Form.

Final Form Expresses the Current Endogenous Variables as Functions of Base Values and All Relevant Current and Lagged Exogenous Variables and Stochastic Disturbance Terms.

Important for Structural Analysis - Determination of Short-Term and Long-Term Multipliers; Reveals the Short-Run and Long-Run Comparative Statics of the Model.
Section 14.4

Types of Structural Models
Types of Structural Models

The position of the zero elements in the $\beta$ matrix indicates which endogenous variables do not appear in different structural equations. Used as a criterion for distinguishing between various types of structures.

(A) If $\beta$ is diagonal, only one endogenous variable appears in each equation. The equations are not simultaneous but seemingly unrelated.

$$
\beta = \begin{bmatrix}
\beta_{11} & \beta_{22} & 0 \\
0 & \cdot & \cdot \\
\beta_{GG} & & \cdot
\end{bmatrix}
$$
(B) If $\beta$ is upper or lower triangular, that is if

\[
\beta = \begin{bmatrix}
\beta_{11} & 0 & \cdots & 0 \\
\beta_{21} & \beta_{22} & \cdots & 0 \\
\beta_{G1} & \beta_{G2} & \cdots & \beta_{GG}
\end{bmatrix}
\]

This system is recursive.

(C) If $\beta$ is neither diagonal nor triangular (block or otherwise), we speak of an integrated structure. A system of equations characterized by an integrated structure is called a general interdependent system. Such structures provide the main subject for discussion of simultaneous equation systems.
Section 14.5

Identification Issues
Identification Issues

\[ \Pi = -\beta \Gamma^{-1} \]

Information on the estimated elements of \( \Pi \) not always enough to disentagle the effects of \( \beta \) and \( \Gamma \) in determining \( \Pi \)

The problem of identification \( \rightarrow \) providing enough a priori information to enable determination of structural parameters from the reduced-form parameters \( \beta \) and \( \Gamma \)

\( \Pi \)

Underidentified \( \rightarrow \) no way to determine all elements of \( \beta \) and \( \Gamma \) from \( \Pi \)
Just-identified \( \rightarrow \) exactly 1 way to determine all elements of \( \beta \) and \( \Gamma \) from \( \Pi \)
Over-identified \( \rightarrow \) more than 1 way to determine elements \( \beta \) and \( \Gamma \) from \( \Pi \)

General situation in most econometric models (over-identification)
Use rank and order conditions
Rank and Order Conditions

Order Condition (Necessary Condition)

The number of predetermined variables excluded from the given equation is at least as large as the number of endogenous variables included in the equation less one.

Rank Condition (Necessary & Sufficient Condition)

At least one non-vanishing determinant of rank \((G-1)\) of the coefficients of the variables, endogenous and predetermined, excluded from the structural equation being considered but appearing in the \((G-1)\) other structural equations of the model.
To express the order condition symbolically let:

\[ H = \text{total number of predetermined variables in the system.} \]

\[ G = \text{total number of endogenous variables in the system.} \]

\[ h = \text{number of predetermined variables in the particular structural equation being considered.} \]

\[ g = \text{number of endogenous variables in the particular structural equation being considered.} \]

\[ H-h = g-1 \]

\[ H-h > g-1 \]

\[ H-h < g-1 \]

Order Condition
Order Condition

The order condition for identification is determined for each structural equation prior to estimation. If some structural equations are under-identified, then it will not be possible to estimate all of their structural parameters. In systems of equations where the parameters of the structural equations may be estimated the estimation procedure to be used will depend on whether the structural equations are just-identified or over-identified. Often times for estimation purposes a distinction is not made between over-identified and just-identified. When a model or structural equations of a model are said to be identified it is understood that they are either just-identified or over-identified.
The order condition is a necessary condition which has to be fulfilled if identification is to be achieved. But just because the order condition is met does not assure us that the model or structural equations of the model are identified. If the order condition is not fulfilled, then we know that identification cannot be achieved but the order condition may be fulfilled and we still may end up with the model and some or all structural equations not being identified.
Rank Condition

The rank condition is a necessary and sufficient condition for identification. The rank condition for a structural equation is that there exist at least one non-vanishing determinant of rank (G-1) of the coefficient of the variables, endogenous and predetermined, excluded from the structural equation being considered by appearing in the (G-1) other structural equations of the model or system. When the rank condition is fulfilled we know that the structural equations with which we are dealing are identified. However, notice that there is a rather subtle difficulty with the rank condition. The difficulty is how one determines which coefficients in the determinant are non-zero.
If we know which parameters in each structural equation were non-zero there would be no problem, but in reality we do not possess this information in advance. After estimation we may be in a position to determine which estimates of parameters may be in a position to determine which estimates of parameters may be considered different from zero. So “after the facts” we are in a position to check if the rank condition was fulfilled.
However, if the assumption is made that all structural parameters of the model are different from zero, before estimation one may determine whether or not it is possible to fulfill the rank condition for each structural equation. This entails finding at least one non-vanishing determinant of rank (G-1) of the parameters of the variables, endogenous and predetermined, excluded from the structural equation under consideration but appearing in the other (G-1) structural equations of the model. If such cannot be found for some structural equations, then identification is not possible even though the order condition may have been fulfilled.
Furthermore, these determinants provide some insight about which parameters have to be non-zero for the rank condition to be fulfilled and identification achieved. After estimation you can determine whether the parameters required to be non-zero in order to fulfill the rank condition can be considered non-zero on the basis of their respective estimates.
Section 14.6

Example of Order and Rank Conditions
Specify the rank and order criteria for identification and apply them to the following model:

\[ Y_1 = \gamma_{10} + \gamma_{11}Z_1 + \gamma_{12}Z_2 + u_1 \]

\[ Y_2 = \gamma_{20} + \beta_{21}Y_1 + \beta_{23}Y_3 + \gamma_{22}Z_2 + \gamma_{23}Z_3 + \gamma_{24}Z_4 + u_2 \]

\[ Y_3 = \gamma_{30} + \beta_{32}Y_2 + \gamma_{31}Z_1 + \gamma_{35}Z_5 + \gamma_{36}Z_6 + u_3 \]
$Y_s \rightarrow$ endogenous $(Y_1, Y_2, Y_3)$

$Z_s \rightarrow$ predetermined $(\text{Intercept}, Z_1, Z_2, Z_3, Z_4, Z_5, Z_6)$

$s = 1, 2, 3$ (equations)

$H = 7; \ G = 3$;

Order Conditions

Equation (1) $7 - 2 > 1 - 1$

Equation (2) $7 - 3 > 3 - 1$ \{over identified\}

Equation (3) $7 - 3 > 2 - 1$

Rank Conditions

<table>
<thead>
<tr>
<th></th>
<th>$Y_1$</th>
<th>$Y_2$</th>
<th>$Y_3$</th>
<th>$Z_1$</th>
<th>$Z_2$</th>
<th>$Z_3$</th>
<th>$Z_4$</th>
<th>$Z_5$</th>
<th>$Z_6$</th>
<th>$\text{INT}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>$-\gamma_{11}$</td>
<td>$-\gamma_{12}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\gamma_{10}$</td>
</tr>
<tr>
<td>(2)</td>
<td>$-\beta_{21}$</td>
<td>1</td>
<td>$-\beta_{23}$</td>
<td>0</td>
<td>$-\gamma_{22}$</td>
<td>$-\gamma_{23}$</td>
<td>$-\gamma_{24}$</td>
<td>0</td>
<td>0</td>
<td>$\gamma_{20}$</td>
</tr>
<tr>
<td>(3)</td>
<td>0</td>
<td>$-\beta_{32}$</td>
<td>1</td>
<td>$-\gamma_{31}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$-\gamma_{35}$</td>
<td>$-\gamma_{36}$</td>
<td>$\gamma_{30}$</td>
</tr>
</tbody>
</table>
For Equation (1) Rank = 2
\[
\begin{vmatrix}
1 & -\beta_{23} & -\gamma_{23} & -\gamma_{24} & 0 & 0 \\
-\beta_{32} & 1 & 0 & 0 & -\gamma_{35} & -\gamma_{36}
\end{vmatrix}
\] \( (6^2) = 15 \)

For Equation (2) Rank = 2
\[
\begin{vmatrix}
-\gamma_{11} & 0 & 0 \\
-\gamma_{31} & -\gamma_{35} & -\gamma_{36}
\end{vmatrix}
\] \( (3^2) = 3 \)

For Equation (3) Rank = 2
\[
\begin{vmatrix}
1 & -\gamma_{12} & 0 & 0 \\
-\beta_{21} & -\gamma_{22} & -\gamma_{23} & -\gamma_{24}
\end{vmatrix}
\] \( (4^2) = 6 \)
For equation (1), possibilities to insure the rank condition include:

(1) \[
\begin{vmatrix}
1 & -\beta_{23} \\
-\beta_{32} & 1
\end{vmatrix}
= 1 - \beta_{32}\beta_{23} \neq 0 \Rightarrow \beta_{32}\beta_{23} \neq 1
\]

(2) \[
\begin{vmatrix}
1 & -\gamma_{23} \\
-\beta_{32} & 0
\end{vmatrix}
= -\gamma_{23}\beta_{32} \neq 0 \Rightarrow \gamma_{23} \neq 0 \text{ or } \beta_{32} \neq 0
\]

(3) \[
\begin{vmatrix}
1 & -\gamma_{24} \\
-\beta_{32} & 0
\end{vmatrix}
= -\gamma_{24}\beta_{32} \neq 0 \Rightarrow \gamma_{24} \neq 0 \text{ or } \beta_{32} \neq 0
\]

(4) \[
\begin{vmatrix}
1 & 0 \\
-\beta_{32} & -\gamma_{35}
\end{vmatrix}
= -\gamma_{35} \neq 0 \Rightarrow \gamma_{35} \neq 0
\]

(5) \[
\begin{vmatrix}
1 & 0 \\
-\beta_{32} & -\gamma_{36}
\end{vmatrix}
= -\gamma_{36} \neq 0 \Rightarrow \gamma_{36} \neq 0
\]
\[(6) \begin{vmatrix} -\beta_{23} & -\gamma_{23} \\ 1 & 0 \end{vmatrix} = \gamma_{23} \neq 0\]

\[(7) \begin{vmatrix} -\beta_{23} & -\gamma_{24} \\ 1 & 0 \end{vmatrix} = \gamma_{24} \neq 0\]

\[(8) \begin{vmatrix} -\beta_{23} & 0 \\ 1 & -\gamma_{35} \end{vmatrix} = \beta_{23}\gamma_{35} \neq 0 \Rightarrow \beta_{23} \neq 0 \text{ or } \gamma_{35} \neq 0\]

\[(9) \begin{vmatrix} -\beta_{23} & 0 \\ 1 & -\gamma_{36} \end{vmatrix} = \beta_{23}\gamma_{36} \neq 0 \Rightarrow \beta_{23} \neq 0 \text{ or } \gamma_{36} \neq 0\]

\[(10) \begin{vmatrix} -\gamma_{23} & -\gamma_{24} \\ 0 & 0 \end{vmatrix} = 0\]
(11) \[
\begin{vmatrix}
-\gamma_{23} & 0 \\
0 & -\gamma_{35}
\end{vmatrix} = -\gamma_{23}\gamma_{35} \neq 0 \implies \gamma_{23} \neq 0 \text{ or } \gamma_{35} \neq 0
\]

(12) \[
\begin{vmatrix}
-\gamma_{23} & 0 \\
0 & -\gamma_{36}
\end{vmatrix} = -\gamma_{23}\gamma_{36} \neq 0 \implies \gamma_{23} \neq 0 \text{ or } \gamma_{36} \neq 0
\]

(13) \[
\begin{vmatrix}
-\gamma_{24} & 0 \\
0 & -\gamma_{35}
\end{vmatrix} = -\gamma_{24}\gamma_{35} \neq 0 \implies \gamma_{24} \neq 0 \text{ or } \gamma_{35} \neq 0
\]

(14) \[
\begin{vmatrix}
-\gamma_{24} & 0 \\
0 & -\gamma_{36}
\end{vmatrix} = -\gamma_{24}\gamma_{36} \neq 0 \implies \gamma_{24} \neq 0 \text{ or } \gamma_{36} \neq 0
\]

(15) \[
\begin{vmatrix}
0 & 0 \\
-\gamma_{35} & -\gamma_{36}
\end{vmatrix} = 0
\]
For equation (2), possibilities to insure the rank condition include:

\[
\begin{vmatrix}
-\gamma_{11} & 0 \\
-\gamma_{31} & -\gamma_{35}
\end{vmatrix} = \gamma_{11}\gamma_{35} \neq 0 \Rightarrow \gamma_{11} \neq 0 \text{ or } \gamma_{35} \neq 0 \\
\begin{vmatrix}
-\gamma_{11} & 0 \\
-\gamma_{31} & -\gamma_{36}
\end{vmatrix} = \gamma_{11}\gamma_{36} \neq 0 \Rightarrow \gamma_{11} \neq 0 \text{ or } \gamma_{36} \neq 0 \\
\begin{vmatrix}
0 & 0 \\
-\gamma_{35} & -\gamma_{36}
\end{vmatrix} = 0
\]

Key for equation (2), $\gamma_{11} \neq 0$. 
For equation (3), possibilities to insure the rank condition include:

(1) \[
\begin{vmatrix}
1 & -\gamma_{12} \\
-\beta_{21} & -\gamma_{22}
\end{vmatrix} = -\gamma_{22} - \gamma_{12}\beta_{21} = -\gamma_{22} \neq \gamma_{12}\beta_{21}
\]

(2) \[
\begin{vmatrix}
1 & 0 \\
-\beta_{21} & -\gamma_{23}
\end{vmatrix} = -\gamma_{23} \neq 0 \Rightarrow \gamma_{23} \neq 0
\]

(3) \[
\begin{vmatrix}
1 & 0 \\
-\beta_{21} & -\gamma_{24}
\end{vmatrix} = -\gamma_{24} \neq 0 \Rightarrow \gamma_{24} \neq 0
\]

(4) \[
\begin{vmatrix}
-\gamma_{12} & 0 \\
-\gamma_{22} & -\gamma_{23}
\end{vmatrix} = \gamma_{12}\gamma_{23} \neq 0 \Rightarrow \gamma_{12} \neq 0 \text{ or } \gamma_{23} \neq 0
\]

(5) \[
\begin{vmatrix}
-\gamma_{12} & 0 \\
-\gamma_{22} & -\gamma_{24}
\end{vmatrix} = \gamma_{12}\gamma_{24} \neq 0 \Rightarrow \gamma_{12} \neq 0 \text{ or } \gamma_{24} \neq 0
\]

(6) \[
\begin{vmatrix}
0 & 0 \\
-\gamma_{23} & -\gamma_{24}
\end{vmatrix} = 0
\]
Section 14.7

Common Methods of Estimation
Common Methods of Estimation

(1) Ordinary Least Squares (OLS) Leads to Inconsistent Estimates

(2) Two-stage Least Square Estimation (2SLS) Just or Over-Identified Structural Equations

(3) Three-Stage Least Squares Estimation (3SLS)
Other Possible Estimation Methods Include:

(1) LIML – limited information maximum likelihood
(2) FIML – full information maximum likelihood
(3) GMM – generalized method of moments
(4) K-class estimators
Section 14.8

Simultaneous Equations
Model of Demand and Supply Relationships
Simultaneous Equations Model of Demand and Supply Relationships

Demand Relationship

\[ Y_1 = C_{10} + \beta_{12} Y_2 + T_{11} Z_1 + T_{12} Z_2 + u_1 \]

Supply Relationship

\[ Y_2 = C_{20} + \beta_{21} Y_1 + T_{23} Z_3 + T_{24} Z_4 + u_2 \]

The \( Y \)'s are endogenous variables, the \( Z \)'s are exogenous, and the \( u \)'s are random disturbances.
The variables are:

\[ Y_1 = \text{quantity demanded} = \text{quantity supplied (identity)} \]

\[ Y_2 = \text{own price} \]

\[ Z_1 = \text{price of a substitute} \]

\[ Z_2 = \text{income} \]

\[ Z_3 = \text{price of an input} \]

\[ Z_4 = \text{a measure of weather} \]

\[ C_{10}, C_{20} = \text{intercepts of the demand and supply relationships} \]

\[ \beta' \text{'s are parameters of the endogenous variables} \]

\[ T' \text{'s are parameters of the exogenous variables} \]
Matrix Representation

\[
\begin{bmatrix}
1 & -\beta_{12} \\
-\beta_{21} & 1
\end{bmatrix}
\begin{bmatrix}
Y_1 \\
Y_2
\end{bmatrix}
= 
\begin{bmatrix}
\tau_{11} & \tau_{12} & 0 & 0 & C_{10} \\
0 & 0 & \tau_{23} & \tau_{24} & C_{20} \\
Z_1 & Z_2 & Z_3 & Z_4 & 1
\end{bmatrix}
+ 
\begin{bmatrix}
u_1 \\
u_2
\end{bmatrix}
\]

Multiple endogenous variables in the structural equations of simultaneous equation systems lead to some difficulties of interpretation of structural coefficients.

When interpreting a coefficient, it is unreasonable to assume all other variables in the equation to be nonvariant, since values of other endogenous variables vary simultaneously.

**Example:** Interpretations of elasticities or flexibilities - require *ceteris paribus* assumption--all other things being equal.
In Summary

Not only the endogenous variable under consideration is affected, but other endogenous variables are affected, which in turn influences the endogenous variable under consideration.

To illustrate, suppose $Z_1$ changes by some incremental amount to $dZ_1$. By how much does $Y_1$ change?

Initially by $\tau_{11}$

But a change in $Y_1$ simultaneously leads to a change in $Y_2$, every unit change in $Y_1 \rightarrow \beta_{21}$ change in $Y_2$.

Changes in $Y_2$ lead to simultaneous changes in $Y_1$.

The change in $Y_1$ associated with a unit change in $Y_2$ is $\beta_{12}$.

A set of “reactions” needs to be taken into consideration to get at the net effect of a change on $Z_1$. 
Section 14.9

Analytically-Derived Reduced Forms
Analytically-Derived Reduced Forms

The net impacts of changes in exogenous variables may be conventionally assessed from the analytically derived reduced forms. Also, these forms may be used for predicting the values of endogenous variables since each endogenous variable is expressed as a function of exogenous variables. The analytically derived reduced forms are obtained by solving for the Y’s. The analytically derived reduced forms for the model being considered are:

\[
\begin{bmatrix}
Y_1 \\
Y_2 
\end{bmatrix} = -\begin{bmatrix}
1 & -\beta_{12} \\
-\beta_{21} & 1 
\end{bmatrix}^{-1} \begin{bmatrix}
\tau_{11} & \tau_{12} & 0 & 0 & C_{10} \\
0 & 0 & \tau_{23} & \tau_{24} & C_{20} \\
\end{bmatrix} \begin{bmatrix}
Z_1 \\
Z_2 \\
Z_3 \\
Z_4 \\
1 
\end{bmatrix} + \begin{bmatrix}
1 & -\beta_{12} \\
-\beta_{21} & 1 
\end{bmatrix}^{-1} \begin{bmatrix}
u_1 \\
u_2 
\end{bmatrix}
\]
Thus,

\[ Y_1 = (-1/1 - \beta_{12}\beta_{21})[\tau_{11}Z_1 + \tau_{12}Z_2 - \beta_{12}\tau_{23}Z_3 - \beta_{12}\tau_{24}Z_4 + (\tau_{10} - \beta_{12}\tau_{20})C_{10}] + (u_1 - \beta_{12}u_2 / 1 - \beta_{12}\beta_{21}) \]

And,

\[ Y_2 = (-1/1 - \beta_{12}\beta_{21})[-\beta_{21}\tau_{11}Z_1 - \beta_{21}\tau_{12}Z_2 + \tau_{23}Z_3 + \tau_{24}Z_4 + (\tau_{20} - \beta_{21}\tau_{10})C_{20}] + (u_2 - \beta_{21}u_1 / 1 - \beta_{12}\beta_{21}) \]
The net impacts of changes in exogenous variables on endogenous variables may be conveniently assessed from the analytically-derived reduced forms.

In much policy-related research, interest is in assessing the net impacts of changes in exogenous variables on selected endogenous variables. Furthermore, the interest frequently is in assessing the impacts not only in a single time period, but in evaluating the accumulated impacts for a specified amount of time, and sometimes the total accumulated impacts may be of interest. These interests should be taken into account in the initial phases of modeling, so that the appropriate lagged endogenous and exogenous variables are incorporated into the model.

Theil and Boot 1962

Formulate the system of analytically - derived reduced form equations

\[ Y_t = AY_{t-1} + BX_t + CX_{t-1} + u_t^* \]

Where

- \( Y_t \) is a vector of current endogenous variables
- \( Y_{t-1} \) is a vector of endogenous variables lagged one time period
- \( X_t \) is a vector of current exogenous variables
- \( X_{t-1} \) is a vector of exogenous variables lagged one time period
- \( U_t^* \) is a vector of derived reduced form disturbances where each element denotes the disturbance associated with an individual reduced form equation

\( A \) is the coefficient matrix of the lagged endogenous variables
\( B \) is the coefficient matrix of the current exogenous variables
\( C \) is the coefficient matrix of the lagged exogenous variables
Noteworthy Characteristics

(1) Both lagged endogenous and exogenous variables are included

(2) Each equation need not contain all endogenous and exogenous variables lagged (some elements of A and C may be zero)

(3) If interest is in tracking the effects of changes in exogenous variables through time, then A and/or C must not be null (zero) matrices
Impact Multipliers

Impact multipliers measure the immediate net effect of changes in exogenous variables on endogenous variables. The net effects of changes in exogenous variables in time $t$ on endogenous variables in time $t$. 

$$\frac{\partial Y_t}{\partial X_t} = B \leftarrow \text{Impact Multipliers}$$
Interim Multipliers

Interim multipliers give the net effects of change in exogenous variables in time t on endogenous variables in time t, where t’ > t. The interim multipliers may be expressed as the following derivative:

$$\frac{\partial Y_{t'}}{\partial X_t} \quad t' > t$$

Consider the net effect of changes in exogenous variables on endogenous variables one year later. Thus t’-t = s = 1. We have,

$$Y_{t-1} = AY_{t-2} + BX_{t-1} + CX_{t-2} + u_t^*$$

and by substituting yields:

$$Y_t = A(AY_{t-1} + BX_{t-1} + CX_{t-2} + u_{t-1}^*)BX_t + CX_{t-1} + u_t^*$$

$$Y_t = A^2 Y_{t-2} + BX_t + (AB + C)X_{t-1} + ACX_{t-2} + (u_t^* A_{t-1}^*)$$

Thus, the influence of changes in exogenous variables on endogenous variables one year later is given by:

$$\frac{\partial Y_t}{\partial X_{t-1}} = \frac{\partial Y_{t+1}}{\partial X_t} = (AB + C)$$
To find the influence of changes in exogenous variables on endogenous variables two years later consider \( t' = t = s = 2 \).

\[
Y_{t-2} = AY_{t-3} + BX_{t-2} + CX_{t-3} + u^*_t
\]

Substituting yields:

\[
Y_t = A^2(AY_{t-3} + BX_{t-2} + CX_{t-3} + u^*_t) + BX_t
\]

\[
+ (AB + C)X_{t-1} + ACX_{t-2} + (u^*_t + Au^*_{t-1})
\]

\[
Y_t = A^3Y_{t-3} + A^2BX_{t-2} + A^2CX_{t-3} + A^2u^*_{t-2} + BX_t
\]

\[
+ (AB + C)X_{t-1} + ACX_{t-2} + (u^*_t + Au^*_{t-1})
\]

\[
Y_t = A^3Y_{t-3} + BX_{t-2} + (AB + C)X_{t-1} + A(AB + C)X_{t-2}
\]

\[
+ A^2CX_{t-3} + (u^*_t + Au^*_{t-1} + A^2u^*_{t-2})
\]

continued...
Thus, the influence of changes in exogenous variables on endogenous variables two years later is given by,

$$\frac{\partial Y_t}{\partial X_{t-2}} = A(AB + C)$$

This process could be continued for any length of lag. In general, let $t' – t = s$ and $D^s = A^{s-1} (AB + C)$.

$D^s$ is a measure of the net effects of changes in exogenous variables on endogenous variables $s$ time periods later. Again, note that some lagged endogenous variables have to be included in the model or it will not be possible to assess the interim multipliers.
Total Multipliers

Total multipliers refer to the accumulated let effect of changes in exogenous variables on endogenous variables as the time lapse approaches infinity. That is $t' - t = s \to \infty$. Total multipliers measure the accumulated affects of all possible interim multipliers or they are measuring the net effects of changes in exogenous variables from the time of changes occur until the effects are dissipated as the time elapsed approaches infinity. The total multipliers are the infinite sum:

$$\sum_{t'=t}^{\infty} \frac{\partial X(t')}{\partial X(t)}$$

Provided the sum converges to a finite limit.
In terms of the coefficient matrices of the analytically-derived reduced forms, the total (long-run) multiplier is given by:

$$B + \sum_{s=1}^{\infty} A^{s-1}(AB + C) = B + (I - A)^{-1}(C + AB)$$

Given the analytically-derived form equations, the total accumulated effects of changes in exogenous variables can easily be assessed.
Summary

Impact multipliers, interim multipliers, and total multipliers provide the type of information often sought by policy makers; these multipliers are obtained from the analytically-derived reduced form equations associated with the set of structural equations and estimated coefficients. Thus, the usefulness of structural estimation may not be to obtain estimated structural parameters per se but to obtain various types of impact multipliers which may be helpful in policy evaluation and decision-making.
Section 14.10

Microeconomics Specification of Simultaneous Equation Models
Example: Demand and Price Mark-up Functions for Canned Cling Peaches and Fruit Cocktail, French and King (1986)

(1) \[ \ln \ QMRN \ = \ b_{10} + b_{11} \ln \ PPR + b_{12} \ln \ TDIN \]
\[ + b_{13} D70 + b_{14} T_{14} + b_{15} (T_{14})^2 + u_1 \]

(2) \[ \ln \ QMFN \ = \ b_{20} + b_{21} \ln \ PPF + b_{22} \ln \ TDIN \]
\[ + b_{23} D70 + b_{24} T_{14} + b_{25} (T_{14})^2 + u_2 \]

(3) \[ \ln \ PPR \ = \ b_{30} + b_{31} \ln \ TCR + b_{32} (D74) \ln \ TCR \]
\[ + b_{33} D74 + b_{34} T + b_{35} \ln \ IRR + b_{36} \ln \ QCRN \]
\[ + b_{37} \ln \ RSR + u_3 \]
(4) \[ \ln PPF = b_{40} + b_{41} \ln TCF + b_{42}(D74) \ln TCF \]
\[ + b_{43}D74 + b_{44}T + b_{45} \ln IRF + b_{46} \ln QCFN \]
\[ + b_{47} \ln RSF + u_4 \]

In matrix form, the matrix of coefficients associated with the endogenous variables is given by:

\[
\begin{bmatrix}
1 & 0 & -b_{11} & 0 \\
0 & 1 & 0 & -b_{21} \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\ln QMRN \\
\ln QMFN \\
\ln PPR \\
\ln PPF
\end{bmatrix}
\]

Given this matrix of coefficients, as long as \( b_{11} \) or \( b_{21} \) are not equal to zero, this system constitutes a simultaneous system (more precisely a recursive system).
*Simultaneous Equation Model;

•Demand and Price-Markup Functions for Canned Cling Peaches and Fruit Cocktail;

•French and King Western Journal of Agricultural Economics 11(1)(1986):8-18;

data peachesA;
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1957 1.12 1.21 120.15 61.69 0.65 134.64 74.66
1958 1.18 1.22 97.58 61.17 0.66 115.03 74.42
1959 1.12 1.18 122.96 68.52 0.69 137.85 80.84
1960 1.17 1.26 115.01 65.89 0.71 134.05 83.19
1961 1.15 1.25 125.21 72.89 0.72 143.62 91.38
1962 1.12 1.15 138.08 80.04 0.75 155.18 92.01
1963 1.10 1.16 135.88 67.12 0.78 149.39 78.17
1964 1.19 1.15 145.95 82.73 0.83 173.00 95.20
1965 1.11 1.26 131.71 69.22 0.89 146.21 86.92
1966 1.14 1.16 147.77 84.16 0.95 168.71 97.77
1967 1.13 1.21 118.87 66.59 1.00 134.22 80.86
1968 1.21 1.21 135.93 80.17 1.07 164.02 96.69
1969 1.29 1.26 142.02 78.61 1.14 183.10 98.68
1970 1.26 1.27 124.69 62.12 1.23 157.66 78.96
1971 1.16 1.35 118.98 59.95 1.31 137.71 80.82
1972 1.07 1.17 112.11 66.01 1.40 119.69 77.14
1973 1.06 1.09 102.97 68.33 1.57 109.51 74.18
1974 1.17 1.23 121.59 61.16 1.69 141.98 75.49
1975 1.26 1.24 110.16 62.51 1.84 139.13 77.51
1976 1.22 1.24 108.99 62.26 1.99 133.22 77.27
1977 1.23 1.19 121.27 62.00 2.17 149.18 73.81
1978 1.15 1.13 101.94 56.68 2.40 116.89 64.26
1979 1.19 1.21 101.81 56.89 2.66 121.65 68.87
1980 1.29 1.40 100.20 54.79 2.91 129.36 76.95
1981 1.40 1.47 85.12 48.69 3.23 118.79 71.50
1982 1.28 1.27 86.76 47.46 3.40 110.78 60.16
1983 1.08 1.20 64.58 39.57 3.62 69.39 47.68
1984 1.27 1.19 66.26 37.76 3.94 83.98 45.11
options nodate;

data peaches; merge peachesA peachesB; by year;

   proc means data=peaches n mean median std min max;
       var _all_
   run;

   data operation; set peaches;
   lqmrn=log(qmrn); lqmfn=log(qmfn); lppr=log(ppr); lppf=log(ppf);
   ltdin=log(tdin); ltcr=log(tcr); lirr=log(irr); lqcrn=log(qcrn);
   lrsr=log(rsr); ltcf=log(tcf); lirf=log(irf); lqcfn=log(qcfn);
   lrsf=log(rsf);

* creation of a trend variable;
   t=0;
   if year=1956 then t=1; if year=1957 then t=2;
   if year=1958 then t=3; if year=1959 then t=4;
   if year=1960 then t=5; if year=1961 then t=6;
   if year=1962 then t=7; if year=1963 then t=8;
   if year=1964 then t=9; if year=1965 then t=10;
   if year=1966 then t=11; if year=1967 then t=12;
   if year=1968 then t=13; if year=1969 then t=14;
   if year=1970 then t=15; if year=1971 then t=16;
   if year=1972 then t=17; if year=1973 then t=18;
   if year=1974 then t=19; if year=1975 then t=20;
   if year=1976 then t=21; if year=1977 then t=22;
   if year=1978 then t=23; if year=1979 then t=24;
   if year=1980 then t=25; if year=1981 then t=26;
   if year=1982 then t=27; if year=1983 then t=28;
   if year=1984 then t=29;
* creation of additional exogenous variables;
  
  `d70=0; if year > 1969 then d70=1;`
  
  `d74=0; if year > 1973 then d74=1;`
  
  `intd74ltcf=d74*ltcf;`
  
  `intd74ltcr=d74*ltcr;`
  
  `t14=0; if year > 1969 then t14=t-14;`
  
  `t14sq=t14*t14;`
  
  `proc print data=operation; var year t d70 d74 t14; run;`
  
  * estimation by OLS;`

  `proc reg data=operation; model lqmrn=lppr ltdin d70 t14 t14sq / dw dwprob; run;`

  `proc reg data=operation; model lqmfn=lppf ltdin d70 t14 t14sq / dw dwprob; run;`

  `proc reg data=operation; model lppr=ltcr intd74ltcr d74 t lirr lqcrn lrsr / dw dwprob; run;`

  `proc reg data=operation; model lppf=ltcf intd74ltcf d74 t lirf lqcfn lrsf / dw dwprob; run;`
```
proc syslin data=operation out=peachesanal3sls outest=peaches3sls;
endogenous lqmrn lqmfn lppr lppf;
instruments ltdin d70 t14 t14sq ltcr intd74ltcr d74 t lirr lqcrn lrsr ltcf intd74ltcf lirf lqcfn lrsf;
qmrreq: model lqmrn=lppr ltdin d70 t14 t14sq / dw overid;
       output p=lqmrn_3sls;
qmfneq: model lqmfn=lppf ltdin d70 t14 t14sq / dw overid;
       output p=lqmfn_3sls;
ppeq: model lppr=ltcr intd74ltcr d74 t lirr lqcrn lrsr / dw overid;
      output p=lppr_3sls;
ppfeq: model lppf=ltcf intd74ltcf d74 t lirf lqcfn lrsf / dw overid;
      output p=lppf_3sls;
run;
```
data sim; set peachesanal;
proc print; var year lqmrn lqmrn_3sls;
run;
proc corr; var lqmrn lqmrn_3sls;
run;
proc print; var year lqmfn lqmfn_3sls;
run;
proc corr; var lqmfn lqmfn_3sls;
run;
proc print; var year lppr lppr_3sls;
run;
proc corr; var lppr lppr_3sls;
run;
proc print; var year lppf lppf_3sls;
run;
proc corr; var lppf lppf_3sls;
run;
proc simlin est=peaches3sls;
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  exogenous ltdin d70 t14 t14sq ltcr intd74ltcr d74 t lirr
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run;
The MEANS Procedure

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The REG Procedure
OLS ESTIMATES
Dependent Variable: lqmrn

Number of Observations Read          29
Number of Observations Used          29

Analysis of Variance

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Root MSE 0.07328  R-Square 0.8957
Dependent Mean 4.71948  Adj R-Sq 0.8730
Coeff Var 1.55276
| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > |t| |
|----------|----|--------------------|----------------|---------|-------|
| Intercept| 1  | 6.04166            | 0.33719        | 17.92   | <.0001|
| lppr     | 1  | -0.69698           | 0.21016        | -3.32   | 0.0030|
| ltdin    | 1  | 0.39795            | 0.10414        | 3.82    | 0.0009|
| d70      | 1  | -0.22884           | 0.07611        | -3.01   | 0.0063|
| t14      | 1  | 0.06816            | 0.02908        | 2.34    | 0.0281|
| t14sq    | 1  | -0.00537           | 0.00116        | -4.62   | 0.0001|
The REG Procedure

Dependent Variable: lqmrn

Durbin-Watson D                2.665
Pr < DW                       0.8292
Pr > DW                       0.1708
Number of Observations            29
1st Order Autocorrelation       -0.344

NOTE: Pr<DW is the p-value for testing positive autocorrelation, and Pr>DW is the p-value for testing negative autocorrelation.
The REG Procedure

OLS ESTIMATES

Dependent Variable: lqmfn

Number of Observations Read          29
Number of Observations Used          29

Analysis of Variance

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Root MSE              0.04025  R-Square   0.9644
Dependent Mean        4.13689  Adj R-Sq   0.9567
Coef Var              0.97307
### Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > |t| |
|----------|----|--------------------|----------------|---------|------|---|
| Intercept| 1  | 6.02993            | 0.24367        | 24.75   | <.0001 |
| lppf     | 1  | -0.91356           | 0.13185        | -6.93   | <.0001 |
| ltdin    | 1  | 0.43074            | 0.05772        | 7.46    | <.0001 |
| d70      | 1  | -0.26844           | 0.04197        | -6.40   | <.0001 |
| t14      | 1  | 0.08286            | 0.01543        | 5.37    | <.0001 |
| t14sq    | 1  | -0.00534           | 0.00063027     | -8.48   | <.0001 |
The REG Procedure

Dependent Variable: lqmfn

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NOTE: Pr<DW is the p-value for testing positive autocorrelation, and Pr>DW is the p-value for testing negative autocorrelation.
The REG Procedure
OLS ESTIMATES
Dependent Variable: lppr

Number of Observations Read          29
Number of Observations Used          29

Analysis of Variance

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Root MSE          0.02975
Dependent Mean    1.97581
Coef Var          1.50576
R-Square          0.9968
Adj R-Sq          0.9957
| Variable        | DF | Parameter Estimate | Standard Error | t Value | Pr > |t| |
|-----------------|----|--------------------|----------------|---------|-------|---|
| Intercept       | 1  | -0.74919           | 0.15861        | -4.72   | 0.0001|
| ltcr            | 1  | 1.22063            | 0.11677        | 10.45   | <.0001|
| intd74ltcr      | 1  | -0.05237           | 0.08419        | -0.62   | 0.5407|
| d74             | 1  | 0.05800            | 0.17264        | 0.34    | 0.7402|
| t               | 1  | -0.01315           | 0.00265        | -4.96   | <.0001|
| lirr            | 1  | -0.07181           | 0.01579        | -4.55   | 0.0002|
| lqcrn           | 1  | -0.19568           | 0.07382        | -2.65   | 0.0149|
| lrsr            | 1  | -0.36048           | 0.11329        | -3.18   | 0.0045|
The REG Procedure

OLS ESTIMATES

Dependent Variable: lppr

Durbin-Watson D                1.551
Pr < DW                       0.0102
Pr > DW                       0.9898
Number of Observations            29
1st Order Autocorrelation  0.204

NOTE: Pr<DW is the p-value for testing positive autocorrelation,
and Pr>DW is the p-value for testing negative autocorrelation.
The REG Procedure
OLS ESTIMATES
Dependent Variable: lppf

Number of Observations Read 29
Number of Observations Used 29

Analysis of Variance

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Root MSE 0.03137
Dependent Mean 2.18848
Coeff Var 1.43339
R-Square 0.9960
Adj R-Sq 0.9946
| Variable          | DF | Parameter Estimate | Standard Error | t Value | Pr > |t| |
|-------------------|----|--------------------|----------------|---------|-------|---|
| Intercept         | 1  | -0.24849           | 0.22376        | -1.11   | 0.2793|   |
| ltcf              | 1  | 0.74403            | 0.17265        | 4.31    | 0.0003|   |
| intd74ltcf        | 1  | -0.06189           | 0.10709        | -0.58   | 0.5694|   |
| d74               | 1  | 0.17741            | 0.21137        | 0.84    | 0.4107|   |
| t                 | 1  | -0.00415           | 0.00393        | -1.05   | 0.3036|   |
| lirf              | 1  | -0.07130           | 0.02583        | -2.76   | 0.0117|   |
| lqcfn             | 1  | -0.50375           | 0.07811        | -6.45   | <.0001|   |
| lrsf              | 1  | 0.17299            | 0.13632        | 1.27    | 0.2183|   |
The REG Procedure

Dependent Variable: lppf

Durbin-Watson D 1.547
Pr < DW 0.0135
Pr > DW 0.9865
Number of Observations 29
1st Order Autocorrelation 0.224

NOTE: Pr<DW is the p-value for testing positive autocorrelation, and Pr>DW is the p-value for testing negative autocorrelation.
The SYSLIN Procedure
Two-Stage Least Squares Estimation

Model QMRNEQ
Dependent Variable lqmrn

Analysis of Variance

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Root MSE 0.07333
R-Square 0.89597
Dependent Mean 4.71948
Adj R-Sq 0.87335
Coeff Var 1.55383
### Parameter Estimates

| Variable  | DF | Parameter Estimate | Standard Error | t Value | Pr > |t| |
|------------|----|--------------------|----------------|---------|-------|
| Intercept  | 1  | 6.101333           | 0.342720       | 17.80   | <.0001|
| lppr       | 1  | -0.73432           | 0.213632       | -3.44   | 0.0022|
| ltdin      | 1  | 0.398169           | 0.104208       | 3.82    | 0.0009|
| d70        | 1  | -0.22721           | 0.076177       | -2.98   | 0.0067|
| t14        | 1  | 0.071754           | 0.029321       | 2.45    | 0.0225|
| t14sq      | 1  | -0.00541           | 0.001164       | -4.64   | 0.0001|
The SYSLIN Procedure

Two-Stage Least Squares Estimation

Model QMFNEQ
Dependent Variable lqmfn

Analysis of Variance

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Root MSE 0.04052  R-Square 0.96419
Dependent Mean 4.13689  Adj R-Sq 0.95640
Coeff Var 0.97937
Parameter Estimates

| Variable  | DF | Estimate  | Standard Error | t Value | Pr > |t| |
|-----------|----|-----------|----------------|---------|-------|---|
| Intercept | 1  | 6.162796  | 0.255353       | 24.13   | <.0001|
| lppf      | 1  | -0.98562  | 0.138200       | -7.13   | <.0001|
| ltdin     | 1  | 0.434944  | 0.058132       | 7.48    | <.0001|
| d70       | 1  | -0.26503  | 0.042277       | -6.27   | <.0001|
| t14       | 1  | 0.088504  | 0.015819       | 5.59    | <.0001|
| t14sq     | 1  | -0.00538  | 0.000635       | -8.47   | <.0001|
The SYSLIN Procedure

Two-Stage Least Squares Estimation

Model PPReq
Dependent Variable lppr

Analysis of Variance

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Root MSE 0.02975   R-Square 0.99679
Dependent Mean 1.97581   Adj R-Sq 0.99573
Coeff Var 1.50576
| Variable       | DF | Parameter Estimate | Standard Error | t Value | Pr > |t| |
|----------------|----|--------------------|----------------|---------|-------|
| Intercept      | 1  | -0.74919           | 0.158609       | -4.72   | 0.0001|
| ltcr           | 1  | 1.220635           | 0.116773       | 10.45   | <.0001|
| intd74ltcr     | 1  | -0.05237           | 0.084194       | -0.62   | 0.5407|
| d74            | 1  | 0.058004           | 0.172642       | 0.34    | 0.7402|
| t              | 1  | -0.01315           | 0.002649       | -4.96   | <.0001|
| lirr           | 1  | -0.07181           | 0.015786       | -4.55   | 0.0002|
| lqcrn          | 1  | -0.19568           | 0.073817       | -2.65   | 0.0149|
| lrsr           | 1  | -0.36048           | 0.113288       | -3.18   | 0.0045|
The SYSLIN Procedure

Two-Stage Least Squares Estimation

Model PPFEQ
Dependent Variable lppf

Analysis of Variance

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Root MSE           | 0.03137  | R-Square       | 0.99598    |
Dependent Mean     | 2.18848  | Adj R-Sq       | 0.99464    |
Coef Var           | 1.43339  |               |            |
## Parameter Estimates

| Variable          | DF | Parameter Estimate | Standard Error | t Value | Pr > |t| |
|-------------------|----|--------------------|----------------|---------|------|---|
| Intercept         | 1  | -0.24849           | 0.223764       | -1.11   | 0.2793|
| ltcf              | 1  | 0.744035           | 0.172648       | 4.31    | 0.0003|
| intd74ltcf        | 1  | -0.06189           | 0.107085       | -0.58   | 0.5694|
| d74               | 1  | 0.177407           | 0.211372       | 0.84    | 0.4107|
| t                 | 1  | -0.00415           | 0.003932       | -1.05   | 0.3036|
| lirf              | 1  | -0.07130           | 0.025835       | -2.76   | 0.0117|
| lqcfn             | 1  | -0.50375           | 0.078114       | -6.45   | <.0001|
| lrsf              | 1  | 0.172987           | 0.136315       | 1.27    | 0.2183|
The SYSLIN Procedure
Three-Stage Least Squares Estimation

Cross Model Covariance

<table>
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<tr>
<th></th>
<th>QMRNEQ</th>
<th>QMFNEQ</th>
<th>PPREQ</th>
<th>PPFEQ</th>
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Cross Model Correlation

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<th>PPFEQ</th>
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<td>0.29182</td>
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Non-zero cross-correlations of disturbance terms. So 3SLS adds information beyond OLS, 2SLS.
## Cross Model Inverse Correlation

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<tr>
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## Cross Model Inverse Covariance

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### System Weighted MSE
0.9714

### Degrees of freedom
88

### System Weighted R-Square
0.9920

---

**Model**
QMRNEQ

**Dependent Variable**
lqmrm

---

**The SYSLIN Procedure**

**Three-Stage Least Squares Estimation**

**Parameter Estimates**

| Variable | DF | Estimate | Standard Error | t Value | Pr > |t| |
|----------|----|----------|----------------|---------|-------|---|
| Intercept | 1  | 6.201060 | 0.334989       | 18.51   | <.0001|
| lppr     | 1  | -0.79948 | 0.208676       | -3.83   | 0.0009|
| ltdin    | 1  | 0.382954 | 0.103763       | 3.69    | 0.0012|
| d70      | 1  | -0.20854 | 0.075503       | -2.76   | 0.0111|
| t14      | 1  | 0.076572 | 0.028906       | 2.65    | 0.0143|
| t14sq    | 1  | -0.00531 | 0.001158       | -4.59   | 0.0001|
Durbin-Watson: 2.62448
Number of Observations: 29
First-Order Autocorrelation: -0.32359

Test for Overidentifying Restrictions

<table>
<thead>
<tr>
<th>Num DF</th>
<th>Den DF</th>
<th>F Value</th>
<th>Pr &gt; F</th>
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<tbody>
<tr>
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Model: QMFNEQ
Dependent Variable: lqmfn
Rank Condition satisfied

Parameter Estimates

| Variable   | DF | Parameter Estimate | Standard Error | t Value | Pr > |t| |
|------------|----|--------------------|----------------|---------|------|---|
| Intercept  | 1  | 6.164389           | 0.241893       | 25.48   | <.0001|
| lppf       | 1  | -0.98632           | 0.130463       | 7.56    | <.0001|
| ltdin      | 1  | 0.435547           | 0.056532       | 7.70    | <.0001|
| d70        | 1  | -0.25483           | 0.039212       | 6.50    | <.0001|
| t14        | 1  | 0.084646           | 0.015114       | 5.60    | <.0001|
| t14sq      | 1  | -0.00513           | 0.000611       | 8.41    | <.0001|
Durbin-Watson: 2.006508
Number of Observations: 29
First-Order Autocorrelation: -0.02168

The SYSLIN Procedure

Three-Stage Least Squares Estimation

Test for Overidentifying Restrictions

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<th>Den DF</th>
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Model: PPREQ
Dependent Variable: lppr

Rank Condition satisfied at 0.10 level of significance
## Parameter Estimates

| Variable      | DF | Parameter Estimate | Standard Error | t Value | Pr > |t| |
|----------------|----|--------------------|----------------|--------|------|---|
| Intercept      | 1  | -0.71085           | 0.154368       | -4.60  | 0.0002 |
| ltcr           | 1  | 1.196653           | 0.111687       | 10.71  | <0.0001 |
| intd74ltcr     | 1  | -0.02371           | 0.082853       | -0.29  | 0.7776 |
| d74            | 1  | 0.003675           | 0.170351       | 0.02   | 0.9830 |
| t              | 1  | -0.01248           | 0.002584       | -4.83  | <0.0001 |
| lirr           | 1  | -0.08473           | 0.014707       | -5.76  | <0.0001 |
| lqcrn          | 1  | -0.17703           | 0.069409       | -2.55  | 0.0186 |
| lrsr           | 1  | -0.36080           | 0.105376       | -3.42  | 0.0026 |
Durbin-Watson 1.389357
Number of Observations 29
First-Order Autocorrelation 0.296214

Test for Overidentifying Restrictions

Num DF | Den DF | F Value | Pr > F
--------|--------|---------|--------
9 - K-N_i | 12 - T-K | 5.27   | 0.0048 |

K = 17, T = 29
N_i = 8
Model PPFEQ
Dependent Variable lppf
Rank Condition satisfied
### Parameter Estimates

| Variable    | DF | Parameter Estimate | Standard Error | t Value | Pr > |t| |
|-------------|----|--------------------|----------------|---------|------|---|
| Intercept   | 1  | -0.22095           | 0.215167       | -1.03   | 0.3162 |
| ltcf        | 1  | 0.715030           | 0.162512       | 4.40    | 0.0002 |
| intd74ltcf  | 1  | -0.05409           | 0.103974       | -0.52   | 0.6084 |
| d74         | 1  | 0.172293           | 0.206825       | 0.83    | 0.4142 |
| t           | 1  | -0.00365           | 0.003744       | -0.98   | 0.3401 |
| lirf        | 1  | -0.06834           | 0.024266       | -2.82   | 0.0103 |
| lqcfn       | 1  | -0.51203           | 0.073584       | -6.96   | <.0001 |
| lrsf        | 1  | 0.244298           | 0.126655       | 1.93    | 0.0674 |
Durbin-Watson 1.561122
Number of Observations 29
First-Order Autocorrelation 0.213946

Test for Overidentifying Restrictions

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Rank Condition satisfied at 0.10 level of significance

$N_i = 8$

$K = 17$

$T = 29$
### Predicted Values Based on 3SLS Estimates

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<td>1969</td>
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| 15  | 1970 | 4.82583| 4.76575

*continued...*
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<td>1984</td>
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The CORR Procedure

2 Variables: lqmrn  lqmrn_3sls

Simple Statistics

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<th>Std Dev</th>
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Maximum Label
4.99566
4.95657  Predicted Values
Pearson Correlation Coefficients, N = 29
Prob > |r| under H0: Rho=0

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Predicted Values: <.0001

\[ R^2 = (0.94573)^2 = 0.8944 \]
Predicted Values Based on 3SLS Estimates

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<th>3sls</th>
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The CORR Procedure

2 Variables: lqmfn lqmfn_3sls

Simple Statistics

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Maximum Label
4.43272
4.43791 Predicted Values

Pearson Correlation Coefficients, N = 29
Prob > |r| under H0: Rho=0

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Predicted Values

R² = (0.9817)² = 0.9637
### Predicted Values Based on 3SLS Estimates

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The CORR Procedure

2 Variables: lppr  lppr_3sls

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Maximum  Label
2.91506
2.89687 Predicted Values

\[ R^2 = (0.99833)^2 = 0.9967 \]

Pearson Correlation Coefficients, N = 29
Prob > |r| under H0: Rho=0

\[
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  lppr & lppr_3sls \\
  lppr  & 1.00000 & 0.99833 \\
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The CORR Procedure

2 Variables: lppf    lppf_3sls

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Maximum Label

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3.03513 Predicted Values

$R^2 = (0.99795)^2 = 0.9959$

Pearson Correlation Coefficients, N = 29

Prob > |r| under H0: Rho=0

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## Summary

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**KEY POINT:** Reduction in standard error attributed to 3SLS procedure.
## Summary

### Dependent Variable: InQMFN

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**KEY POINT:** Reduction in standard error attributed to 3SLS procedure.
## Summary

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| R²           | 0.9968             | 0.9968         | 0.9967             |                |                    |                |
| DW           | 1.551              |                |                    |                |                    | 1.389          |

**KEY POINT:** Reduction in standard error attributed to 3SLS procedure.
## Summary

**Dependent Variable: lnPPF**

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| | R²         | 0.9960 | 0.9960 | 0.9959 |
| | DW         | 1.547  | 1.561  |        |

**KEY POINT:** Reduction in standard error attributed to 3SLS procedure.
The SIMLIN Procedure
Based on 3SLS Estimates
Inverse Coefficient Matrix for Endogenous Variables

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<td>-0.005311</td>
<td>-0.9567</td>
<td>0.0190</td>
</tr>
<tr>
<td>lqmfn</td>
<td>0.4355</td>
<td>-0.2548</td>
<td>0.0846</td>
<td>-0.005134</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>lppr</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.1967</td>
<td>-0.0237</td>
</tr>
<tr>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

% change in qmrn due to a 1% change in tdin:

\[
\text{[exp(-0.2085)-1] x 100%}
\]

(18.8%) reduction in qmrn after 1970 relative to before 1970.
<table>
<thead>
<tr>
<th>Variable</th>
<th>d74</th>
<th>t</th>
<th>lirr</th>
<th>lqcrn</th>
<th>lrsr</th>
<th>ltcf</th>
</tr>
</thead>
<tbody>
<tr>
<td>lqmrn</td>
<td>-0.002938</td>
<td>0.009976</td>
<td>0.0677</td>
<td>0.1415</td>
<td>0.2885</td>
<td>0</td>
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<tr>
<td>lqmfn</td>
<td>-0.1699</td>
<td>0.003605</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-0.7052</td>
</tr>
<tr>
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<td>0.003675</td>
<td>-0.0125</td>
<td>-0.0847</td>
<td>-0.1770</td>
<td>-0.3608</td>
<td>0</td>
</tr>
<tr>
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<td>0.1723</td>
<td>-0.003655</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.7150</td>
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<table>
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<th>Variable</th>
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<th>lirf</th>
<th>lqcfn</th>
<th>lrsf</th>
<th>Intercept</th>
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<td>6.7694</td>
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<tr>
<td>lqmfn</td>
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<td>6.3823</td>
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<tr>
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<tr>
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<td>-0.0683</td>
<td>-0.5120</td>
<td>0.2443</td>
<td>-0.2209</td>
</tr>
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</table>

(1) These coefficients are the Impact Multipliers.
(2) Use this analytically-derived reduced form to generate forecasts.
Section 7.11

Final Form of the System
Final Form of the System

If Matrix $A$ is such that the limit of $A^s$ for $s \to \infty$ is a zero matrix, then the final form of the equation system is given by:

$$Y_t = BX_t + \sum_{s=1}^{\infty} A^{s-1}(C + AB)X_{t-s} + \sum_{s=0}^{\infty} A^s u^*_t$$

Question of Interest: The stability of the system and the influence of past and present values of the exogenous variables on the current values of the endogenous variables.

Final Form of the System - Special importance in determining the stability of the system; manipulate reduced form equations to eliminate all lagged endogenous variables except those corresponding to the endogenous variable on the LHS.
Final Form of the Equation System - demonstrates how the time paths of the exogenous variables determine the time path of each endogenous variable.

Stability of a System - After a departure from equilibrium, will the system eventually return to equilibrium? -- In general, we say that a system is stable if, in a situation where the values of the exogenous variables are held constant through time, the mean values of the endogenous variables eventually settle down to constant levels. The actual values of the endogenous variables will fluctuate because of the effect of the stochastic disturbances. Hence, a system is considered unstable if, for constant values of the exogenous variables, the mean values of the endogenous variables fail to eventually settle down.
Section 14.12

Determining Whether a System is Stable or Not
Determining Whether a System is Stable or Not

-- The sums of each set of dynamic multipliers must be finite
-- If the exogenous variables are held constant and the disturbances are disregarded, the final form becomes a linear nonhomogenous difference equation. The system is stable if the absolute value of the largest root is smaller than one. In other words, the matrix $A^s$ converges to the zero (or null) matrix. That is,

$$- A^s \rightarrow 0 \text{ as } s \rightarrow \infty$$

All characteristics roots $\lambda_i$ of $A$ must be less than 1 in modulus (Absolute Value)

Stability conditions are important because multiplier analysis is relevant only if the model is stable.
For a stable system, for $A^s$ to approach a null matrix as $s$ increases, it is a necessary condition that none of the characteristic roots be greater than 1. If the roots are complex, any modulus must be less than 1. The dominant latent root determines the behavior of the system. A positive dominant root implies monotonic convergence. If the dominant root is negative, the system oscillates each period. A complex dominant root implies cycles.

Characteristic Root $C + di$  \[ M = \sqrt{c^2 + d^2} \]

To find the length of a cycle compute $\cos R = C/M$

Angle between real part of the complex number and the modulus. The length of a cycle is $360/R$. The length of a cycle is the reciprocal of the frequency.
Section 14.13

Example of Stability Condition: Klein Model
Example of Stability Condition: Klein Model

A model of the United States economy for the period 1921 to 1941 known as Klein’s Model I, has been estimated as follows:

Consumption: \[ C_t = 16.555 + 0.017P_t + 0.216P_{t-1} + 0.810W_t + u_{1t}; \]

Investment: \[ I_t = 20.278 + 0.150P_t + 0.616P_{t-1} - 0.158K_{t-1} + u_{2t}; \]

Private Wages: \[ W_t^* = 1.500 + 0.439E_t + 0.147E_{t-1} + 0.130A_t + u_{3t}; \]

Income: \[ Y_t + T_t = C_t + I_t + G_t; \]

Income: \[ Y_t = P_t + W_t; \]

Capital: \[ K_t = K_{t-1} + I_t; \]

Wages: \[ W_t = W_t^* + W_t^{**}; \]

Private Product: \[ E_t = Y_t + T_t - W_t^{**}. \]
### Endogenous Variables:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Consumption</td>
</tr>
<tr>
<td>I</td>
<td>Investment</td>
</tr>
<tr>
<td>W*</td>
<td>Private Wage Bill</td>
</tr>
<tr>
<td>P</td>
<td>Profit</td>
</tr>
<tr>
<td>Y</td>
<td>National Income</td>
</tr>
<tr>
<td>K</td>
<td>End-of-Year Capital Stock</td>
</tr>
<tr>
<td>W</td>
<td>Total Wage Bill</td>
</tr>
<tr>
<td>E</td>
<td>Profit Product</td>
</tr>
</tbody>
</table>

### Exogenous Variables:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>W**</td>
<td>Government Wage Bill</td>
</tr>
<tr>
<td>T</td>
<td>Indirect Taxes</td>
</tr>
<tr>
<td>A</td>
<td>Time in Years (1931 = 1)</td>
</tr>
<tr>
<td>G</td>
<td>Government Expenditures</td>
</tr>
</tbody>
</table>
Matrix Form

\[
\begin{bmatrix}
1 & 0 & 0 & -0.17 & 0 & 0 & -0.81 & 0 \\
0 & 1 & 0 & -0.15 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & -0.439 \\
-1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 1 & 0 & -1 & 0 \\
0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \\
\end{bmatrix}_{8 \times 8} = \begin{bmatrix}
C \\
I \\
W^* \\
P \\
Y \\
K \\
W \\
E \\
\end{bmatrix}_{8 \times 1} \\
\begin{bmatrix}
16.555 & 0 & 0 & 0 & 0 \\
20.278 & 0 & 0 & 0 & 0 \\
1.5 & 0 & 0 & .13 & 0 \\
0 & 0 & -1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
I \\
W^{**} \\
T \\
A \\
G \\
\end{bmatrix}_{8 \times 5} \\
\begin{bmatrix}
X_t \\
Y_t \\
G \\
\end{bmatrix}_{5 \times 1}
\]
\[ H y_t = G X_t + I Y_{t-1} + u_t \]
Analytically-Derived Reduced Form of the Klein Model

\[ y_t = H^{-1}Iy_{t-1} + H^{-1}GX_t + H^{-1}u_t \]

\[ A = H^{-1}I \quad H_{ut}^{-1} = u_t^* \]

\[ B = H^{-1}G \]

\[ C = H^{-1}O = 0 \]

Impact Multipliers → Elements of B
Interim Multipliers → Elements of \( A^{s-1}(AB) \)
Total Multipliers → \( B + (I - A)^{-1}(AB) \)

continued...
Characteristic Roots of $A$ (other than zero)

\[ \lambda_1 = .310 \]
\[ \lambda_2 = .708 - .298i \]
\[ \lambda_3 = .708 + .298i \]

\[ \cos R = \frac{.708}{.768} \]

length of cycle is \[ \frac{360}{R} \]

All roots less than 1 system stable

Dominant Root Complex \( \Rightarrow \) Cycles
Section 14.14

Specification, Estimation, and Simulation of a Macroeconomic Dynamic Simultaneous Equation Model
Specification, Estimation, and Simulation of a Dynamic Macroeconomic Simultaneous Equation Model

(1) \( GDP = C + INR + IR + IIN + G \) (identity)

(2) \( C = f_1[YD, TR, WLTH, RS, C_{-1}] + \varepsilon_1 \)

(3) \( IIN = f_2[YD, C, C_{-1}, INV, IIN_{-1}] + \varepsilon_2 \)

(4) \( INV = f_3[GDP, C, INV_{-1}] + \varepsilon_3 \)

Dynamics in this specification due to lagged endogenous variables \([C_{-1}, IIN_{-1}, INV_{-1}]\)
This delineation serves to support the contention that this system is truly simultaneous.
Variables of the Model

**Endogenous**
- C: Consumption
- GDP: Gross Domestic Product
- IIN: Inventory Investment
- INV: Stock of Inventories

**Exogenous**
- G: Government Spending
- YD: Disposable Income
- WLTH: Wealth
- RS: Short-term Interest Rate
- INR: Nonresidential Investment
- IR: Residential Investment
- TR: Federal Government Transfer Payments
Identification Status

Order Conditions

C  H = 10(or 11)*  h = 5(6)*  g = 1  5>0
IIN  H = 10(or 11)*  h = 3(4)*  g = 3  7>2
INV  H = 10(or 11)*  h = 1(2)*  g = 3  9>2

* Including Intercept

All equations over-identified
model over-identified
## Rank Conditions

### Endogenous

<table>
<thead>
<tr>
<th>GDP</th>
<th>C</th>
<th>IIN</th>
<th>INV</th>
<th>G</th>
<th>YD</th>
<th>TR</th>
<th>WLT</th>
<th>RS</th>
<th>INR</th>
<th>IR</th>
<th>CLAG</th>
<th>IINLAG</th>
<th>INVLAG</th>
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<tr>
<td>C</td>
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<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>IIN</td>
<td>0</td>
<td>β_{22}</td>
<td>1</td>
<td>β_{24}</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>γ_{28}</td>
<td>γ_{29}</td>
</tr>
<tr>
<td>INV</td>
<td>β_{31}</td>
<td>β_{32}</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
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<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

### Predetermined
For identification of equation (C) form this submatrix

\[
\begin{bmatrix}
0 & 1 & \beta_{24} & 0 & 0 & 0 & \beta_{29} & 0 \\
\beta_{31} & 0 & 1 & 0 & 0 & 0 & 0 & \beta_{2,10} \\
1 & 1 & 0 & 1 & 1 & 1 & 0 & 0
\end{bmatrix}
\begin{pmatrix}
8 \\
3
\end{pmatrix}
= \frac{8!}{3!5!} = 56
\]

For identification of equation (IIN) form this submatrix

\[
\begin{bmatrix}
0 & 0 & \gamma_{13} & \gamma_{14} & \gamma_{15} & 0 & 0 & 0 \\
\beta_{31} & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_{2,10} \\
1 & 1 & 0 & 0 & 0 & 1 & 1 & 0
\end{bmatrix}
\begin{pmatrix}
8 \\
3
\end{pmatrix}
= 56
\]

For identification of equation (INV) form this submatrix

\[
\begin{bmatrix}
0 & 0 & \gamma_{12} & \gamma_{13} & \gamma_{14} & \gamma_{15} & 0 & 0 & \gamma_{18} & 0 \\
1 & 0 & \gamma_{22} & 0 & 0 & 0 & 0 & \gamma_{28} & \gamma_{29} \\
1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0
\end{bmatrix}
\begin{pmatrix}
10 \\
3
\end{pmatrix}
= \frac{10!}{3!7!} = 120
\]
For equation (C)

\[ |D_1| = \begin{vmatrix} 0 & 1 & \beta_{24} \\ \beta_{31} & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ \beta_{31} & 0 \\ 1 & 1 \end{vmatrix} = 1 + \beta_{24} \beta_{31} \neq 0 \]

For equation (IIN)

\[ |D_1| = \begin{vmatrix} 0 & 0 & \gamma_{13} \\ \beta_{31} & 0 & 0 \\ 1 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ \beta_{31} & 0 \\ 1 & 1 \end{vmatrix} = \gamma_{13} \beta_{31} \neq 0 \]

For equation (INV)

\[ |D_1| = \begin{vmatrix} 0 & 0 & \gamma_{12} \\ 1 & 0 & \gamma_{22} \\ 1 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{vmatrix} = \gamma_{12} \neq 0 \]
* Simultaneous Equation Example;
* Lexicon of Macroeconomic Variables;
* C refers to personal consumption expenditures (billions of 1958 dollars);
* G refers to government purchases of goods and services (billions of 1958 dollars);
* GDP refers to gross domestic product (billions of 1958 dollars);
* IIN refers to inventory investment, change in business inventories (billions of 1958 dollars);
* INV refers to level of business inventories (billions of 1958 dollars);
* INR refers to fixed nonresidential investment (billions of 1958 dollars);
* IR refers to fixed residential investment in nonfarm structures (billions of 1958 dollars);
* RS refers to nominal short-term interest rates, market yield on 3-month Treasury bills (%);
* TR refers to federal government transfer payments (billion of 1958 dollars);
* WLTH refers to index of real household wealth;
* YD refers to disposable income (billions of 1958 dollars);
* Analysis of endogenous variables GDP, C IIN, and INV;
data macro;
input YEAR QUARTER C G GDP IIN INV INR IR RS TR WLTH YD;
q1=0; q2=0; q3=0; q4=0;
if quarter=1 then q1=1;
if quarter=2 then q2=1;
if quarter=3 then q3=1;
if quarter=4 then q4=1;
lagc=lag1(c); lagiin=lag1(iin); laginv=lag1(inv);
datalines;

* Simultaneous System;

proc syslin data=macro out=pmacro3sls 3sls outest=macro3sls;
endogenous c gdp iin inv;
instruments g ir rs tr wlth yd inr lagc lagiin laginv;
Keynesian: identity gdp=c+inr+ir+iin+g;
consumptioneq: model c=yd tr wlth rs lagc / dw overid;
output p=c_consumptioneq3sls r=rc_consumptioneq3sls;
iineq: model iin=yd c lagc inv lagiin / dw overid;
output p=iin_iineq3sls r=riin_iineq3sls;
inveq: model inv=gdp c laginv / dw overid;
output p=inv_inveq3sls r=rinv_inveq3sls;
run;
data sim; set pmacro3sls;
proc print; var year quarter c c_consumptioneq3sls;
run;
proc print; var year quarter iin iin_iineq3sls;
run;
proc print; var year quarter inv inv_inveq3sls;
run;
proc corr; var c c_consumptioneq3sls;
run;
proc corr; var iin iin_iineq3sls;
run;
proc corr; var inv inv_inveq3sls;

proc simlin est=macro3sls interim=4 total;
endog c gdp iin inv;
exogenous g ir rs tr wth yd inr;
lagged lagc c 1 lagiin iin 1 laginv inv 1;
run;
The MEANS Procedure

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>Median</th>
<th>Std Dev</th>
<th>Minimum</th>
<th>Maximum</th>
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<td>415.2681818</td>
<td>409.7000000</td>
<td>98.1198429</td>
<td>279.1000000</td>
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<tr>
<td>G</td>
<td>88</td>
<td>124.1227273</td>
<td>135.5000000</td>
<td>22.2270506</td>
<td>84.6000000</td>
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<td>141.6496559</td>
<td>434.2000000</td>
<td>901.2000000</td>
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<tr>
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<td>5.4700000</td>
<td>4.8513987</td>
<td>-13.1000000</td>
<td>17.5600000</td>
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<tr>
<td>INV</td>
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<td>162.4284091</td>
<td>164.2000000</td>
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<tr>
<td>WLTH</td>
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<tr>
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<td>576.1000000</td>
<td>124.7608891</td>
<td>382.4000000</td>
<td>793.8000000</td>
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</tbody>
</table>

88 quarterly observations
The AUTOREG Procedure
Dependent Variable C

Ordinary Least Squares Estimates

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Value</th>
<th>Prob</th>
<th>Label</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSE</td>
<td>489.339018</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MSE</td>
<td>6.04122</td>
<td></td>
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<tr>
<td>SBC</td>
<td>423.952579</td>
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</tr>
<tr>
<td>MAE</td>
<td>1.88155309</td>
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<td></td>
</tr>
<tr>
<td>MAPE</td>
<td>0.45030639</td>
<td></td>
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</tr>
</tbody>
</table>

Absence of serial correlation

Miscellaneous Statistics

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Value</th>
<th>Prob</th>
<th>Label</th>
</tr>
</thead>
<tbody>
<tr>
<td>Durbin h</td>
<td>1.1373</td>
<td>0.1277</td>
<td>Pr &gt; h</td>
</tr>
</tbody>
</table>
The AUTOREG Procedure
Dependent Variable    IIN

Ordinary Least Squares Estimates

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Value</th>
<th>Prob</th>
<th>Label</th>
</tr>
</thead>
<tbody>
<tr>
<td>Durbin h</td>
<td>2.0354</td>
<td>0.0209</td>
<td>Pr &gt; h</td>
</tr>
</tbody>
</table>

| Variable | DF | Estimate | Standard Error | t Value | Pr > |t| |
|----------|----|----------|----------------|---------|-------|
| Intercept| 1  | -7.1507  | 1.6503         | -4.33   | <.0001|
| YD       | 1  | 0.2944   | 0.0312         | 9.43    | <.0001|
| C        | 1  | -0.5531  | 0.0980         | -5.64   | <.0001|
| lagc     | 1  | 0.4069   | 0.0994         | 4.09    | <.0001|
| INV      | 1  | -0.5676  | 0.0805         | -7.05   | <.0001|
| lgiin    | 1  | 0.1245   | 0.0773         | 1.61    | 0.1112|

Total R-Square          0.7072

serial correlation evident
The AUTOREG Procedure

Dependent Variable INV

Ordinary Least Squares Estimates

SSE  36.0695176  DFE  83
MSE  0.43457  Root MSE  0.65922
SBC  188.158917  AIC  178.295285
MAE  0.51079262  AICC  178.78309
MAPE  0.32331984  Regress R-Square  0.9997
         Total R-Square  0.9997

Miscellaneous Statistics

Statistic        Value      Prob        Label
Durbin h       3.4348    0.0003       Pr > h

Standard                 Approx
Variable        DF     Estimate        Error    t Value    Pr > |t|
Intercept        1      -1.6881       0.3840      -4.40      <.0001
GDP              1       0.0619     0.004959      12.49      <.0001
C                1      -0.0436     0.008147      -5.36      <.0001
laginv           1       0.8847       0.0142      62.09      <.0001

Presence of serial correlation
The SYSLIN Procedure
Two-Stage Least Squares Estimation

Model                 CONSUMPTIONEQ
Dependent Variable                C

Analysis of Variance

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Root MSE             2.45789
Dependent Mean       416.83333
Coeff Var            0.58966
R-Square              0.99940
Adj R-Sq              0.99937
| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > |t| |
|----------|----|--------------------|----------------|---------|-------|-----|
| Intercept| 1  | 6.068307           | 2.855905       | 2.12    | 0.0366|
| YD       | 1  | 0.271845           | 0.039806       | 6.83    | <.0001|
| TR       | 1  | 0.730647           | 0.123116       | 5.93    | <.0001|
| WLTH     | 1  | 3.877187           | 2.143968       | 1.81    | 0.0743|
| RS       | 1  | -1.08037           | 0.285631       | -3.78   | 0.0003|
| lagc     | 1  | 0.550002           | 0.066517       | 8.27    | <.0001|
### The SYSLIN Procedure
Two-Stage Least Squares Estimation

Model: IINEQ
Dependent Variable: IIN

#### Analysis of Variance

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- Root MSE: 3.09525
- R-Square: 0.62431
- Dependent Mean: 5.14506
- Adj R-Sq: 0.60112
- Coeff Var: 60.15975
## Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > |t| |
|----------|----|--------------------|----------------|---------|-------|---|
| Intercept| 1  | -4.99643           | 1.929395       | -2.59   | 0.0114|
| YD       | 1  | 0.226602           | 0.038162       | 5.94    | <.0001|
| C        | 1  | -0.08169           | 0.147567       | -0.55   | 0.5814|
| lgc      | 1  | -0.02215           | 0.143290       | -0.15   | 0.8775|
| INV      | 1  | -0.46766           | 0.093973       | -4.98   | <.0001|
| lagiin   | 1  | 0.249380           | 0.091665       | 2.72    | 0.0080|
The SYSLIN Procedure
Two-Stage Least Squares Estimation

Model INVEQ
Dependent Variable INV

Analysis of Variance

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Root MSE 0.66696
Dependent Mean 163.01609
Coeff Var 0.40914

R-Square 0.99969
Adj R-Sq 0.99967
Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > |t| |
|----------|----|--------------------|----------------|---------|-------|
| Intercept| 1  | -1.42773           | 0.391350       | -3.65   | 0.0005|
| GDP      | 1  | 0.056640           | 0.005107       | 11.09   | <.0001|
| C        | 1  | -0.03223           | 0.008496       | -3.79   | 0.0003|
| laginv   | 1  | 0.874733           | 0.014529       | 60.21   | <.0001|
The SYSLIN Procedure
Three-Stage Least Squares Estimation

Cross Model Covariance

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Cross Model Correlation

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### Cross Model Inverse Correlation

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### Cross Model Inverse Covariance

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System Weighted MSE: 0.8151
Degrees of freedom: 245
System Weighted R-Square: 0.9999

Model: CONSUMPTI
Dependent Variable: C
The SYSLIN Procedure
Three-Stage Least Squares Estimation

Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > |t| |
|----------|----|--------------------|----------------|---------|-------|
| Intercept| 1  | 7.156179           | 2.665609       | 2.68    | 0.0088|
| YD       | 1  | 0.274646           | 0.036785       | 7.47    | <.0001|
| TR       | 1  | 0.721728           | 0.112296       | 6.43    | <.0001|
| WLTH     | 1  | 1.838694           | 1.936007       | 0.95    | 0.3451|
| RS       | 1  | -0.73364           | 0.262790       | -2.79   | 0.0065|
| lags     | 1  | 0.550337           | 0.061026       | 9.02    | <.0001|

Durbin-Watson           1.693445
Number of Observations  87
First-Order Autocorrelation  0.105552
Test for Overidentifying Restrictions

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Model: IINEQ
Dependent Variable: IIN

Parameter Estimates

| Variable  | DF | Estimate | Standard Error | t Value | Pr > |t| |
|-----------|----|----------|----------------|---------|------|---|
| Intercept | 1  | -5.97175 | 1.821170       | -3.28   | 0.0015 |
| YD        | 1  | 0.274991 | 0.029149       | 9.43    | <.0001 |
| C         | 1  | -0.07163 | 0.065893       | -1.09   | 0.2802 |
| lagc      | 1  | -0.05880 | 0.054099       | -1.09   | 0.2803 |
| INV       | 1  | -0.55574 | 0.071866       | -7.73   | <.0001 |
| lagiin    | 1  | 0.032398 | 0.034043       | 0.95    | 0.3441 |

Durbin-Watson: 1.450175
Number of Observations: 87
First-Order Autocorrelation: 0.259707
The SYSLIN Procedure
Three-Stage Least Squares Estimation

Test for Overidentifying Restrictions

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Model: INVEQ
Dependent Variable: INV

Parameter Estimates

| Variable | DF | Parameter Estimate | Standard Error | t Value | Pr > |t| |
|----------|----|-------------------|----------------|---------|------|---|
| Intercept| 1  | -1.37202          | 0.390726       | -3.51   | 0.0007 |
| GDP      | 1  | 0.055192          | 0.005096       | 10.83   | <.0001|
| C        | 1  | -0.03012          | 0.008342       | -3.61   | 0.0005|
| laginv   | 1  | 0.874686          | 0.013563       | 64.49   | <.0001|

Rank condition satisfied for IINEQ
Durbin-Watson 1.293388
Number of Observations 87
First-Order Autocorrelation 0.336732

Test for Overidentifying Restrictions

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Rank condition satisfied for INVEQ
## Summary: Equation C

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## Summary: Equation IIN

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Key Points:

(1) Note reductions in standard errors of 3SLS parameter estimates over 2SLS parameter estimates due to correlations among disturbance terms of the respective stochastic equations.

(2) OLS, 2SLS are limited-information estimation techniques; where possible use full-information estimation techniques like 3SLS.
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Within-sample prediction

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The CORR Procedure

2 Variables: C c_consumptioneq3sls

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176
### Pearson Correlation Coefficients

**Prob > |r| under H0: Rho=0**

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The CORR Procedure

2 Variables:  IIN   iin_iineq3sls

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<td>0.75774</td>
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<tr>
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<td>1.00000</td>
</tr>
</tbody>
</table>

Prob > |r| under H0: Rho=0
Number of Observations

88             87
87             87

R² = 0.5742
The CORR Procedure

2 Variables: INV  inv_inveq3sls

Simple Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>Std Dev</th>
<th>Sum</th>
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<tbody>
<tr>
<td>INV</td>
<td>88</td>
<td>162.42841</td>
<td>37.13565</td>
<td>14294</td>
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<td>inv_inveq3sls</td>
<td>87</td>
<td>163.01609</td>
<td>36.93173</td>
<td>14182</td>
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Minimum     Maximum     Label
111.30000   223.20000
111.71783   223.98986

Predicted Values

Pearson Correlation Coefficients

<table>
<thead>
<tr>
<th></th>
<th>INV</th>
<th>inv_inveq3sls</th>
</tr>
</thead>
<tbody>
<tr>
<td>INV</td>
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</tbody>
</table>

R² = 0.99968

Predicted Values

Number of Observations

<table>
<thead>
<tr>
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<th>inv_inveq3sls</th>
</tr>
</thead>
<tbody>
<tr>
<td>88</td>
<td>87</td>
</tr>
<tr>
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</tbody>
</table>
The SIMLIN Procedure

Inverse Coefficient Matrix for Endogenous Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>C</th>
<th>IIN</th>
<th>INV</th>
<th>GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>1.0000</td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>GDP</td>
<td>0.9170</td>
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<td>-0.5392</td>
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<td>-0.0830</td>
<td>0.9702</td>
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<tr>
<td>INV</td>
<td>0.0205</td>
<td>0.0535</td>
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</table>
Reduced Form for Lagged Endogenous Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>lagc</th>
<th>lagiin</th>
<th>laginv</th>
<th>lagGDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>GDP</td>
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<td>0.0314</td>
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</tr>
<tr>
<td>IIN</td>
<td>-0.1027</td>
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<tr>
<td>INV</td>
<td>0.008130</td>
<td>0.001735</td>
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</table>

Interpretation: $C_t$ for example

$$C_t = 7.1532 + 0.5503C_{t-1} - 0.7336RS_t + 0.7217TR_t + 1.8387WLTH_t + 0.2746YD_t$$
### Reduced Form for Exogenous Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>G</th>
<th>IR</th>
<th>RS</th>
<th>TR</th>
<th>WLTH</th>
<th>YD</th>
<th>INR</th>
<th>Intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0</td>
<td>0</td>
<td>-0.7336</td>
<td>0.7217</td>
<td>1.8387</td>
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<td>0</td>
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<tr>
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<td>0.9702</td>
<td>0.9702</td>
<td>-0.6727</td>
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<tr>
<td>IIN</td>
<td>-0.0298</td>
<td>-0.0298</td>
<td>0.0609</td>
<td>-0.0599</td>
<td>-0.1527</td>
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<td>-1.5043</td>
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</tbody>
</table>

### Interpretation:

\[
\frac{\partial C_t}{\partial RS_t} = -0.7336 \quad \frac{\partial C_t}{\partial WLTH_t} = 1.8387
\]

\[
\frac{\partial C_t}{\partial TR_t} = 0.7217 \quad \frac{\partial C_t}{\partial YD_t} = 0.2746
\]
<table>
<thead>
<tr>
<th>Variable</th>
<th>G</th>
<th>IR</th>
<th>RS</th>
<th>TR</th>
<th>WLTH</th>
<th>YD</th>
<th>INR</th>
<th>Intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
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<td>0.0000000</td>
<td>-0.4037516</td>
<td>0.3971932</td>
<td>1.011901</td>
<td>0.1511478</td>
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<td>3.938307</td>
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<tr>
<td>GDP</td>
<td>-0.0261911</td>
<td>-0.0261911</td>
<td>-0.3193722</td>
<td>0.3141845</td>
<td>0.800425</td>
<td>0.1210015</td>
<td>-0.0261911</td>
<td>3.735025</td>
</tr>
<tr>
<td>IIN</td>
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<td>-0.0261911</td>
<td>0.0843794</td>
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<td>-0.211475</td>
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<td>-0.203282</td>
</tr>
<tr>
<td>INV</td>
<td>0.0453933</td>
<td>0.0453933</td>
<td>-0.0186193</td>
<td>0.0183168</td>
<td>0.046664</td>
<td>0.0199301</td>
<td>0.0453933</td>
<td>-1.228258</td>
</tr>
</tbody>
</table>

Interpretation:

\[
\frac{\partial C_{t+1}}{\partial RS_t} = -0.4038 \\
\frac{\partial C_{t+1}}{\partial TR_t} = 0.3972 \\
\frac{\partial C_{t+1}}{\partial WLTH_t} = 1.0119 \\
\frac{\partial C_{t+1}}{\partial YD_t} = 0.1511 
\]
### Interim Multipliers for Interim 2

<table>
<thead>
<tr>
<th>Variable</th>
<th>G</th>
<th>IR</th>
<th>RS</th>
<th>TR</th>
<th>WLTH</th>
<th>YD</th>
<th>INR</th>
<th>Intercept</th>
</tr>
</thead>
<tbody>
<tr>
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<td>-0.2221993</td>
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<tr>
<td>GDP</td>
<td>-0.0222323</td>
<td>-0.0222323</td>
<td>-0.1692847</td>
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<td>0.0573062</td>
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<tr>
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<td>-0.0222323</td>
<td>0.0529146</td>
<td>-0.0520551</td>
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<tr>
<td>INV</td>
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### Interim Multipliers for Interim 3

<table>
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<th>RS</th>
<th>TR</th>
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<th>YD</th>
<th>INR</th>
<th>Intercept</th>
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</thead>
<tbody>
<tr>
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<td>0.0000000</td>
<td>-0.1222844</td>
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<tr>
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<tr>
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<td>-0.0188463</td>
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<td>-0.0328804</td>
<td>-0.0837670</td>
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### Interim Multipliers for Interim 4

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<th>TR</th>
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<th>YD</th>
<th>INR</th>
<th>Intercept</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.0139060</td>
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</table>
### Total Multipliers

\( \mathbf{B}(\mathbf{I-A})^{-1}\mathbf{AB} \)

<table>
<thead>
<tr>
<th>Variable</th>
<th>G</th>
<th>IR</th>
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<th>TR</th>
<th>WLTH</th>
<th>YD</th>
<th>INR</th>
<th>Intercept</th>
</tr>
</thead>
<tbody>
<tr>
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<td>12.83567</td>
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<td>-.2018900</td>
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<td>0.3515110</td>
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<td>0.3515110</td>
<td>-9.12002</td>
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</tbody>
</table>

Cumulative or long-run impact of changes in G, IR, RS, TR, WLTH, YD, and INR on C, GDP, IIN, INV

Cumulative impact of a unit change in YD on
C = 0.6108
GDP = 0.7159
IIN = 0.1051
INV = 0.1685
Key Points

(1) Use analytically-derived reduced forms for forecasting, both within-sample as well as out-of-sample.

(2) May graph dynamic multipliers

EXAMPLE:

<table>
<thead>
<tr>
<th>Impact of YD on C</th>
<th>Impact of YD on C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contemporaneous</td>
<td>0.2746</td>
</tr>
<tr>
<td>1 Quarter Ahead</td>
<td>0.1511</td>
</tr>
<tr>
<td>2 Quarters Ahead</td>
<td>0.0832</td>
</tr>
<tr>
<td>3 Quarters Ahead</td>
<td>0.0458</td>
</tr>
<tr>
<td>4 Quarters Ahead</td>
<td>0.0252</td>
</tr>
</tbody>
</table>

Impact of YD on C

Quarters Ahead

continued...
(3) May convert impact, interim, and long-run (total) multipliers into elasticities
Consider impact multiplier of YD on C = 0.2746
SR elasticity then is 0.2746(YD/C) = 0.3730
LR elasticity is 0.6108(YD/C) = 0.8297
Check on Stability Conditions

\[ A = \begin{bmatrix}
.5503 & 0 & 0 & 0 \\
.4476 & .0314 & -.4716 & 0 \\
-.1027 & .0314 & -.4716 & 0 \\
.0081 & .0017 & .8487 & 0 
\end{bmatrix} \]

Solve \( |A - \lambda I| = 0 \)

Eigenvalues of A
(1) 0.5149
(2) -0.5365
(3) \( \approx 0 \) (very small)
(4) exactly 0

\[ \therefore \text{model stable} \]
\[ \text{dominant root negative } \Rightarrow \text{oscillatory pattern} \]