Name: **KEY Capps**

UIN: ____________________________

Class Time (please circle): Section 501 or Section 502
Monday, Dec. 12 Friday, Dec. 9
8am to 10am 12:30pm to 2:30pm

Instructions:

1. Please provide your name and UIN.

2. Circle the correct date and class time.

3. To get full credit on answers on this exam, be clear, rigorous, and thorough in your responses.

4. You cannot get credit (full or partial) unless something is written.

5. Sign the Aggie Pledge.

   “On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work.”

__________________________  ________________________
Signature                  Date
(15 pts) 1. A demand function for John Deere farm equipment is given by:

\[ Q = 800 - 4P, \] where \( Q \) is quantity of farm equipment and \( P \) is the price of farm equipment. Total cost of John Deere farm equipment is given by \( TC = 8000 + 100Q. \)

(3pts) (a) Calculate the own-price elasticity when \( P = $50. \) Show all work.

\[ \frac{dQ}{dP} \frac{P}{Q} = -4 \frac{P}{Q} \quad \text{when} \quad P = 50, \quad Q = 600; \]

\[ \text{So, own-price elasticity} = \frac{-4(50)}{600} = -\frac{200}{600} = -\frac{1}{3}. \]

(2pts) (b) Derive the inverse demand function.

\[ P = 800 - 4Q \]

So,

\[ 4P = 800 - Q \]

\[ \boxed{P = 200 - \frac{1}{4}Q} \]

(3pts) (c) Derive the expression for profit for John Deere. Express you answer in terms of \( Q. \)

\[ \Pi = \Pi_c - TC = PQ - TC = (200 - \frac{1}{4}Q)(Q - 800) - 100Q \]

\[ \Pi = 200Q - \frac{1}{4}Q^2 - 8000 - 100Q \]

\[ \boxed{\Pi = 100Q - \frac{1}{4}Q^2 - 8000} \]

(5pts) (d) What value of \( Q \) maximizes profit for John Deere? Please indicate first-order and second-order conditions to substantiate your answer.

\[ \frac{d\Pi}{dQ} = 100 - \frac{1}{2}Q = 0 \]

\[ \boxed{Q = 200} \]

\[ \text{So, } \frac{d^2\Pi}{dQ^2} = -\frac{1}{2} < 0 \quad \text{which indicates a maximum.} \]

(2pts) (e) What is the maximum level of profit?

\[ \text{When } Q = 200 \]

\[ \Pi = 100(200) - \frac{1}{4}(200)^2 - 8000 \]

\[ \boxed{\Pi = 2,000} \]
(20pts) 2. Using 60 observations, we estimate the demand function for Kellogg’s Raisin Bran as follows:

\[
\ln Q_{RBR} = 0.45 - 0.78 \ln P_{RBR} + 0.65 \ln P_{GMC} + 0.43 \ln P_{KFF} \\
(0.21) \quad (0.01) \quad (0.03) \quad (0.02)
\]

\[
+ 0.32 \ln I + 0.04 \ln A, \\
(0.04) \quad (0.01)
\]

where \( R^2 = 0.90. \)

\( Q_{RBR} \) denotes the quantity of Kellogg’s Raisin Bran sold, \( P_{RBR} \) denotes the price of Kellogg’s Raisin Bran, \( P_{GMC} \) denotes the price of General Mills Cheerios, \( P_{KFF} \) represents the price of Kellogg’s Frosted Flakes, \( I \) represents U.S. disposable income, and \( A \) represents the level of advertising expenditures associated with Kellogg’s Raisin Bran.

P-values are given in parentheses. The level of significance chosen for this analysis is 0.05.

(2pts) (a) What is the \( \bar{R}^2 \)?

(3pts) (b) Which explanatory variables are statistically significant? All explanatory variables since all p-values except the intercept are < 0.05, then the are statistically significant.

(2pts) (c) True or False. The demand for Kellogg’s Raisin Bran is elastic.

(2pts) (d) If General Mills raises the prices of Cheerios by 4 percent, then the quantity of Kellogg’s Raisin Bran sold rises by \( 2.6 \) percent, all other factors invariant.

(2pts) (e) True or False. Both General Mills Cheerios and Kellogg’s Frosted Flakes are substitutes for Kellogg’s Raisin Bran. Cross-price elasticity 0.65 and 0.43 are greater than zero indicating of substitutes.

(2pts) (f) Based on the estimation coefficient of income, what type of good is Kellogg’s Raisin Bran? Income elasticity 0.32

(2pts) (g) If the level of advertising expenditures were to double, then the quantity of Kellogg’s Raisin Bran sold rises by \( 4 \) percent, all other factors invariant.

(2pts) (h) What is the name of the estimation technique used to obtain the estimated coefficients of the explanatory variables? OLS

(3pts) (i) What test statistic would you use to test the null hypothesis that all the estimated coefficients associated with the explanatory variables are jointly equal to zero? Specify the degrees-of-freedom with this test statistic.

\( H_0: \) all explanatory variables except the intercept are jointly equal to zero.

\( H_1: \) at least one coefficient is not zero.
(17pts) 3. Technologies, Inc., considers profit scenarios for the next fiscal year.

<table>
<thead>
<tr>
<th>Business Conditions</th>
<th>Profit ( \bar{\Pi} )</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excellent</td>
<td>$30,000</td>
<td>0.4</td>
</tr>
<tr>
<td>Conventional (Typical)</td>
<td>$25,000</td>
<td>0.5</td>
</tr>
<tr>
<td>Abysmal</td>
<td>$6,000</td>
<td>0.1</td>
</tr>
</tbody>
</table>

\[ U = \sqrt{\bar{\Pi}} \]
\[ 173.2051 \]
\[ 158.1139 \]
\[ 77.4597 \]

(3pts) (a) Calculate the expected value of profit for Technologies, Inc., for the next fiscal year.

\[ E[\bar{\Pi}] = 0.4 \left( \frac{30,000}{2} \right) + 0.5 \left( \frac{25,000}{2} \right) + 0.1 \left( \frac{6,000}{2} \right) = \frac{112,000}{2} = \frac{22,500}{2} \]

(3pts) (b) Calculate the standard deviation of profit for Technologies, Inc., for the next fiscal year.

\[ \text{Variance} (\bar{\Pi}) = 0.4 \left( \frac{30,000}{2} \right)^2 + 0.5 \left( \frac{25,000}{2} \right)^2 + 0.1 \left( \frac{6,000}{2} \right)^2 \]

\[ = 12,000 \text{, 90,000} \]

Standard deviation = \[ \sqrt{\text{Variance}(\bar{\Pi})} = \sqrt{6788.96} = 82.705 \]

(2pts) (c) Calculate the coefficient of variation of profit for Technologies, Inc., for the next fiscal year.

\[ \text{Coefficient of variation} = \frac{\text{Standard deviation}}{E[\bar{\Pi}]} = \frac{82.705}{22,500} = 0.00367 \]

(3pts) (d) Suppose the utility function for the CFO (Chief Financial Officer) is given by \( U = \sqrt{\bar{\Pi}} \), where \( \bar{\Pi} \) corresponds to \( \Pi \). Is the CFO a risk taker, risk averter, or neutral to risk? Why?

\[ \text{The CFO is a risk averter} \]

\[ MU_{\bar{\Pi}} = \frac{1}{2 \bar{\Pi}} \]

(3pts) (e) Calculate the expected value of utility for the CFO.

\[ \text{EV of utility of } \bar{\Pi} = 0.4 \left( \frac{\sqrt{30,000}}{2} \right)^2 + 0.5 \left( \frac{\sqrt{25,000}}{2} \right)^2 + 0.1 \left( \frac{\sqrt{6,000}}{2} \right)^2 \]

\[ = 69.28 + 79.06 + 7.75 = 156.0899 \]

(3pts) (f) What is the risk premium?

\[ Risk \text{ Premium} = \text{EV}(U(\Pi)) - \bar{\Pi} \]

\[ = 156.0899 - 17.75 = 138.3399 \]

\[ = 138.3399 \]
(8pts) 4. Suppose that for the quantity sold (Q) of a product named MERIDIAN, developed recently by Blunders Company, may be expressed as a function of price (P) and advertising (A) as follows:

\[ Q = 5000 - 10P + 40A + PA - 0.8A^2 - 0.5P^2 \]

(a) Find the values of P and A that maximize Q. Show all work. Do not check on SOC.

\[
\begin{align*}
\frac{\partial Q}{\partial P} &= -10 + A - P = 0 \\
\frac{\partial Q}{\partial A} &= 40 + P = 0
\end{align*}
\]

so \( P = 10 \) and \( 40 + P = 1.6A \)

or \( P = -10 \) and \( 40 + A = 1.6A \)

30 = 6A

(2pts) (b) What is the optimal level of Q, given your answer in (a)?

\[ Q = 5000 \times 10 \left( 40 + 40 \left( 50 \right) \right) \left( \frac{g}{g} \right) - 8 \left( 500 \right)^2, 5 \left( 40 \right)^2 \]

(9pts) 5. Let \( x \) correspond to the number of tractors produced on assembly line 1 and let \( y \) correspond to the number of tractors produced on assembly line 2 for Massey Ferguson. The firm wishes to minimize weekly total costs (TC) given by:

\[ TC = 3x^2 + 6y^2 - xy \]

Due to contracts with dealers, Massey Ferguson must produce exactly 20 tractors per week.

\[ \text{constraint} \quad x + y = 20 \]

(a) Find the optimal amounts of tractors to produce on assembly line 1 (x) and on assembly line 2 (y), subject to the equality constraint.

\[
\begin{align*}
\frac{\partial TC}{\partial x} &= 6x - y + \lambda x + \lambda y \leq 0 \\
\frac{\partial TC}{\partial y} &= 12y - x + \lambda x + \lambda y \leq 0
\end{align*}
\]

(b) What is the Lagrangian multiplier?

\[
\begin{align*}
\lambda &= y - 6x = 7 - 6(13) = -71 \\
\lambda &= x - 12y = 13 - 12(1) = -71
\end{align*}
\]

(2pts) (c) If Massey Ferguson were to alter the number of tractors produced each week from 20 to 18, what would be the change in total costs?

\[ \text{Total costs will fall by roughly } \frac{142}{2} \]

\[ -71 \times 2 \]
(15pts) 6. Suppose Woodstone Industries produces two different types of products $P_1$ and $P_2$. The input requirements and resource limits are listed below.

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Input Required for One Unit of Output</th>
<th>Resource Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_1$</td>
<td>$P_2$</td>
</tr>
<tr>
<td>$X$</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$Y$</td>
<td>12</td>
<td>8</td>
</tr>
</tbody>
</table>

The per unit profit for $P_1$ and $P_2$ is respectively $500$ and $600$.

(2pts) (a) Provide the objective function for Woodstone Industries.

$$\max \pi = 500P_1 + 600P_2$$

(4pts) (b) Provide the constraints for the profit maximization problem.

$$P_1 + 2P_2 \leq 80$$
$$12P_1 + 8P_2 \leq 480$$
$$P_1 \geq 0, P_2 \geq 0$$

(4pts) (c) Graphically provide the feasible solution space and the feasible solution boundary for this problem.

(3pts) (d) Find the optimal values of $P_1$ and $P_2$ for Woodstone Industries to manufacture.

- at point $A$: $P_1 = 40$ and $P_2 = 0$
- at point $B$: $P_1 = 20$ and $P_2 = 30$
- at point $C$: $P_1 = 0$ and $P_2 = 40$

$$\pi_A = 40(500) + 0(600) = 20,000$$
$$\pi_B = 20(500) + 30(600) = 19,000$$
$$\pi_C = 40(600) = 24,000$$

So, optimal values of $P_1$ and $P_2$ are $P_1 = 20$ and $P_2 = 30$

(2pts) (e) Find the maximum sales for Woodstone Industries.

$$\max \pi = 24,000$$

When $P_2 = 30$
When $P_1 = 20$
(10pts) 7. Branded Products, Inc., based in San Francisco, CA, is a leading manufacturer of household laundry detergent and bleach products. About a year ago, Branded Products rolled out its new CleanX detergent in 30 regional markets, following its success in test markets. Specifically, you, as an analyst for Branded Products, Inc., estimate a demand model given by:

\[ Q_{\text{CleanX}} = f(P_{\text{CleanX}}, P_{ \text{PG}}, AD, I), \]

where

- \( Q_{\text{CleanX}} \) = number of cases of CleanX
- \( P_{\text{CleanX}} \) = price per case of CleanX
- \( P_{\text{PG}} \) = competitor (Proctor & Gamble) price per case
- \( AD \) = amount of advertising expenditures spent in each regional market
- \( I \) = average household income in each regional market.

Details of the regression analysis are as follows:

<table>
<thead>
<tr>
<th>Regression Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R^2 )</td>
</tr>
<tr>
<td>Standard error</td>
</tr>
<tr>
<td>Observations</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Standard Error</th>
<th>t Stat</th>
<th>P Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>807.938</td>
<td>137.846</td>
<td>5.86</td>
</tr>
<tr>
<td>Price, ( P )</td>
<td>-0.034</td>
<td>0.457</td>
<td>-11.02</td>
</tr>
<tr>
<td>Competitor price, ( P_{\text{PG}} )</td>
<td>4.860</td>
<td>1.006</td>
<td>-4.83</td>
</tr>
<tr>
<td>Advertising, ( AD )</td>
<td>0.328</td>
<td>0.104</td>
<td>3.14</td>
</tr>
<tr>
<td>Household income, ( I )</td>
<td>0.009</td>
<td>0.001</td>
<td>7.99</td>
</tr>
</tbody>
</table>

(5pts) (a) Use the regression model estimation results to forecast the demand in Houston, TX, (a new market), where the price per case is $145, the competitor (Proctor & Gamble) price is $100, the amount of advertising expenditures is $900, and the average household income for the Houston, TX, market is $51,786. Show all work.

\[ Q_{\text{CleanX}}(\text{Houston}) = 807.938 - 5.034(145) + 4.86(100) + 0.328(900) + 0.009(51,786) \]

\[ Q_{\text{CleanX}}(\text{Houston}) = 1,325.282 \]

(5pts) (b) Construct a 95% confidence interval for your forecast in (a). A critical value of the t-distribution is needed to answer this problem. What is the degrees-of-freedom associated with this t-distribution? For the construction of this confidence interval, use a value of 2.

Confidence interval = point forecast + \( t(\frac{\text{standard error}}{\text{degrees of freedom}}) \)

\[ 1,325.282 \pm 70.92 = \left[ 1,254.362, 1,396.202 \right] \]
(3pts) 8. Suppose we wish to analyze the Mexican peso to U.S. dollar exchange rate. The exchange rate deals with the number of pesos per U.S. dollars. In conducting this analysis, we estimate the following model.

\[ \text{PesoUS}_t = -2.6 + 0.8\text{PesoUS}_{t-1} + 0.6\text{PesoUS}_{t-2} \]

The variable PesoUS, represents the Mexican peso to U.S. dollar exchange rate.

(i) What is the technical name of this model?

\[ \text{AR}(2) \]

\[ \text{Autoregressive model of order 2} \]

(2pts) ii. Suppose that the peso to U.S. dollar exchange rate for September 2011 was 11, and the peso to U.S. dollar exchange rate for October 2011 were 10. Using the model, forecast the exchange rate for November 2011. Show all work.

\[ \begin{align*}
\text{Exchange rate for November 2011} &= -2.6 + 0.8(10) + 0.6(11) \\
\text{Exchange rate for November 2011} &= -2.6 + 8 + 6.6 \\
\text{Exchange rate for November 2011} &= 12.
\end{align*} \]

(3pts) 9. List three metrics of forecast accuracy.

(a) \(MSE/NMSE\)

(b) \(MAE\)

(c) \(MAPE\)