1. \( Q = \frac{1}{a} p^{-1/b} \)

\[
\frac{dQ}{dp} = \frac{1}{a} \left( -\frac{1}{b} \right) p^{-1/b - 1} = \frac{1}{ab} p^{-1/b}
\]

(a) own-price elasticity

\[
\frac{dQ}{dp} \cdot \frac{p}{Q} = -\frac{1}{ab} p^{-\frac{1}{b}} p^{-1} \left( \frac{p}{a} p^{-\frac{1}{b}} \right) = -\frac{1}{b}, \quad b > 0
\]

\(-\frac{1}{b}\) is the own-price elasticity and is constant.

(b) \( b \) must be > 0 to insure that the own-price elasticity is negative; \( a \) must be > 0 to insure that \( Q > 0 \).

2. \( Q_s = 200 + 3p \)

\( Q_d = 400 - p \).

(a) Equilibrium condition \( Q_s = Q_d \)

\[ 200 + 3p = 400 - p \]

\[ 4p = 200 \]

\[ p = 50 \]

\( Q_s = Q_d = 350 \)

(b) when \( p = 60 \), \( Q_s = 380 \) and \( Q_d = 340 \)

since \( Q_s > Q_d \), excess supply

(c) when \( p = 40 \), \( Q_s = 320 \) and \( Q_d = 360 \)

since \( Q_d > Q_s \), excess demand
2. (d) own-price elasticity of demand ($\epsilon_d$)

\[ Q_d = 400 - p \]
\[ \frac{dQ_d}{dp} = -1 \]
\[ \epsilon_d = \frac{dQ_d}{dp} \frac{p}{Q_d} = -\frac{p}{400} = -\frac{5}{350} = -\frac{1}{7} \]

at equilibrium

(e) own-price elasticity of supply ($\epsilon_s$)

\[ Q_s = 200 + 3p \]
\[ \frac{dQ_s}{dp} = 3 \]
\[ \epsilon_s = \frac{dQ_s}{dp} \frac{p}{Q_s} = \frac{3p}{200 + 3p} = \frac{150}{350} = \frac{3}{7} \]

at equilibrium

3. \[ Q = 10 - p \rightarrow p = 10 - Q \]

(a) \[ R(Q) = PQ = (10 - Q)Q = 10Q - Q^2 \]

revenue is a quadratic function of $Q$

marginal revenue function \[ \frac{dR(Q)}{dQ} = 10 - 2Q \]

(b) \[ \max R(Q) = 10Q - Q^2 \]

Foc \[ 10 - 2Q = 0 \]
\[ \frac{dR(Q)}{dQ} = 10 - 2Q = 0 \]
\[ Q = 5 \]

Soc \[ -2 = -2 \]
\[ \frac{d^2R(Q)}{dQ^2} = -2 \]

$Q = 5$ serves to maximise $R(Q)$

when $Q = 5$, $p = 5$

The maximum total revenue is £25.
3(c). When total revenue is a maximum, marginal revenue is 0.

4. \( Q = 400 - 1200P + 8A + 55\text{Pop} + 800P^0 \)
   \( P = 1.50, \quad A = 4,000, \quad \text{Pop} = 40, \quad P^0 = 4. \)

(a) \( Q = 400 - 1,200(1.50) + 8(4,000) + 55(40) + 800(4) \)
   \( Q = 400 - 1,800 + 32,000 + 2,200 + 800 \)
   \( Q = 2,400 \)

(b) Own-price elasticity
   \( \frac{dQ}{dP} \frac{P}{Q} = -1200 \frac{P}{Q} = -1200 \frac{1.50}{2,400} = -0.75 \)

Advertising elasticity
   \( \frac{dQ}{dA} \frac{A}{Q} = \frac{8A}{Q} \frac{1,000}{2,400} = \frac{1}{3} = 0.3333 \)

(c) Given that the own-price elasticity is negative and inelastic, to increase total revenue, it is advisable that McPablo's Food Shops raise price.
5. \( Q = 800 - 4P \quad TC = 8000 + 100Q \).

(a) own-price elasticity of demand
\[
\frac{dQ}{dP} \cdot \frac{P}{Q} = -4P
\]
when \( P = $50 \quad Q = 600 \)
so, the own-price elasticity of demand is equal to
\[
-\frac{4(50)}{600} = -\frac{200}{600} = -\frac{1}{3}
\]

(b) inverse demand function
\[
Q = 800 - 4P \Rightarrow 4P = 800 - Q
P = 200 - 1.25Q
\]

(c) \( TR = PQ = \left(200 - 1.25Q\right)Q = 200Q - 25Q^2 \)
\( TC = 8000 + 100Q \)
\( \pi = TR - TC = 200Q - 25Q^2 - 8000 - 100Q \)
\( \pi = -25Q^2 - 100Q + 200Q - 8000 \)
\( \pi = -25Q^2 - 100Q + 100 \)

(d) \( FOC \quad \frac{d\pi}{dQ} = 0 = 100 - 5Q \)
so \( C \quad \frac{d^2\pi}{dQ^2} = -5 < 0 \Rightarrow \text{maximum.} \)
\( Q = 200 \) solves the FOC and corresponds to the value of \( Q \) which maximizes \( T \).
5. (e) \[ Q = 200, \quad P = 150 \]

(f) \[
\Pi = -6,000 + 100Q - 25Q^2
\]

\[
\Pi' = -6,000 + 100(200) - 25(200)^2
\]

\[
\Pi' = -6,000 + 20,000 - 10,000
\]

\[
\Pi' = 2,000
\]

6. \[ Q_{Yoplait} = 0.5 P_{Yoplait} + P_{Dannon} + P_{Horizon Organic} I \]

(a) Own-price elasticity of Yoplait = \(-1.5\)

\[
-1.5 = \frac{-\Delta Q_{Yoplait}}{\% \Delta P_{Yoplait}}
\]

so if \% \Delta P_{Yoplait} = 8, then \% \Delta Q_{Yoplait} = -12

TRUE!

(b) **TRUE!**

a price reduction for Yoplait yogurt will increase the number of units sold. **And** a price reduction will increase sales revenue because demand is elastic.

(c) **TRUE!**

Cross-price elasticity of Yoplait with Dannon is 0.8

Cross-price elasticity of Yoplait with Horizon Organic is 0.4

both cross-price elasticities are positive => substitution
(d) False!
Annual competitor to Yoplait is Dannon

(e) Cross-price elasticity of Yoplait w.r.t Dannon
\[
\frac{\% \Delta P_{\text{Yoplait}}}{\% \Delta P_{\text{Dannon}}} = 0.8
\]
\[
\% \Delta P_{\text{Dannon}} = 5 \text{ then } \% \Delta P_{\text{Yoplait}} = 4
\]
FALSE!

(f) Income elasticity for Yoplait is 0.3 \Rightarrow \text{ Necessity}
TRUE!

(g) a 1 percent increase in the price of Yoplait
\[
\Rightarrow \text{ a } -1.5 \text{ percent change in the quantity of Yoplait}
\text{ a 10 percent increase in advertising expenditure} \Rightarrow
\text{ a 1.0 percent increase in the quantity of Yoplait}
\]
Net result: a 0.5 decline in the quantity of Yoplait
FALSE!

Note: needed a 15 percent increase in advertising expenditure to offset a 1 percent increase in the price of Yoplait
7. $Q_a = \alpha + \beta p_a + \gamma p_b + \eta y$

(a) $\beta < 0$ to insure a downward sloping demand curve.
$\eta > 0$ to insure a normal good.
$\gamma > 0$ to insure that good b is a substitute for good a.

(b) $p_a = 2$, $Q_a = 4$, $\beta = -\frac{1}{2}$

Own-price elasticity
\[
\frac{dQ_a}{dp_a} \frac{p_a}{Q_a} = \frac{\beta p_a}{Q_a} = \frac{(-\frac{1}{2}) \cdot 2}{4} = -\frac{1}{4}.
\]

inelastic demand —
To increase revenue, all other factors held constant.
raise price.