Problem Set #2

1. Verify the demand function. \( Q = \frac{1}{a} p^{-\frac{b}{1}} \), where \( a > 0, b > 0 \), has constant demand elasticity.
   a) What is the own-price elasticity of demand?
   b) The parameters \( a \) and \( b \) of the demand function are restricted to be positive. Why?

2. Suppose that the demand and supply relationships of a specific type of laptop computer manufactured by Dell are given as:

\[
Q_s = 200 + 3p \\
Q_d = 400 - p
\]

a) Solve for the equilibrium price and quantity of the number of laptops.
b) What is the excess demand or excess supply when \( p = 60 \)?
c) What is the excess demand or excess supply when \( p = 40 \)?
d) At equilibrium, what is the own-price elasticity of demand?
e) At equilibrium, what is the own-price elasticity of supply?

3. Given the demand function \( Q = 10 - p \).
   a) Write down the revenue function and marginal revenue function.
   b) Plot the total revenue curve. What value of \( P \) and \( Q \) serves to maximize total revenue?
   c) When total revenue is a maximum, what is marginal revenue?

4. Management of McPablo’s Food Shops has completed a study of weekly demand for its “old-fashioned” tacos in 53 regional markets. The study revealed that

\[
Q = 400 - 1,200P + 0.8A + 55Pop + 800P^0,
\]

where \( Q \) is the number of tacos sold per store per week, \( A \) is the level of local advertising expenditures (in dollars), \( Pop \) denotes the local population (in thousands), and \( P^0 \) is the average taco price of local competitors. For the typical McPablo’s outlet, \( P = $1.50 \), \( A = $1,000 \), \( Pop = 40 \), and \( P^0 = $1 \).

a) Estimate the weekly sales for the typical McPablo’s outlet?
b) What is the own-price elasticity for tacos? What is the advertising elasticity?
c) Should McPablo’s raise its taco price? Why or why not?
5. A demand function for John Deer farm equipment is given by:

\[ Q = 800 - 4P, \] where \( Q \) is quantity of farm equipment and \( P \) is the price of farm equipment. Total cost of John Deer equipment is given by \( TC = 8000 + 100Q \).

a) Calculate the own-price elasticity when \( P = \$50 \).

b) Derive the inverse demand function.

c) Profit equals Total Revenue (TR) minus Total Cost (TC). Derive the expression for profit for John Deer. Express your answer in terms of \( Q \).

d) What value of \( Q \) maximizes profit for John Deer? Please indicate the first-order and second-order conditions to substantiate your answer.

e) What value of \( P \) maximizes profit?

f) What is the maximum level of profit?

6. Suppose that the demand function for Yoplait yogurt is given by the following:

\[ Q_{\text{Yoplait}} = 5P_{\text{Yoplait}}^{-1.5} P_{\text{Dannon}}^{0.8} P_{\text{HorizonOrganic}}^{0.4} I^{0.3} A^{0.1}, \]

where \( Q_{\text{Yoplait}} \) is the quantity demanded of Yoplait yogurt; \( P_{\text{Dannon}} \) is the price of Dannon yogurt; \( P_{\text{HorizonOrganic}} \) is the price of Horizon Organic yogurt, and \( I \) is consumer income. \( A \) corresponds to advertising expenditures for Yoplait yogurt. Indicate whether each of the following statements is true or false and provide explanations.

a) An 8 percent increase in the price of Yoplait yogurt, all other factors held constant, yields a 12 percent decrease in the quantity demanded of Yoplait yogurt.

b) A price reduction for Yoplait yogurt will increase both the number of units sold and sales revenue.

c) Dannon and Horizon Organic brands are substitutes for Yoplait yogurt.

d) Horizon Organic yogurt is the principal competitor to Yoplait yogurt.

e) If the price of Dannon yogurt increases by 5 percent, all other factors held constant, the quantity demanded of Yoplait yogurt increases by 8 percent.

f) Yoplait yogurt is a necessity.

g) To offset a 1 percent increase in the price of Yoplait yogurt, a 10 percent increase in advertising expenditures is required, holding all other factors constant.
7. Suppose the demand function for good \( a \) is given by

\[
Q_a = \alpha + \beta p_a + \gamma p_b + \eta Y
\]

where \( p_a \) and \( p_b \) correspond to the price of good \( a \) and \( b \) respectively, and \( Y \) denotes income.

a) It is known that good \( a \) has a downward sloping demand curve, is a normal good, and is a close substitute to good \( b \). With this prior information, what should be the respective signs of \( \beta, \gamma \), and \( \eta \)?

b) Suppose that \( p_a = 2, Q_a = 4, \) and \( \beta = -\frac{1}{2} \). Other things being equal, will a lower or higher price increase revenue? Why?