Aquarius Products, Inc. has just completed development of a new line of skin-care products. Preliminary market research indicates two feasible marketing strategies: (1) creating general consumer acceptance through media advertising or (2) creating distributor acceptance through intensive personal selling. Sales estimates associated with each marketing alternative are as follows:

<table>
<thead>
<tr>
<th>Media Advertising Strategy</th>
<th>Personal Selling Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>Sales ($)</td>
</tr>
<tr>
<td>0.1</td>
<td>500,000</td>
</tr>
<tr>
<td>0.4</td>
<td>1,500,000</td>
</tr>
<tr>
<td>0.4</td>
<td>2,500,000</td>
</tr>
<tr>
<td>0.1</td>
<td>3,500,000</td>
</tr>
</tbody>
</table>

(a) Assume that the company has a 50 percent profit margin on sales (i.e., profits equal one-half of sales revenue). Calculate expected profits for each strategy.

\[
\text{EV} (\Pi_{\text{MAD}}) = \frac{1}{2} (0.1)(500,000) + \frac{1}{2} (0.4)(1,500,000) + \frac{1}{2} (0.4)(2,500,000) + (0.1)(3,500,000) + (0.4)(1,000,000)
\]

\[
\text{EV} (\Pi_{\text{PSD}}) = \frac{1}{2} (0.1)(500,000) + \frac{1}{2} (0.4)(1,500,000) + \frac{1}{2} (0.4)(2,500,000) + (0.1)(3,000,000)
\]

(b) Calculate the standard deviation of the profit distribution associated with each strategy. Which strategy appears to be riskier?

\[
\sigma_{\Pi_{\text{MAD}}} = \sqrt{\frac{1}{2} (0.1) (250,000 - 500,000)^2 (1) + \frac{1}{2} (0.4) (750,000 - 500,000)^2 (1) + \frac{1}{2} (0.4) (1,250,000 - 500,000)^2 (1) + \frac{1}{2} (0.1) (1,500,000 - 350,000)^2 (1)}
\]

\[
\sigma_{\Pi_{\text{PSD}}} = \sqrt{\frac{1}{2} (0.1) (200,000 - 500,000)^2 (1) + \frac{1}{2} (0.4) (700,000 - 500,000)^2 (1) + \frac{1}{2} (0.4) (1,200,000 - 500,000)^2 (1) + \frac{1}{2} (0.1) (1,000,000 - 300,000)^2 (1)}
\]

Calculate the coefficient of variation of the profit distribution associated with each strategy. In terms of minimizing relative risk, which strategy should Aquarius Products, Inc. pursue?

\[
CV_{\text{MAD}} = \frac{\sigma_{\Pi_{\text{MAD}}}}{EV_{\Pi_{\text{MAD}}}} = \frac{403,112.89}{9,000,000} = 0.04421
\]

\[
CV_{\text{PSD}} = \frac{\sigma_{\Pi_{\text{PSD}}}}{EV_{\Pi_{\text{PSD}}}} = \frac{413,490.17}{9,000,000} = 0.046
\]

To minimize relative risk, Aquarius Products, Inc. needs to pursue a personal selling strategy.

media advertising strategy is more risky than the personal selling strategy
(d) Assume that management's utility function resembles the one illustrated in the following figure. Calculate expected utility for each strategy. Which strategy should the marketing manager recommend?

The Relation Between Total Profit and Utility for Aquarius Products, Inc.

Utility of profit (utils)

<table>
<thead>
<tr>
<th>Profit ($000)</th>
<th>U(150,000) = 50</th>
<th>U(175,000) = 82.5</th>
<th>U(215,000) = 95</th>
<th>U(275,000) = 100</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \frac{AU}{AW} = \frac{23.5}{250} = 0.094 )</td>
<td>( \frac{AU}{WT} = \frac{12.5}{500} = 0.025 )</td>
<td>( \frac{AU}{WT} = \frac{5.5}{500} = 0.011 )</td>
<td>( \frac{AU}{WT} = \frac{7.5}{250} = 0.03 )</td>
</tr>
</tbody>
</table>

Media advertising strategy maximizes expected utility

\( \text{EV} \left[ U(\text{tags}) \right] = (0.1)(50) + (0.4)(82.5) + (0.4)(95) + (0.1)(100) = 86 \)

\( \text{EV} \left[ U(\text{PPS}) \right] = (0.3)(70) + (0.4)(82.5) + (0.3)(95) = 81 + 33 + 29 = 83 \)
<table>
<thead>
<tr>
<th></th>
<th>City</th>
<th>Scenario (State of Nature)</th>
<th>Annual M Contribution</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dallas</td>
<td>A</td>
<td>$100,000</td>
<td>.3</td>
</tr>
<tr>
<td></td>
<td>Dallas</td>
<td>B</td>
<td>$309,000</td>
<td>.7</td>
</tr>
<tr>
<td></td>
<td>Las Vegas</td>
<td>A</td>
<td>$69,000</td>
<td>.6</td>
</tr>
<tr>
<td></td>
<td>Las Vegas</td>
<td>B</td>
<td>$1,000,000</td>
<td>.4</td>
</tr>
</tbody>
</table>

\[ \text{EV}(\Pi_D) = 0.3 (100,000) + 0.7 (309,000) = 249,000 \]

\[ \text{EV}(\Pi_{LV}) = 0.6 (69,000) + 0.4 (1,000,000) = 436,000 \]

choose Las Vegas on the basis of the expected value of profit.

\[ \text{Var}(\Pi_D) = \sigma_{\Pi_D}^2 = 0.3 \left[ (100,000 - 249,000)^2 \right] + 0.7 \left[ (309,000 - 249,000)^2 \right] \]

\[ \sigma_{\Pi_D} = \text{Square root of the above expression} = \$91,651.51 \]

\[ \text{Var}(\Pi_{LV}) = \sigma_{\Pi_{LV}}^2 = 0.6 \left[ (69,000 - 436,000)^2 \right] + 0.4 \left[ (1,000,000 - 436,000)^2 \right] \]

\[ \sigma_{\Pi_{LV}} = \text{Square root of the above expression} = \$460,504.07 \]

choose Dallas on the basis to minimize absolute risk.

\[ \text{Relative risk} = \text{Coefficient of variation} \quad (CV) = \frac{\sigma}{\text{EV}} \]

\[ CV_{\Pi_D} = \frac{\sigma_{\Pi_D}}{\text{EV}(\Pi_D)} = \frac{\$91,651.51}{\$249,000} = 0.3819 \]

\[ CV_{\Pi_{LV}} = \frac{\sigma_{\Pi_{LV}}}{\text{EV}(\Pi_{LV})} = \frac{\$460,504.07}{\$436,000} = 1.0562 \]

choose Dallas on the basis to minimize relative risk.
2. \( U = \sqrt{\pi} \)

(c) Dallas

<table>
<thead>
<tr>
<th>( \pi )</th>
<th>( U(\pi) )</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100,000</td>
<td>516.2278</td>
<td>.3</td>
</tr>
<tr>
<td>$300,000</td>
<td>547.7226</td>
<td>.7</td>
</tr>
</tbody>
</table>

\[ EV(U(\pi_D)) = 478.27 \]

Las Vegas

<table>
<thead>
<tr>
<th>( \pi )</th>
<th>( U(\pi) )</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$60,000</td>
<td>244.9490</td>
<td>.6</td>
</tr>
<tr>
<td>$1,000,000</td>
<td>1,000,000</td>
<td>.4</td>
</tr>
</tbody>
</table>

\[ EV(U(\pi_W)) = 544.97 \]

choose Las Vegas for \( U(\pi) = \sqrt{\pi} \)

(d) Risk premium for Las Vegas.

\[ EV(\pi_W) = \left[ EV(U(\pi_W)) \right]^2 = \$436,000 - \$299,176 = \$136,824 \]

Risk premium for Dallas

\[ EV(\pi_D) = \left[ EV(U(\pi_D)) \right]^2 = \$240,000 - \$228,716 = \$11,284 \]

(e) \( U = \ln \pi \)

(c) Dallas

<table>
<thead>
<tr>
<th>( \pi )</th>
<th>( U(\pi) )</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100,000</td>
<td>11.5129</td>
<td>.3</td>
</tr>
<tr>
<td>$300,000</td>
<td>12.6155</td>
<td>.7</td>
</tr>
</tbody>
</table>

\[ EV(U(\pi_D)) = 12.2820 \]

Las Vegas

<table>
<thead>
<tr>
<th>( \pi )</th>
<th>( U(\pi) )</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$60,000</td>
<td>11.0021</td>
<td>.6</td>
</tr>
<tr>
<td>$1,000,000</td>
<td>13.8155</td>
<td>.4</td>
</tr>
</tbody>
</table>

\[ EV(U(\pi_W)) = 12.1275 \]

choose Dallas for \( U(\pi) = \ln \pi \)
2. (c) Risk premium for Las Vegas

What value of \( \Pi \) equals \( \ln \Pi = 12.1275 \)?

\[
\Pi = e^{12.1275} = $184,880
\]

So, risk premium equals

\[
EV(\Pi_{LV}) - = $184,880 = $436,000 - $184,880 = $251,120
\]

Risk premium for Dallas

\[
\Pi = e^{12.2820} = $215,767
\]

So, risk premium equals

\[
EV(\Pi_D) = $215,767 = $249,000 - $215,767 = $24,233
\]

3. (a) Albuquerque

<table>
<thead>
<tr>
<th>State of Nature</th>
<th>( \Pi )</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Failure</td>
<td>$100,000</td>
<td>.6</td>
</tr>
<tr>
<td>Success</td>
<td>$200,000</td>
<td>.4</td>
</tr>
</tbody>
</table>

\[
EV(\Pi_A) = $140,000
\]

IN ALBUQUERQUE START UP COSTS = $1.2 MILLION

RISK-FREE ANNUAL OPPORTUNITY COST OF INVESTMENT CAPITAL

EQUALS 10% OF START UP COST = $120,000

CERTAINTY EQUIVALENT IS

\[
\frac{$120,000}{$140,000} = e_A = 0.857
\]

(b) Santa Fe

<table>
<thead>
<tr>
<th>State of Nature</th>
<th>( \Pi )</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Failure</td>
<td>$40,000</td>
<td>.3</td>
</tr>
<tr>
<td>Success</td>
<td>$60,000</td>
<td>.7</td>
</tr>
</tbody>
</table>

\[
EV(\Pi_{SF}) = $54,000
\]

IN SANTA FE START UP COSTS = $1.7 MILLION

RISK-FREE ANNUAL OPPORTUNITY COST OF INVESTMENT CAPITAL

EQUALS 10% OF START UP COST = $170,000

CERTAINTY EQUIVALENT IS

\[
\frac{$170,000}{$54,000} = 3.1481 = e_{SF}
\]
3. IF WE ASSUME THAT THE MANAGEMENT OF TEX-MEX IS RISK AVERSE AND THAT THE CERTAINTY EQUIVALENT METHOD IS USED IN DECISION-MAKING, THEN THE MORE ATTRACTIVE OUTLET IS IN ALBUQUERQUE.

\( \delta_b < 1 \ \Rightarrow \ \text{RISK AVERSION} \)

\( \delta_{SE} > 1 \ \Rightarrow \ \text{RISK TAKING OR RISK LOVING} \)

A. \( U(y) \), UTILITY OF INCOME

\[ U(50) = 10 \]
\[ U(100) = 15 \]
\[ U(150) = 18 \]

Examine \( MU_{50 \to 100} = \frac{5}{50} = 0.1 \)

\( MU_{100 \to 150} = \frac{3}{50} = 0.06 \)

As \( y \) increases, \( MU(y) \) decreases \( \Rightarrow \) RISK AVERSION

5. (a) worst possible outcome for Project A is \( -$500 \)
   
   worst possible outcome for Project B is \( $20 \)
   
   by maximin strategy, choose Project B

(b) for status quo state of nature, best outcome is \( $1,000 \)
   for recession best outcome is \( $20 \)
   for boom best outcome is \( $2,000 \)

   opportunity loss or regret matrix is given by:

<table>
<thead>
<tr>
<th>States of Nature</th>
<th>Project A</th>
<th>Project B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Status Quo</td>
<td>$4,000 - $4,000 = $0</td>
<td>$1,000 - $200 = $800</td>
</tr>
<tr>
<td>Recession</td>
<td>$20 - (-$200) = $220</td>
<td>$20 - $20 = $0</td>
</tr>
<tr>
<td>Boom</td>
<td>$2,000 - $2,000 = $0</td>
<td>$2,000 - $1,500 = $500</td>
</tr>
</tbody>
</table>

   Maximum Regret:
   
   Project A: \( $320 \)
   Project B: \( $800 \)

   Opportunity Loss

   by a minimax regret decision rule, choose Project A.

   Note: probabilities do not enter game-theoretic strategies.