Problem Set #3

1. Aquarius Products, Inc, has just completed development of a new line of skin-care products. Preliminary market research indicates two feasible marketing strategies: (1) creating general consumer acceptance through media advertising or (2) creating distributor acceptance through intensive personal selling. Sales estimates associated with each marketing alternative are as follows:

<table>
<thead>
<tr>
<th>Media Advertising Strategy</th>
<th>Personal Selling Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Probability</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
</tr>
</tbody>
</table>

(a) Assume that the company has a 50 percent profit margin on sales (i.e., profits equal one-half of sales revenue). Calculate expected profits for each strategy.

\[
\begin{align*}
EV(\text{Mas}) &= 0.1(250,000) + 0.4(750,000) + 0.4(1,250,000) + 0.1(1,750,000) \\
EV(\text{Mas}) &= 750,000
\end{align*}
\]

\[
\begin{align*}
EV(\text{PSS}) &= 0.3(500,000) + 0.4(750,000) + 0.3(1,000,000) \\
EV(\text{PSS}) &= 750,000
\end{align*}
\]

(b) Calculate the standard deviation of the profit distribution associated with each strategy. Which strategy appears to be riskier?

Riskier strategy is PSS

\[
\begin{align*}
Var(\text{Mas}) &= 0.1(250,000 - EV(\text{Mas}))^2 + 0.4(750,000 - EV(\text{Mas}))^2 + 0.4(1,250,000 - EV(\text{Mas}))^2 + 0.1(1,750,000 - EV(\text{Mas}))^2 \\
Var(\text{Mas}) &= 1,63 \times 10^4 \\
\sigma_{\text{Mas}} &= \sqrt{1,63} = 403.113
\end{align*}
\]

\[
\begin{align*}
Var(\text{PSS}) &= 0.3(500,000 - EV(\text{PSS}))^2 + 0.4(750,000 - EV(\text{PSS}))^2 + 0.3(1,000,000 - EV(\text{PSS}))^2 \\
Var(\text{PSS}) &= 3.75 \times 10^4 \\
\sigma_{\text{PSS}} &= \sqrt{3.75} = 193.649
\end{align*}
\]

(c) Calculate the coefficient of variation of the profit distribution associated with each strategy. In terms of minimizing relative risk, which strategy should Aquarius Products, Inc, pursue?

\[
\begin{align*}
CV(\text{PSS}) &= \frac{\sigma_{\text{PSS}}}{EV(\text{PSS})} = \frac{193.649}{750,000} = 0.2582 \\
CV(\text{Mas}) &= \frac{\sigma_{\text{Mas}}}{EV(\text{Mas})} = \frac{403.113}{750,000} = 0.0538
\end{align*}
\]

In order to minimize relative risk, Aquarius Products, Inc, should pursue PSS.
Answers to Problem Set #3

1. (a) $EV (\Pi_{MAS}) = $1,000,000
   $EV (\Pi_{PSS}) = $750,000

(b) $\sigma_{MAS} =$ 403,113
$\sigma_{PSS} =$ 193,649
MAS is the riskier strategy

(c) $CV_{MAS} = \frac{\sigma_{MAS}}{EV(\Pi_{MAS})} = 0.4031$
$CV_{PSS} = \frac{\sigma_{PSS}}{EV(\Pi_{PSS})} = 0.2582$
Pursue PSS

(d) $EV [U(\Pi_{MAS})] = 86$
$EV [U(\Pi_{PSS})] = 81$
Recommend MAS $\Rightarrow$ maximizes expected value of utility

MAS $\rightarrow$ Media Advertising Strategy
PSS $\rightarrow$ Personal Selling Strategy
(d) Assume that management's utility function resembles the one illustrated in the following figure. Calculate expected utility for each strategy. Which strategy should the marketing manager recommend?

\[\text{Expected Utility for MAS = 86}\]
\[\begin{align*}
&= 0.1 U(\$250,000) + 0.4 U(\$750,000) + 0.4 U(\$1,250,000) + 0.1 U(\$1,750,000) \\
&= 0.1 (50) + 0.4 (82.5) + 0.4 (95) + 0.1 (100)
\end{align*}\]

\[\text{Expected Utility for PSS = 81}\]
\[\begin{align*}
&= 0.3 U(\$500,000) + 0.4 U(\$750,000) + 0.3 U(\$1,000,000) \\
&= 0.3 (70) + 0.4 (82.5) + 0.3 (90)
\end{align*}\]
2. Fish Daddy’s is a chain of seafood restaurants. The company has a limited amount of capital for expansion and must carefully weigh available alternatives. Currently, the company is considering opening restaurants in Baltimore, Maryland, or Sacramento, California. Projections for these two potential outlets under two possible scenarios (A and B), their contributions to profit and their respective probabilities are given as follows:

<table>
<thead>
<tr>
<th>City</th>
<th>Scenario Contribution</th>
<th>Annual Profit (( \pi ))</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baltimore</td>
<td>A</td>
<td>$100,000</td>
<td>0.4</td>
</tr>
<tr>
<td>Baltimore</td>
<td>B</td>
<td>$200,000</td>
<td>0.6</td>
</tr>
<tr>
<td>Sacramento</td>
<td>A</td>
<td>$80,000</td>
<td>0.7</td>
</tr>
<tr>
<td>Sacramento</td>
<td>B</td>
<td>$300,000</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Calculate the expected value, standard deviation, and coefficient of variation for the annual contribution to profit for each city location. Specify the choice of location given this information based on these criteria. Show all work.

**Expected Value:**

\[
 EV_B = 0.4 \times 100,000 + 0.6 \times 200,000 = 160,000 \\
 EV_S = 0.7 \times 80,000 + 0.3 \times 300,000 = 146,000
\]

Choose Baltimore because \( EV_B > EV_S \)

**Standard Deviation:**

\[
 s_B = \sqrt{\frac{\sigma_B^2}{2}} = \sqrt{0.4 \left[ \frac{100,000 - 160,000}{2} \right]^2 + 0.6 \left[ \frac{200,000 - 160,000}{2} \right]^2} = 48,788.80 \\
 s_S = \sqrt{\frac{\sigma_S^2}{2}} = \sqrt{0.7 \left[ \frac{80,000 - 146,000}{2} \right]^2 + 0.3 \left[ \frac{300,000 - 146,000}{2} \right]^2} = 146,67
\]

Choose Baltimore because \( s_B < s_S \)

**Coefficient of Variation:**

\[
 CV_B = \frac{s_B}{EV_B} = \frac{48,788.80}{160,000} = 0.3062 \\
 CV_S = \frac{s_S}{EV_S} = \frac{146,67}{146,000} = 0.6905
\]

Choose Baltimore because \( CV_B < CV_S \).
3. Tex-Mex, Inc., is a rapidly growing chain of Mexican food restaurants. The company has a limited amount of capital for expansion and must carefully weigh available alternatives. Currently, the company is considering opening restaurants in Santa Fe or Albuquerque, New Mexico. Projections for these two potential outlets are as follows. Each restaurant would require a capital expenditure of $700,000, plus land acquisition costs of $500,000 for Albuquerque and $1 million for Santa Fe. The company uses the 10% yield on risk-less U.S. Treasury bills to calculate the risk-free annual opportunity cost of investment capital.

<table>
<thead>
<tr>
<th>City</th>
<th>Outcome</th>
<th>Annual profit contribution</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Albuquerque</td>
<td>Failure</td>
<td>$100,000</td>
<td>0.5</td>
</tr>
<tr>
<td>Albuquerque</td>
<td>Success</td>
<td>$200,000</td>
<td>0.5</td>
</tr>
<tr>
<td>Santa Fe</td>
<td>Failure</td>
<td>$60,000</td>
<td>0.5</td>
</tr>
<tr>
<td>Santa Fe</td>
<td>Success</td>
<td>$40,000</td>
<td>0.5</td>
</tr>
</tbody>
</table>

(a) Calculate the minimum certainty equivalent adjustment for each restaurant's cash flows that would justify investment in each outlet.

Capital Expenditure Requirements
$1,200,000 for Albuquerque
$1,700,000 for Santa Fe

Opportunity Cost (Risk-Free) of Investment Capital
$120,000
$170,000

EV (Albuquerque) = $152,000; EV (Santa Fe) = $50,000

\[
\alpha = \frac{\text{Certainty Equivalent}}{\text{Expected Value}} = \frac{\$120,000}{\$152,000} = 0.78 \text{ and } \frac{\$170,000}{\$50,000} = 3.4
\]

\( \alpha \) represents the certainty equivalent adjustment factor.

(b) Assume that the management of Tex-Mex is risk averse and uses the certainty equivalent method in decision-making. Which is the more attractive outlet? Why?

risk adverse \( \alpha < 1 \Rightarrow \text{Albuquerque} \)
risk loving \( \alpha_{gr} > 1 \Rightarrow \text{Santa Fe} \)
4. Technomics, Inc. considers profit scenarios for the next fiscal year.

<table>
<thead>
<tr>
<th>Business Conditions</th>
<th>Profit</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excellent</td>
<td>$250,000</td>
<td>0.3</td>
</tr>
<tr>
<td>Conventional (Typical)</td>
<td>$160,000</td>
<td>0.6</td>
</tr>
<tr>
<td>Abysmal</td>
<td>$50,000</td>
<td>0.1</td>
</tr>
</tbody>
</table>

(a) Calculate the expected value of profit for Technomics, Inc. for the next fiscal year.

\[
E[V(\Pi)] = 0.3(\$250,000) + 0.6(\$160,000) + 0.1(\$50,000) = \$176,000
\]

(b) Suppose the utility function for the CFO (Chief Financial Officer) is given by

\[
U(\Pi) = \sqrt{\Pi}
\]

where \( \Pi \) corresponds to profit.

(i) Calculate the MU of profit for this decision-maker.

\[
MU(\Pi) = \frac{1}{2\sqrt{\Pi}}
\]

(ii) Is the CFO a risk taker, risk averter, or neutral to risk?

Because \( MU(\Pi) \downarrow \Pi \to \infty \), the CFO is a risk averter.

(iii) If \( U = 3\Pi^2 + 6\Pi \), is the CFO a risk taker, risk averter, or neutral to risk?

\[
MU(\Pi) = 6\Pi + 6 \quad \text{note that } MU(\Pi) \uparrow \Pi \uparrow
\]

Consequently, with this utility function, the CFO is a risk taker.

(c) Calculate the expected value of utility for the CFO when \( U = ln\Pi \). Show all work. Repeat this exercise when \( U = \sqrt{\Pi} \).

(d) What is the risk premium when \( U = ln\Pi \). Show all work. Repeat this exercise when \( U = \sqrt{\Pi} \).
4(c) Expected Value of Utility

<table>
<thead>
<tr>
<th>Business Conditions</th>
<th>Profit</th>
<th>$U = \ln \pi$</th>
<th>$\sqrt{U}$</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excellent</td>
<td>$250,000$</td>
<td>12.429216</td>
<td>500</td>
<td>.3</td>
</tr>
<tr>
<td>Conventional (Typical)</td>
<td>$160,000$</td>
<td>11.982929</td>
<td>400</td>
<td>.6</td>
</tr>
<tr>
<td>Abysmal</td>
<td>$50,000$</td>
<td>10.819778</td>
<td>223.6068</td>
<td>.1</td>
</tr>
</tbody>
</table>

When $U = \ln \pi$

\[
\text{Expected Value of Utility} = \frac{.3}{12.429216} + \frac{.6}{11.982929} + \frac{.1}{10.819778} = 12.0005
\]

When $U = \sqrt{\pi}$

\[
\text{Expected Value of Utility} = .3(500) + .6(400) + .1(223.6068) = 412.36068
\]

4(d) Risk Premium = $\text{EV}(\pi^*) - \pi^*$

$\pi^*$ is the value of profit where either $\ln \pi = 12.0005$ or where $\sqrt{\pi} = 412.36068$

When $U = \ln \pi$, then $\pi^* = \exp(12.0005) = \$162,836.21$

When $U = \sqrt{\pi}$, then $\pi^* = (412.36068)^2 = \$170,041.33$

When $U = \ln \pi$, Risk Premium is $\$176,000 - \$162,836.21 = \$13,163.79$

When $U = \sqrt{\pi}$, Risk Premium is $\$176,000 - \$170,041.33 = \$5,958.67$

It is of interest to note that if $U = \pi^2$ (a utility function for a risk taker)

the Risk Premium can be shown to be $\$176,000 - \$185,364.51 = -\$9,364.51$

Moral of the story for a risk taker, Risk Premium must be < 0.
5. Suppose two projects offer the following payoff matrix:

<table>
<thead>
<tr>
<th>State of Economy</th>
<th>Project A</th>
<th>Project B</th>
<th>Probability of the State of the Economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recession</td>
<td>$4,000</td>
<td>$3,000</td>
<td>0.1</td>
</tr>
<tr>
<td>Normal</td>
<td>$5,000</td>
<td>$6,000</td>
<td>0.7</td>
</tr>
<tr>
<td>Boom</td>
<td>$6,000</td>
<td>$8,000</td>
<td>0.2</td>
</tr>
</tbody>
</table>

(a) What project should be undertaken if a maximin decision rule is used?

\[
\text{Worst outcome for Project A} \rightarrow \$4,000 \\
\text{Worst outcome for Project B} \rightarrow \$3,000 \\
\text{Maximin = "best of worst outcome"} \\
\text{Choose Project A}
\]

(b) What project should be undertaken if a minimax regret decision rule is used?

Your answer must provide the opportunity loss or regret matrix.

\[
\begin{array}{ccc}
\text{Loss} & \text{Project A} & \text{Project B} \\
\text{Recession} & 0 & \$1,000 & \text{minimax regret} \\
\text{Normal} & \$1,000 & 0 & \\
\text{Boom} & \$2,000 & \$1,000 & \\
\text{maximin regret} & \$2,000 & \$1,000 & \\
\end{array}
\]

(c) Would your answer to (a) or (b) change if the probabilities associated with the state of the economy were to change to 0.2 (recession), 0.5 (normal), and 0.3 (boom) respectively?

\[\text{NO}\]

Maximum and minimax regret decision rules do not use probabilities