A class of differential demand systems

Pedro Duarte Neves*

Universidade Católica Portuguesa, Palma de Cima, 1600 Lisboa, Portugal

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Abstract
The Rotterdam, Almost Ideal Demand System, CBS, and NBR models constitute a class of differential demand systems. They correspond to alternative assumptions made concerning the constancy of certain parameters. This paper discusses the restrictions that they impose on the evolution of demand elasticities over time.

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1. Different parameterizations of the budget differential equation

The basic equations of the Rotterdam model [Theil (1965)] read as

\[ w_i \, d \ln q_i = \mu_i (d \ln x - d \ln P) + \sum_j \pi_{ij} \, d \ln p_j, \quad i = 1, \ldots, n, \]

in which the summation is taken over all \( n \) goods and where \( w_i \) is the budget share of commodity \( i \), \( q_i \) is the quantity of good \( i \), \( p_i \) is the unit price of good \( i \), \( x \) is total expenditure, and \( P \) is the Divisia price index, implicitly defined as \( d \ln P = \sum_j w_j \, d \ln p_j \). The parameter \( \mu_i \) is the (constant) marginal budget share of good \( i \); negativity of the substitution effect imposes \( \pi_{ii} < 0 \). The left-hand side of (1) corresponds to the quantity component of the change in the budget share. The total differential of the budget share \( w_i \) is given by

\[ dw_i = w_i \, d \ln q_i + w_i \, d \ln p_i - w_i \, d \ln x. \]

Defining the parameters \( b_i = \mu_i - \pi_i \), \( w_i \) and \( l_{ij} = \pi_{ij} \), \( w_i w_i + w_j \delta_{ij} \), \( \delta_{ij} \) being the Kronecker delta which is unity if \( i = j \) and zero otherwise, one is able to derive from (1) and (2) a simplified first-differences version of the Almost Ideal (AI) Demand System [Deaton and Muellbauer (1980)]:

\[ dw_i = b_i (d \ln x - d \ln P) + \sum_j l_{ij} \, d \ln p_j, \quad i = 1, \ldots, n. \]

The AI parameterization imposes that the difference between the marginal and the actual budget shares remains constant over the sample period. Homothetic preferences correspond to \( b_i = 0 \) for each \( i \). Negativity of the substitution effect does not necessarily impose negativity of \( l_{ij} \).

Expressions (1) and (3) correspond to different parameterizations of the budget differential

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equation, where ‘parameterization’ refers to the assumptions made concerning the constancy of the relevant parameters. In this way, the parameters \( \mu_i \) and \( \pi_{ij} \) are treated as constants in the Rotterdam model, in the same way as the parameters \( b_i \) and \( l_{ij} \) are taken to be constant in the AI model.

Selecting the Rotterdam parameters \( \pi_{ij} \) and the AI parameters \( b_i \), Keller and van Driel (1985) proposed the CBS system:

\[
 w_i (d \ln q_i - d \ln Q) = b_i (d \ln x - d \ln P) + \sum_j \pi_{ij} d \ln p_j , \quad i = 1, \ldots, n ,
\]

where \( d \ln Q = d \ln x - d \ln P \) is a quantity index. The left-hand side of (4) corresponds to the weighted change in the volume share \( q_i/Q \). The CBS system shares income coefficients with the AI model and price coefficients with the Rotterdam model.

Finally, Neves (1987) proposed the NBR system:

\[
 w_i (d \ln q_i + d \ln p_i - d \ln P) = \mu_i (d \ln x - d \ln P) + \sum_j l_{ij} d \ln p_j , \quad i = 1, \ldots, n ,
\]

treating \( \mu_i \) and \( l_{ij} \) as constants. This system has the Rotterdam income coefficients and the AI price coefficients. The left-hand side corresponds to the weighted change in real expenditure on commodity \( i \).

Expressions (1), (3), (4) and (5) correspond to alternative parameterizations of the budget share differentials. The right-hand sides of the four demand systems contain the same variables. However, since the left-hand sides are different, the interpretation of the associated parameters is not the same.

2. Implications of the alternative models

Table 1 shows the complete set of elasticities for each model. Since the observed shares vary over the sample period, it is possible to obtain estimates of these elasticities for each time unit. It is now important to analyse to what extent the evolution of these elasticities over time is constrained by the alternative constancy assumptions.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Demand elasticities</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model</strong></td>
<td>( \eta_i )</td>
</tr>
<tr>
<td>Rotterdam</td>
<td>( \mu_i/w_i )</td>
</tr>
<tr>
<td>AI</td>
<td>( 1 + b_i/w_i )</td>
</tr>
<tr>
<td>CBS</td>
<td>( 1 + b_i/w_i )</td>
</tr>
<tr>
<td>NBR</td>
<td>( \mu_i/w_i )</td>
</tr>
</tbody>
</table>
2.1. Income elasticities ($\eta_i$)

The Rotterdam and the NBR models imply a constant marginal propensity to spend on each commodity. This assumption corresponds to linear Engel curves. The sign of the income elasticity, $\eta_i$, is determined by that of $\mu_i$. If the estimated $\mu_i$ is negative, $i$ is an inferior good ($\eta_i < 0$) over all the sample period. If the estimated $\mu_i$ is positive, $i$ is allowed to change from a luxury ($\eta_i > 1$) into a necessity ($0 < \eta_i < 1$), depending on the values of the observed shares.

The AI and CBS models impose constancy of the parameter $b_i$, implying Engel curves of the PIGLOG type. The sign of $b_i$ determines whether $\eta_i$ is larger than one (luxury) or not (necessity or inferior good). Both constancy assumptions appear to be too restrictive. However, with the usual level of aggregation of goods in empirical work, inferiority is rarely observed, reducing the practical importance of the limitation of a constant $\mu_i$ in the Rotterdam model and in the NBR model. From this point of view, the assumption of a constant $\mu_i$ seems to be less adequate, since commodity $i$ is not allowed to change from a luxury into a necessity or vice versa.

2.2. Compensated cross-price elasticities ($\varepsilon_{ij}^*, i \neq j$)

The Rotterdam and the CBS models share the (constant) price coefficients $\pi_{ij}$. The sign of this parameter determines if goods $i$ and $j$ are net complements ($\pi_{ij} < 0$) or net substitutes ($\pi_{ij} > 0$). This is somewhat too restrictive as one would like a specification that does not impose the same Hicksian classification between two commodities over all the sample period. As Table 1 shows, the assumption of constant $l_{ij}$'s is not so restrictive since the sign of the compensated cross-price elasticity, $\varepsilon_{ij}^*$, is determined by the sign of $l_{ij} + w_i/w_j$ and this sign is allowed to vary over the sample period. In this way, a pair of goods can turn from (Hicksian) complements into (Hicksian) substitutes. From this point of view, the AI and the NBR models should be preferred to the Rotterdam and CBS models.

2.3. Own-price elasticities ($\varepsilon_{ii}$ and $\varepsilon_{ii}^*$)

A final issue concerns the evolution of own-price elasticities over time. Take the case of a commodity with an increasing budget share over the sample period. If this turns out to be the case, the parameterization of the Rotterdam model implies that the uncompensated, $\varepsilon_{ii}$, and the compensated, $\varepsilon_{ii}^*$, own-price elasticities are forced to increase over time. In effect, it is easy to prove that $\partial \varepsilon_{ii}/\partial w_i > 0$ and that $\partial \varepsilon_{ii}^*/\partial w_i > 0$. The same is true for the own-price compensated elasticity of the CBS model.

This is a direct consequence of the constancy assumptions of the model and does not necessarily reflect in an adequate way the observed behaviour. In effect, unless we have grounds for believing that increasing budget shares imply less elastic (more rigid) demands, then we have good reasons for considering the Rotterdam and the CBS parameterizations as too restrictive. From this point of view, the assumption of a constant $l_{ii}$ is more plausible and constitutes an advantage of the AI and the NBR models. The sign of $\partial \varepsilon_{ii}/\partial w_i$ is determined by the sign of (minus) $l_{ii}$ in the AI model and by the sign of $(w_i^2 - l_{ii})$ in the NBR model. In this way, an increasing budget share over the sample period does not necessarily imply increasing own-price elasticities.

3. Conclusions

Differential demand systems are commonly used in macroeconometric model-building. For the purpose of simulation of consumer behaviour, one would like to specify approximations to the
unknown demand functions that do not condition a priori the set of demand elasticities. Therefore, it is crucial to be aware of the type of restrictions imposed on price and income elasticities by some of the most popular demand systems used in the empirical analysis of consumer behaviour.

The Rotterdam, AI, CBS, and NBR models constitute alternative parameterizations of the budget differential equation. They correspond to alternative assumptions made concerning the constancy of certain parameters. These assumptions tend to impose unwanted restrictions on the description of consumer behaviour. As far as the evolution of elasticities over time is concerned, the NBR model seems to be the least restrictive parameterization.

References