An Approach to Specifying and Estimating Nonreversible Functions

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An Approach to Specifying and Estimating Nonreversible Functions

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This note concerns the specification and estimation of nonreversible linear functions in economic research. Commonly suggested applications involve short- or long-run supply curves that, because of asset fixity or other rigidities, are more output responsive for price increases than for price decreases. The theoretical reasoning behind such functions is not at issue here, only the empirical issues likely to arise.

In 1971, Wolffram proposed an ingenious method of dealing with nonreversibilities through partitioning (or segmenting) the variables involved. Tweeten and Quance further discussed the idea but generally endorsed the Wolffram method. Interested readers are referred to those papers, since their essentials are not repeated. In what follows, a rather simple alternative approach for specifying linear nonreversibilities is suggested. This method is consistent with the Wolffram technique but is operationally clearer. In addition, some results applying this technique in two empirical supply studies are discussed briefly to provide insight into some of the issues likely to arise.

**Specification**

Since nonreversibilities are most commonly expressed in terms of asymmetrical changes from a previous position in time (or possibly space), identification of the starting point or initial observation is central to the analysis. Yet this first observation has no independent explanatory power, since the differential effects to be measured hinge on changes from the previous position, not its level. This key point was not emphasized by either Wolffram or Tweeten and Quance.

Imagine that the variable $Y$ depends upon the values taken by $X$ and that both are time-series variables. The hypothesis to be examined is that one-unit increases in $X$ from period to period have a different absolute impact on $Y$ than do one-unit decreases in $X$. Such a relationship can be written

$$\Delta Y_i = a_o + a_1 \Delta X_i + a_2 \Delta X_i^2$$

for $i = 1, 2, \ldots, T$, where $\Delta Y_i = Y_i - Y_{i-1}$; $\Delta X_i = X_i - X_{i-1}$ if $X_i > X_{i-1}$ and $= 0$ otherwise; $\Delta X_i^2 = X_i - X_{i-1}$ if $X_i < X_{i-1}$ and $= 0$ otherwise; $X_o$ is the initial value of $X$; and $Y_o$ is the initial value of $Y$.

Other variables, segmented or not, could be added to this basic specification, and $a_o$ might be zero, positive, or negative. A nonreversibility occurs in $\Delta Y$ if $a_i \neq a_o$. To link equation (1) to the initial position, note that the value of $Y$ at any point $t$ is

$$Y_t = Y_o + \sum_{i=1}^{t} \Delta Y_i,$$

for $i = 1, 2, \ldots, t, t + 1, \ldots, T$, where $T$ is the total number of observations beyond the initial value. The difference between the current and the initial value of $Y$ is the sum of the period-to-period changes that have occurred. So,

$$Y_t - Y_o = \sum_{i=1}^{T} \Delta Y_i.$$

Inserting equation (1) into equation (3) and simplifying,

$$Y_t - Y_o = a_o t + a_1 (\Sigma \Delta X_i) + a_2 (\Sigma \Delta X_i^2).$$

Letting $Y_i^o, R_i^o$, and $D_i^o$ equal $Y_t - Y_o, \Sigma \Delta X_i$, and $\Sigma \Delta X_i^2$, respectively,

$$Y_i^o = a_o t + a_1 R_i^o + a_2 D_i^o,$$

where $R_i^o$ is the sum of all period-to-period increases in $X$ from its initial value up to period $t$, and $D_i^o$ is the similar sum of all period-to-period decreases in $X$. The variable $R_i^o$ is always positive, and $D_i^o$ is always negative. If $a_o$ is not zero, it appears in equation (5) as a trend coefficient. If any other variables affect $\Delta Y$ in equation (1), they also would be entered in equation (5) as deviations from their initial values.

The computation of $R_i^o$ and $D_i^o$ for a hypothetical set of ten observations on a variable $X$ are illustrated in Table 1. The hypothetical data are the same as used by Wolffram. Interested readers will note that both methods of segmenting the variable $X$ are basically comparable. However, the method proposed here flows directly from considering period-to-period changes and does not require, as does the Wolffram method, an implicit change in the sign of the $a_o$ coefficient in order to compare it with $a_1$. If a positive (negative) net relation exists between $Y$ and $X$, both $a_1$ and $a_2$ are positive (negative) in the formulation proposed here using $R_i^o$ and $D_i^o$. In addition, expressing all variables as deviations from
initial values eliminates the problem of how to deal with the equation intercept, a question of some complexity when actual values of the variables are employed (Wolffram, Tweeten and Quance).

Thus, equation (5) is a simple estimating function for a nonreversibility to which other variables and stochastic properties can be added. Of special interest would be the test for \( a_1 = a_2 \) to confirm or refute the hypothesis of nonreversibility. By extension, other functional forms that can be linearized for estimation (such as logarithmic, semilogarithmic, etc.) also can be adapted to nonreversibility analysis using this basic framework and this technique of partitioning variables.

### Empirical Applications

The hypothesis of nonreversibility in supply response to price change was tested using previously estimated supply equations for two quite different U.S. farm commodities: milk and pinto beans. Only those results that pertain to the nonreversibility question are discussed. A complete presentation of the two supply studies would far exceed the objectives of this note.

A fully reversible milk supply function was estimated by OLS using annual data from the 1947–72 period. The estimated function displays significant (5.0% level or stronger) coefficients with expected signs on lagged milk prices received by farmers, lagged sugar beet prices, weather proxies, and a measure of price instability for the previous three years. Reversible equations with various combinations and forms of these variables display \( R^2 \) values from 0.90 to 0.94. Individual regression coefficients are significant at beyond the 5% level, and expected signs emerge on all variables. The one-year lagged price response has \( t \)-values of 5.79 to 7.41, depending on the particular specification used. At the point of means, the estimated price elasticity is between 0.99 and 1.02.

As with the milk supply analysis, the data are transformed into deviations from initial values (1953), and the lagged pinto bean price is segmented. In none of the various specifications did statistically significant evidence of nonreversibility emerge. The estimated coefficients are very similar to one another for either rising or falling prices, and both have \( t \)-values between 4.0 and 6.0. The estimated price elasticities at the means are very close to 1.0 for both rising and falling prices. Moreover, the segmentation does not appreciably alter estimated coefficients on the other variables.

While certainly not conclusive, these two illustrations suggest that hypothesis of nonreversibility is tenable in agricultural supply work. Even in cases where the hypothesis is not supported, the results for other explanatory variables may not be substantially altered when data are segmented and transformed to explore nonreversibility.

In using the techniques suggested here, the investigator should remain aware of a few problems that might emerge. First, the segmentation and data transformation consume two degrees of freedom: one for the added price variable and one for the loss of explanatory power in the initial observation. Second, intercorrelations among explanatory variables might be intensified. When a variable is segmented into increasing and decreasing components, it is possible that the two segments will be highly correlated with each other or with other trend-like variables in the analysis. On the other hand, if the variable to be segmented (a price, for example) is

### Table 1. Hypothetical Example of Segmented Variable \( X_i \)

<table>
<thead>
<tr>
<th>Variable (( X_i ))</th>
<th>( X' )</th>
<th>( X'' )</th>
<th>( R^* )</th>
<th>( D^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_0 = 10 )</td>
<td>-</td>
<td>-</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>( X_1 = 13 )</td>
<td>3</td>
<td>0</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>( X_2 = 11 )</td>
<td>0</td>
<td>-2</td>
<td>3</td>
<td>-2</td>
</tr>
<tr>
<td>( X_3 = 14 )</td>
<td>3</td>
<td>0</td>
<td>6</td>
<td>-2</td>
</tr>
<tr>
<td>( X_4 = 17 )</td>
<td>3</td>
<td>0</td>
<td>9</td>
<td>-2</td>
</tr>
<tr>
<td>( X_5 = 12 )</td>
<td>0</td>
<td>-5</td>
<td>9</td>
<td>-7</td>
</tr>
<tr>
<td>( X_6 = 9 )</td>
<td>0</td>
<td>-3</td>
<td>9</td>
<td>-10</td>
</tr>
<tr>
<td>( X_7 = 16 )</td>
<td>7</td>
<td>0</td>
<td>16</td>
<td>-10</td>
</tr>
<tr>
<td>( X_8 = 20 )</td>
<td>4</td>
<td>0</td>
<td>20</td>
<td>-10</td>
</tr>
<tr>
<td>( X_9 = 18 )</td>
<td>0</td>
<td>-2</td>
<td>20</td>
<td>-12</td>
</tr>
</tbody>
</table>
already highly correlated with other variables (other prices, for example), then segmentation may well reduce the intercorrelation problem. It is impossible to say in advance which tendency, if either, will prevail.

In both the milk and pinto bean supply studies, some intercorrelations were increased between the price segments and other variables as compared with those using actual prices. But because of the high overall $R^2$'s, no real estimation problems emerged. Similarly, no estimation problems occurred in these two examples because of strong correlation between the two segments; the correlation coefficients $r$ were $-0.78$ and $-0.87$ between the segments of milk and pinto bean prices, respectively. The segmentation process did not alleviate any high intercorrelations between the actual prices and other variables, since none were present initially. On balance, each of these commodities provided a favorable environment for an empirical test of nonreversibility in supply response. Other investigations surely might be less fortunate.

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References


