EXACT AGGREGATION AND A REPRESENTATIVE CONSUMER*

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I. INTRODUCTION

This paper discusses macroeconomic demands in economies comprised of individuals who each possess utility-derived demand systems of the form,

\[ q^i = a^i(P) + b^i(P)x + c^i(P)g(x,P,\eta), \]

where \( q^i \) is an individual's demand for good \( i \), \( P \) is the vector of prices \( (p^1, \ldots, p^n) \), \( x \) is an individual's total expenditures (income for short), \( \eta \) is a taste parameter that may vary across individuals, \( g \) is any function of \( x, P, \) and \( \eta \), and \( a^i, b^i, \) and \( c^i \) are any functions of prices (indices are superscripts, because subscripts will denote differentiation). Individuals here can represent groups like households or communities, as long as they jointly maximize utility.

Equation (1) is the most general form of a utility-derived demand system having Engel curves that are linear in income and one function of income. Such Engel curves are convenient for empirical work because they can be estimated using ordinary least squares, have the ordinary linear model nested within them, and are parsimonious (see, e.g., Leser [1963]).

Equation (1) demands are also convenient for aggregation, because all nonlinear effects are embodied in the single term \( g \). Virtually all empirical studies of aggregate data that have explicitly considered both aggregation and individual utility maximization have employed specifications that are special cases of equation (1). These include all linear Engel curve models; Deaton and Muellbauer's [1980] AIDS model; Jorgenson, Lau, and Stoker's [1982] TRANSLOG models; the models of Berndt, Darrough, and Diewert [1977]; and the polynomial IAGL models estimated by Lewbel [1988]. Equation (1) encompasses all Homothetic, Quasi-homothetic, PIGL, PIGLOG, and QES models, among others, as special cases. All of these cited models are in fact simpler than equation (1) in that they do not have \( g \) depending on \( P \).

Aggregate, or per capita, demands in the economy are derived by averaging equation (1) across individuals. This paper describes

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the ways in which these aggregate models resemble representative consumer models. Since the assumption of a representative consumer is commonly employed in macroeconomic work, it is useful to know how demands that arise from aggregation of individuals differ from those that arise from the maximization of a representative consumer's utility function.

Implicitly define a function $\mu$ as the solution to the equation,

$$g(X, P, \mu) = E(g(x, P, \eta)),$$

where $E(\cdot)$ denotes taking the mean across all individuals in the economy, and $X = E(x)$ is mean (per capita) income in the economy. Differentiability and local monotonicity of $g$ with respect to $\eta$ is sufficient to guarantee existence of $\mu$. Notice that $\mu$ depends on $P$ and on the distributions of $x$ and $\eta$ across all individuals in the economy. Let $Q^i = E(q^i)$; i.e., $Q^i$ is mean (per capita) demand for good $i$ in the economy. Averaging equation (1) across consumers and applying equation (2) gives

$$Q^i = a^i(P) + b^i(P)X + c^i(P)g(X, P, \mu),$$

which closely resembles equation (1). In fact, if $u_n(q^1, \ldots, q^n)$ is the utility function of an individual (having tastes indicated by $\eta$) leading to equation (1), then per capita demands in the economy will equal those arising from the maximization of $u_n(Q^1, \ldots, Q^n)$, subject to the economy-wide budget constraint $\Sigma_i P_i Q^i = X$. Therefore, the aggregate demands in the economy equal those of a representative consumer having a taste parameter equal to $\mu$.

Just as $\eta$ may parameterize how a household's utility depends on the distribution of members of the individual household, $\mu$ can be interpreted as parameterizing how the representative consumer's utility depends on the distribution of all the members of the economy.

This representative consumer is a purely mathematical result and need not have economic content. For example, there is no reason to equate the representative consumer's utility level with any measure of social welfare.\textsuperscript{1} Nevertheless, having aggregate demands equal those arising from utility maximization is a convenience for both empirical and theoretical macro models. This paper provides a large class of nonlinear models in which the popular macro assumption of a representative consumer can be formally rationalized.

The representative taste parameter construction of equations

\textsuperscript{1} In particular, it is not assumed here that society is maximizing any kind of social welfare function.
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(2) and (3) is analogous to Muellbauer's [1975, 1976] idea of a representative income level, but has a number of advantages over Muellbauer's proposal. In particular, the representative taste parameter model yields aggregate demand equations based on actual mean income. Also, the class of models given by equation (1) is larger than the class to which Muellbauer's construction can be applied.

A difficulty with the representative consumer interpretation of equation (3) is that \( \mu \) depends on \( P \) and \( X \) in general, and so may not be legitimately included in the utility function. When individual demands are nonlinear in \( x \), aggregate demands depend on the distribution of \( x \) across individuals. In the models considered here, \( Q_i \) depends on the distribution through \( \mu \). Having \( \mu \) independent of \( P \) and \( X \) will require an assumption concerning the distribution of \( x \). The assumption that will be employed is "mean scaling," a generalization of proportional distribution movement.

The next section contains some results regarding equation (1) and utility theory. Section III discusses the representative taste parameter \( \mu \), and shows when it will not depend on \( P \) and \( X \). Section IV considers the estimation of aggregate models arising from equations (1) and (2), and Section V concludes.

II. SOME UTILITY FUNCTIONS AND CHARACTERIZATIONS

Let \( u = v(x,P) \) be the consumer's indirect utility function. In all that follows, superscripts will index functions, except that the superscript \( k \) will be an exponent. Subscripts \( i, j, \) and \( n \) will denote derivatives with respect to prices (e.g., \( A_j \) means \( \partial A(P)/\partial P^j \), while \( a^i \) would be any function relating to good \( i \)). The derivative of a function of one variable with respect to that variable is denoted by the usual prime.

A demand system is defined in the usual way. Demand equations are derivable (via Roy's identity) from some twice differentiable, homogeneous of degree zero indirect utility function \( v(x,P) \). Extending earlier work by Gorman [1961, 1981], Muellbauer [1975], and Howe, Pollak, and Wales [1979], Lewbel [1987a] proves the following theorem characterizing all demand systems of the form,

\[
q^i = a^i(P) + b^i(P)x + c^i(P)f(x).
\]

THEOREM 1 (A TAXONOMY OF DEMAND SYSTEMS). Assume that demand equations are of the indented form of equation (4) for \( i = 1, \ldots, n \), where \( a^i, b^i, \) and \( c^i \) are arbitrary differentiable functions of \( P \) and \( f \) is an arbitrary differentiable function of \( x \).
Then one of the following eight cases must hold:

Case i: Homothetic demands: $q_i = (B_i/B)x$ and $u = s(x/B)$.

Case ii: Quasi-homothetic demands: $q_i = BA_i + (B_i/B)x$ and $u = s((x/B) - A)$.

Case iii: PIGL demands: $q_i = (B_i/B)x + B^{1-k}C_i x^k$ and $u = s(((x/B)^{1-k}/(1-k)) - C)$.

Case iv: PIGLOG demands: $q_i = ((B_i/B) - C_i \log B)x + C_i \log x$ and $u = s(-((x/B) - A)^{-1} - C)$.

Case v: QUADRATIC (QES) demands: $q_i = (BA_i + (A^2C_i/B) + ((B_i - 2AC_i)/B)x + (C_i/B)x^2 and u = s(\log((x/B) - A) - 1 - C)$.

Case vi: Extended PIGL demands: $q_i = BC_i + (B_i/B)x + h(C)B^{1-k}C_i x^k$ and, when $h(C)$ equals a constant $\lambda$, $u = s(\nu(x/B) - C)$, where $\nu(z) = \int^z (dz/(1 + \lambda z^k))$.

Case vii: Extended PIGLOG demands: $q_i = h(C)BC_i + ((B_i/B) - C_i \log B)x + C_i \log x$ and, when $h(C)$ equals a constant $\lambda$, $u = s(\nu(x/C)$, where $\nu(z) = \int^z (dz/(\lambda + z \log z))$.

Case viii: LINLOG demands: $q_i = (h(C) - \log B)BC_i + (B_i/B)x + BC_i \log x$ and, when $h(C)$ equals a constant $\lambda$, $u = s(\nu(x/B) - C)$, where $\nu(z) = \int^z (dz/(\lambda + \log z))$.

In all these cases, $A$ and $C$ are any twice differentiable, homogeneous of degree zero functions of prices; $B$ is any twice differentiable, homogeneous of degree one function of prices; $h$ is any differentiable function; $s$ is any monotonic function; $h$ is any constant except zero or one; and $\lambda$ is any constant except zero.

In the above models, having $h(C)$ not be constant complicates the demand equations while adding nothing to either income or price flexibility, so demands with $h(C) = \lambda$ are not likely to be of much practical interest. I shall therefore sometimes restrict attention to the case where $h(C) = \lambda$, an arbitrary constant. The assumption $h(C) = \lambda$ has no effect on cases i through v. To be completely consistent with utility maximization, any particular model in Theorem 1 must have monotonicity and concavity imposed on it.

The following corollary can be proved by inspection of Theorem 1.

**Corollary.** Assuming that $h(C) = \lambda$, all demands in the form of equation (4) have indirect utility functions of the canonical form,

\[ u = v((x/B) - A) - C, \]
where \( \psi \) is some differentiable function that may also depend on \( \lambda \), and \( A, B, \) and \( C \) are as in Theorem 1 (\( A \) or \( C \) or both can be zero). Also, \( A \) can be taken to be zero in all cases except the QES.\(^2\)

For any function \( \psi \) the indirect utility function in equation (5) yields

\[
q^i = BA_i + (B_i / B)x + BC_i(1/\psi'),
\]

and thereby provides a convenient way of constructing demand systems in the form of equation (1), where \( g = 1/\psi' \). In fact, this gives a way of constructing demands in the form of equation (1), where \( g \) is any function of \( (x/B) - A \) for any functions \( A(P) \) and \( B(P) \) having appropriate homogeneity. Since \( \psi \) can be any function, this allows for an unlimited variety of Engel curve shapes within the basic framework permitted in equation (1).

III. AGGREGATION BY REPRESENTATIVE TASTE PARAMETERS

In this section conditions are described that make the representative taste parameter \( \mu \) independent of \( P \) and \( X \). By inspection of equation (2), \( \mu \) is independent of prices if \( g \) is independent of prices. Theorem 1 gives every possible demand system for which \( g \) is independent of prices, so for the remainder of this section we can restrict attention to Theorem 1-type demand equations, where \( \eta \) is introduced into the function \( f \) in some way.

Define \( \theta^s \) as the proportion of individuals in the population having income level \( x = sX \); let \( S \) = the set of all values of \( s = x/X \) that are represented in the population (i.e., values of \( s \) for which \( \theta^s \neq 0 \)); and let \( \theta \) be set of all \( \theta^s \) for \( s \in S \). Any discrete distribution can be parameterized in this way. However, a distribution will be defined to be "mean scaled" if, when parameterized in this way, \( \theta \) does not depend on \( X \).\(^3\) Lewbel [1986, 1988] discusses mean scaling in depth. Some results concerning mean scaling from these sources are listed in the Appendix.

The following theorems show how mean scaling can yield

\(^2\) When \( A = 0 \), equation (5) is what Lewbel [1988] calls IAGL utility. IAGL demands correspond to Gorman Polar form subutility [Gorman, 1959] and so can be interpreted as the second stage of a two-stage budgeting process.

\(^3\) Define \( \Phi(x/X, \theta) \) be the distribution function of \( x \), parameterized in terms of \( X \) and \( \theta \). Here \( \Phi(x/X, \theta) \) equals the sum of \( \theta_s \) over all \( s \in S \), \( s < (x/X) \). An equivalent definition of mean scaling would be that \( \theta \) be independent of \( X \) when \( \Phi \) is parameterized such that \( \Phi(x/X, \theta) = \Phi(rx/rX, \theta) \) for any positive constant \( r \). A continuous version of mean scaling can be based on this latter definition.
aggregate demands in an economy that are identical to the demands of a representative agent.

**Theorem 2.** Assume that all individuals in the economy have demands given by any of the cases in Theorem 1 except the QES. Define \( \omega \) by \( \omega(\theta) = \Sigma_s \theta^s f(s) \). There exists a direct utility function \( u_\omega \) (\( u \) depends on the value of \( \omega \)) such that aggregate demands in the economy equal those arising from the maximization of \( u_\omega(Q^1, \ldots, Q^n) \), subject to the budget constraint \( \Sigma_i P_i Q_i = X \).

**Corollary.** Make the same assumptions as in Theorem 2. Then mean scaling is sufficient to yield a representative consumer whose direct utility function \( u_\omega \) does not depend on \( P \) and \( X \). Mean scaling is also necessary, in the sense that if any \( \theta \) does depend on \( X \), there will exist a Theorem 1, non-QES demand system for which \( \omega(\theta) \), and hence \( u_\omega \), depends on \( X \).

**Proof.** The proof of Theorem 2 follows directly from that of Theorem 3, and so is deferred until then. Sufficiency of the Corollary is obvious from Theorem 2. For necessity, suppose that \( \omega(\theta) \) does not depend on \( X \) for all the demand systems. Define \( \omega_k \) by \( \omega_k = \Sigma_s \theta^s s^k \), corresponding to \( \omega \) for PIGL or Extended PIGL-type demands. By selecting a large enough assortment of values for \( k \), we can solve for any \( \theta^s \) in terms of a set of \( \omega_k \) and \( s^k \) terms. Since these terms do not depend on \( X \) by assumption, neither does any \( \theta^s \).

**Theorem 3.** Assume that the distribution of \( x \) is mean scaled. Let \( H = E(\eta) \), and assume that the distributions of \( \eta \) and \( x \) are independent. For each demand system in Theorem 1 except the QES, there exists a way to introduce the taste parameter \( \eta \) into the demand system of each individual in the economy such that \( y_i \), defined by equation (2), depends only on \( H \) and \( \theta \).

**Proof.** For cases i and ii in Theorem 1, \( f \) drops out, and aggregation is automatic. In the remaining cases each individual's indirect utility function is given by \( u = u(x/B, \eta) + C \) for some function \( u \).

a. For cases iii and vi let \( f = \eta x^k \), where \( \eta \) equals any constant except zero. Individual demands can then be written as

\[
q_i = (B_i/B)x + BC_i(1 + h(C)\eta(x/B)^k).
\]

4. The number of positive \( \theta^s \) terms is countable, while the number of values \( k \) can take on is uncountable. A continuous distribution analogue of this theorem is possible, in which the definition of \( \omega_k \) is a uniquely invertible Mellin transform. See Lewbel [1986] for a similar result.
When \( h(C) = \ell \), individual indirect utility functions have \( v(z, \eta) = \int (dz/(\ell + \kappa z \eta^k)) \).

b. For cases iv and vii let \( f = \eta x + x \log x \), where \( \eta \) equals any positive constant. Individual demands can then be written as

\[
q^i = (B_i/B)x + BC_i(h(C) + (x/B)(-\eta + \log (x/B))).
\]

When \( h(C) = \ell \), individual indirect utility functions have \( v(z, \eta) = \int (dz/(\ell + z \eta + z \log z)) \).

c. For case viii let \( f = \eta + \log x \), where \( \eta \) equals any positive constant. Individual demands can then be written as

\[
q^i = (B_i/B)x + BC_i(h(C) + \eta + \log (x/B)).
\]

In each of the above cases aggregate demands have the same functional form and utility function as the corresponding individual demands, except that \( q^i, x, \text{ and } \eta \) are replaced with \( Q^i, X, \text{ and } \mu \), respectively.

It must be verified that the above demand systems are consistent with utility maximization (i.e., that they satisfy Slutsky symmetry, etc.) and that for some value of \( \eta \) they are equivalent to equation (4) demands. Consistency with utility maximization is straightforward to check. Case a above reduces to the corresponding equation (4) demands when \( \eta = 1 \), and the other cases reduce for \( \eta = 0 \). For these values of \( \eta \) Theorem 2 follows as a corollary to this theorem.

Finally, it must be shown that for the above demand systems \( \mu \) as defined by equation (2) in fact depends only on \( H \) and \( \theta \). In case a, \( \mu = H \Sigma s \theta^s \). In case b, \( \mu = H + \Sigma s \theta^s \log s \). In case c, \( \mu = H + \Sigma s \theta^s \log s \).

To verify these equations for \( \mu \), we have that in case a, \( E(F) = E(\eta x^k) = HE(x^k) = H \Sigma s \theta^s X^k = \mu X^k \). For case b, first observe that \( \Sigma s \theta^s s = E(x/X) = E(x/X) = X/X = 1 \). Using this fact and the same logic as case a gives \( E(F) = E(x(\eta + \log x)) = HX + E(x \log x) = HX + \Sigma s \theta^s X \log (sX) = HX + (\Sigma s \theta^s s) X \log X + (\Sigma s \theta^s s \log s) X = X(\mu + \log X) \). Case c is \( E(F) = E(\eta + \log x) = H + E(\log x) = H + \Sigma s \theta^s \log (sX) = \mu + \log X \).

In each case in Theorem 3, \( \mu \) depends only on \( H \) and \( \theta \), and is independent of \( X \) and \( P \). If individuals have any utility-derived demands in the form of equation (4), except QES demands, then by the appropriate inclusion of a taste parameter, aggregate demands will be of the same utility-derived functional form, and will have an
aggregate, or representative, taste parameter that does not depend on aggregate income or on prices.

To illustrate Theorems 2 and 3, consider some examples. Deaton and Muellbauer’s [1980] AIDS model is in the PIGLOG class, and has $C = \sum_i \gamma^i \log P^i$ and $B = \beta'^0 + \sum_i \beta'^i \log P^i + (1/2) \sum_i \sum_j \beta'^{ij} \log P^i \log P^j$, where $B$’s and $\gamma$’s are constants. Using Theorem 3 to introduce $\eta$ into PIGLOG demands having this $B$ and $C$ yields the alternative AIDS specification:

\begin{align}
(P^i q^i / x) &= \eta \gamma^i + \beta^i + \sum_j \beta'^{ij} \log P^j + \gamma^i \log (x/B).
\end{align}

The version of the Translog model used by Jorgenson, for aggregation purposes (see, e.g. Jorgenson, Lau, and Stoker [1982]), is also PIGLOG, having $C = - \log (\sum_i (\gamma^i - \sum_j \beta'^{ij} \log P^j))$ and $B = - (\sum_i (\gamma^i \log P^i - (1/2) \sum_j \beta'^{ij} \log P^i \log P^j))/C$ with $\gamma^i = -1$ and $\sum_i \sum_j \beta'^{ij} = 0$. The latter restriction is what reduces the TRANSLOG from a fractional demand system (see Lewbel [1987b]) to a PIGLOG form. The alternative Translog specification is

\begin{align}
\frac{P^i q^i}{x} &= \frac{\gamma^i + (\sum_j \beta'^{ij}) \eta - (\sum_j \beta'^{ij} \log P^j) + (\sum_j \beta'^{ij}) \log x}{\sum_i (\gamma^i - \sum_j \beta'^{ij} \log P^j)}.
\end{align}

Both these alternative systems arise from the indirect utility function $u = v(x/B, \eta) - C$, where $v(z, \eta) = \int (dz/(\eta z + \log z))$. When $\eta = 0$, this reduces to the ordinary AIDS or Translog model, respectively. Applying Theorem 3, we can estimate equations (10) or (11) with aggregate data by replacing $q^i$ with $Q^i$, $x$ with $X$, and $\eta$ with $\mu = H + \sum \theta^i s \log s$. The aggregate taste parameter $\mu$, which can be estimated using distribution data, is independent of $P$ by construction. If the distribution of $x$ is mean scaled, then $\mu$ is independent of $X$ as well. The resulting aggregate model is the demand system that arises from the indirect utility function $u = v(X/B, \mu) - C$.

Another example is $q^i = BC_i + (B_i/B)x + (C_i/B) \eta x^2$, where $B$ and $C$ are any functions with the appropriate homogeneity. This is a demand system that is quadratic in $x$, but is not in the general QES form. It is an Extended PIGL model with $k = 2$ and $h(C) - \lambda$ absorbed into $\eta$. It is also an example of Lewbel’s [1988] Polynomial IAGL demand system. By Theorem 3 the representative taste parameter in the aggregate form of this model will have the simple form $\mu = (1 + (1/\rho^2)) H$, where $\rho$ is the coefficient of variation of the $x$ distribution.
IV. INFORMATION REQUIREMENTS OF AGGREGATE MODELS

All the aggregation models estimated by Berndt, Darrough, and Diewert [1977] are in the form of equation (4), as is Deaton and Muellbauer's [1980] AIDS model and Jorgenson, Lau, and Stoker's [1982] Translog aggregation model, although their model includes the addition of Barten equivalence scales. In all these studies the authors use data on the income distribution over time to estimate $E(f)$. An alternative (though in some cases observationally equivalent) technique for each of these models would be to use the same distribution data to estimate $\mu$ and then use $X, P$, and $\mu$ to estimate the corresponding representative consumer model. Lewbel [1988] does this for a class of models similar to case vi of Theorem 1.

A problem that all these aggregate nonlinear models face is that they require distribution data on $x$ in every time period. One solution is to use representative taste parameters, and assume that the distribution of individuals is such that $\mu$ is constant over time. An alternative solution to the problem of aggregating nonlinear $x$ functions is provided by Theorem 4.

In addition to incorporating estimates of $E(f)$ in every period, Jorgenson, Lau, and Stoker [1982] added the sophistication of using data from a single cross-section survey to efficiently estimate the equivalence scales in Engel curves. Given data from a single cross section along with time series observations on $Q^i, P$, and $X$, it is possible to estimate aggregate models when individuals have any equation (1) type demands, using the following theorem.

THEOREM 4. Assume that individual demands are in the form of equation (1). Then, for $i = 2, \ldots, n$, aggregate demands satisfy the relationship,

$$Q^i = \left(\frac{c^i}{c^n}\right) Q_n + a^i - \left(\frac{c^i}{c^n}\right) a^n + \left(b^i - \left(\frac{c^i}{c^n}\right) b^n\right) X.$$  

Proof. By equation (1), $q^n - a^n - b^n x - c^n g = 0$. Multiply this expression by $c^i/c^n$, and subtract it from equation (1) to get $q^i = (c^i/c^n) q_n + a^i - (c^i/c^n) a^n + (b^i - (c^i/c^n) b^n) x$. This expression is linear in variables that vary across households, that is, $q^i, q^n, X$.

5. Many studies, e.g., Hildenbrand and Hildenbrand [1985], show that the shape of the income and total consumption distributions have remained quite constant over time. Lewbel [1988] finds little variation in empirical estimates of $\mu$ over time. Also, if the $x$ distribution is lognormal, it can be mean scaled with $\theta$ being the gini coefficient, which, in industrialized countries, has been found to remain relatively constant over time while real mean income has risen. See also the result about mean scaling and random walks in the Appendix.
so that taking the mean of it does not require any distribution information. Equation (12) is the mean of this expression.

Equation (12) depends only on $P$, $X$, and aggregate quantities, and so apparently could be estimated without appeal to any distribution or cross-section data. The problem is that the vector $(c^1, \ldots, c^n)$ is only identified up to some scaling factor. However, if aggregate estimates of equation (12) are combined with estimates of equation (1) — Engel curves from a single cross section — the model may be fully identified.\footnote{This situation is analogous to the underidentification of the linear form of Houthakker's [1960] Addilog model, which, like Theorem 4, combines pairs of demand system equations to cancel out nonlinear terms.}

Although Theorem 4 is fairly trivial mathematically, it has the important implication that aggregate demands can be estimated for equation (1) type demand systems even when they do not satisfy the conditions required for ordinary exact aggregation, i.e., even when $g$ depends on prices.

When good distribution data are available, equation (12) can be expected to yield less efficient estimates than direct estimation of aggregate demands employing estimates of $E(g)$ when that is possible. However, even in this case equation (12) can check whether measurement errors in $E(g)$ or $\mu$ are biasing aggregate estimates, since Theorem 4 requires no assumptions about how distributions are shaped, or about how they change over time.

When $g$ depends on prices, it is not definite from Theorem 4 alone that any unknown constant parameters that appear inside $g$ can be identified. However, in general they will be identified, because such parameters will also generally appear in $a^i$, $b^i$, and $c^i$. For example, when the indirect utility function takes the form of equation (5) for any function $v$, then $g = 1/\nu'$ depends on prices only through $B$ and $C$, and parameters in $B$ and $C$ are identified because they appear in $a^i$, $b^i$, and $c^i$, as equation (6) shows.

V. Conclusions

This paper discussed the aggregation of demands that are linear in income and an arbitrary function of income, showed how the resulting aggregate demands can be interpreted as representative consumer macro models, and described a technique for esti-
mating such models without distribution information from more than one time period.

Attempts to rationalize representative consumer models (especially for purposes other than social welfare measurement) may seem like a quixotic endeavor. Macroeconomists (and many applied microeconomists and econometricians) routinely assume the existence of one, seeing it as a necessary (though acceptable) evil required for the sake of tractability. Many mathematical economists are unwilling to accept the existence of any structure in macro demands. Some econometricians employ exact aggregation methods to specify macro demand equations, but reject any representative agent interpretation of the result as unnecessary if not erroneous.

Representative consumer models are typically employed when one wants to ignore the complications caused by aggregation. As a result, those economists that do consider aggregation explicitly will tend to actively disparage representative consumer models, in part because doing so justifies their emphasis on aggregation issues.

It is a fact that the use of a representative consumer assumption in most macro work is an illegitimate method of ignoring valid aggregation concerns. However, the representative consumer framework vastly simplifies a great deal of macro work and thought, and so is not likely to be abandoned.

The solution this paper proposes is to discover what assumptions regarding both functional forms of demands and distributions of agents are required to make the demand equations arising from exact aggregation equal to those arising from utility maximization by a representative agent. For example, instead of simply assuming a representative consumer, a macroeconomist may explicitly deal with the aggregation problem by assuming an equation (4) type demand system and a mean scaled income distribution. In this way one can both deal explicitly with aggregation and have a representative consumer model at one’s disposal. More generally, any demands in the form of equation (1) may be assumed, as long as it is recognized that a parameter which depends on distributions will be present in the resulting macro model. In either case, aggregation is formally dealt with, and an untestable representative consumer assumption is replaced with testable assumptions regarding distributions of individuals and functional forms of micro models.

APPENDIX: RESULTS CONCERNING MEAN SCALING

First consider mean scaling intuitively. If \( \theta \) were constant over time, then if \( X \) increased by \( r \) percent in a period every individual’s
income would have increased by the same \( r \) percent. This is proportional distribution movement, and is a special case of mean scaling. More generally, distribution changes over time can be thought of as a combination of proportional distribution movement and of redistribution (to account for nonproportional changes in the shape of the distribution). Unlike proportional distribution movement, which assumes no redistribution, mean scaling assumes only that the proportional change and the redistribution are independent of each other. In mean scaling, \( \theta \) is not required to be constant, but only independent of \( X \).

The following properties of mean scaled distributions can all be proved formally. See Lewbel [1986, 1988] for details.

I. Mean scaling imposes no restriction on the distribution of \( x \) in a single time period. It only restricts changes in the shape of the distribution over time. It therefore requires multiple cross sections to confirm or deny whether the distribution of \( x \) is mean scaled.

II. The distribution of \( x \) is mean scaled if and only if the ratio of \( X \) to any quantile of the distribution is independent of \( X \). These ratios are constant if and only if there is proportional distribution movement. This result provides a way to test for mean scaling, given multiple observations of quantiles of the distribution.

III. If for each individual, \( \log x \) is a random walk over time, and the distribution of the random innovations is independent of \( X \), then the distribution of \( x \) will be mean scaled. This is relevant because in most applications of demand systems \( x \) is actually total expenditures, and under rational expectations the marginal utility of expenditures evolves as a random walk.

**References**


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