Flexible Specification of Mixed Demand Systems

Giancarlo Moschini and Anuradha Vissa

Common flexible functional forms are not very useful for modeling mixed demand systems because derivation of mixed demands requires closed forms for both direct and indirect utility functions. In this paper we study the alternative of using a direct approximation to mixed demands. We develop a new approach to Slutsky relations for mixed demand functions allowing formulation of a Rotterdam mixed demand model. The resulting mixed demand system is illustrated with an application to meat demand in Canada.

Key words: duality, meat demand, mixed demand, Rotterdam model, Slutsky symmetry.

Empirical studies in applied demand analysis using systems of demand equations typically rely on one of two assumptions: either prices are assumed to be predetermined or quantities are assumed to be predetermined. The first of these assumptions leads to quantity-dependent or direct demand systems, such as the (direct) translog (Christensen, Jorgenson, and Lau), the almost ideal demand system (Deaton and Muellbauer), or the (direct) differential or Rotterdam model (Theil). Direct demands are the usual representation of preferences for the individual consumer, who is typically taken as making optimal consumption decisions for given prices and income. Its use at the aggregate level is equivalent to assuming that supplies are perfectly elastic and that demands adjust to clear the market. Such a condition may hold for aggregate (market) data when one is modeling the demand of tradeable goods in the case of a small open economy, or when prices are administratively set (e.g., public utilities). The alternative, of assuming that quantities are predetermined and that prices adjust to clear the market, leads to price-dependent or inverse demand systems, such as the (inverse) translog (Christensen, Jorgenson, and Lau) or (inverse) Rotterdam model (Theil; Barten and Bettendorf). The inverse demand approach may be useful when analyzing the demand for perishable products defined over a short period of time.

In addition to the two polar cases of direct and inverse demand functions, another class of models allows one to sidestep estimation of both demand and supply functions in a simultaneous equations framework. This class is the case of 'mixed demand' functions (Chavas): prices of some goods are predetermined such that the respective quantities demanded adjust to clear the market, whereas for the remaining set of goods quantities supplied are predetermined and prices must adjust to clear the market. Despite its obvious potential for application by combining the two polar cases previously discussed, the mixed demand approach has been virtually ignored in empirical work.¹

A possible reason for the scarcity of applications of mixed demand systems is that knowledge of both direct and indirect utility functions is required to characterize the demand properties. Therefore, commonly used flexible functional forms (FFFs), such as those underlying the Translog and Almost Ideal systems, cannot be used to specify a mixed demand system because these FFFs do not have a closed form dual representation. A flexible mixed demand system can be specified nonetheless by approximating

¹ An exception is Heien (1977). Dahlgran also estimates a demand system with quantity and prices on the right-hand side, although he does not relate his study to the theory of mixed demands.
the mixed demand equations directly through a differential approach, leading to a Rotterdam mixed demand system. Such is the strategy used here. The crucial issue in this context is the specification of cross-equation symmetry restrictions. We develop a new and simple approach to the derivation of Slutsky relationships for mixed demands, using the concept of virtual or shadow prices of the related area of rationed demand (Gorman; Neary and Roberts). The proposed mixed Rotterdam specification is illustrated with an application to Canadian meat demand, which seems particularly suited to the requirement of a mixed system because of the institutional feature of supply management in the poultry sector.

Theory of Mixed Demands

Let \( x_A = (x_1, x_2, \ldots, x_m) \) denote the vector of commodities chosen optimally and let \( x_B = (x_{m+1}, x_{m+2}, \ldots, x_n) \) denote the vector of commodities in fixed quantity whose price is optimally determined. If \( p_i \) denotes the nominal price of good \( i \), and \( y \) is total expenditure on \( x_A \) and \( x_B \) (income, for short), then \( v_i = p_i/y \) is the corresponding normalized price, and \( v_A \) and \( v_B \) are the vectors of normalized prices of the two subsets of goods. Mixed demands are then derived from the constrained optimization problem (Samuelson, Chavas):

\[
\max_{x_A, x_B} \{ U(x_A, x_B) - V(v_A, v_B) \mid v_A \cdot x_A + v_B \cdot x_B = 1 \}
\]

where \( U(\ldots) \) and \( V(\ldots) \) are the direct and indirect utility functions, quasiconcave and quasi-convex in their respective arguments. The solution to (1) gives Marshallian mixed demands \( x_A(v_A, x_B, 1) \) and \( v_B(v_A, x_B, 1) \). Clearly, at the optimum, \( U[x_A(v_A, x_B, 1), x_B] = V[v_A, v_B(v_A, x_B, 1)] = \tilde{V}(v_A, x_B, 1) \), where \( \tilde{V} \) is the mixed utility function.

The mixed demand functions \( x_A(v_A, x_B, 1) \) and \( v_B(v_A, x_B, 1) \) satisfy the typical restrictions of consumer theory. First, they satisfy the adding up condition \( v_A \cdot x_A + v_B \cdot x_B = 1 \) implied by the budget constraint. Second, the homogeneity condition implies that \( x_A(v_A, x_B, 1) \) is homogeneous of degree zero in nominal prices and income, that is \( x_A(v_A, x_B, 1) = x_A(p_A, x_B, y) \). Similarly, the optimal nominal prices of group B are homogeneous of degree one in \( (p_A, y) \), implying that \( v_B(v_A, x_B, 1) \) is homogeneous of degree zero in \( (p_A, y) \). Hence \( p_A = y \cdot v_B(\cdot) = p_B(p_A, x_B, y) \). It follows also that the mixed utility function is homogeneous of degree zero in \( p_A \) and \( y \), meaning that \( \tilde{V}(v_A, x_B, 1) = \tilde{V}(p_A, x_B, y) \).

The symmetry restrictions can be illustrated in terms of the compensated mixed demand functions. As shown by Chavas, the compensated mixed demands are the same as the compensated demands under rationing, although mixed demands should be carefully distinguished from rationed demands (in the case of the latter, some markets do not clear). Compensated rationed demand can be characterized in terms of the restricted cost function \( C(p_A, x_B, u) \) defined as (Gorman; Deaton)

\[
C(p_A, x_B, u) = \min \{ p_A : x_A \mid U(x_A, x_B) = u \}
\]

It is known that \( C(\cdot) \) is nondecreasing in \( u \) and \( p_A \), nonincreasing in \( x_B \), and homogeneous of degree one and concave in \( p_A \). In addition, it can be shown that the restricted cost function \( C(\cdot) \) is convex in \( x_B \) if the utility function is quasi-concave (Moschini). From the derivative property, the partial derivatives of \( C(\cdot) \) with respect to \( p_A \) give the compensated mixed demands for goods in the \( A \) group; that is, they give the solutions to problem (2). Moreover, the partial derivatives with respect to \( x_B \) give (the negative of) the compensated shadow or virtual prices of group B goods. These shadow prices are the compensated price-dependent demand functions of \( x_B \). Hence

\[
\frac{\partial C}{\partial p_i} = x'_i(p_A, x_B, u) \quad i \in A
\]

\[
\frac{\partial C}{\partial x_k} = -p'_k(p_A, x_B, u) \quad k \in B
\]

Compensated demand functions \( x'_i(p_A, x_B, u) \) are homogeneous of degree zero in \( p_A \), whereas the compensated price-dependent demand functions \( p'_k(p_A, x_B, u) \) are homogeneous of degree one in \( p_A \). Curvature and symmetry conditions imply that the matrix of partial derivatives \( [\partial x_A/\partial p_A] \) is symmetric and negative semidefinite, the matrix of partial derivatives \( [\partial p_B/\partial x_B] \) is symmetric and negative semidefinite, and \( \partial^2 C/\partial p_i \partial x_k = \partial^2 C/\partial x_k \partial p_i = -\partial^2 p_k/\partial p_i \) for all \( i \in A, k \in B \). These three conditions
imply that the Hessian of the restricted cost function is skew symmetric.

To make such restrictions operational, the compensated mixed demand functions, \(x^i(p_A, x_B, u)\) and \(p^i(p_A, x_B, u)\), must be related to the Marshallian mixed demand functions, \(x_i(p_A, x_B, y)\) and \(p_i(p_A, x_B, y)\). As mentioned above, the distinguishing feature of mixed demands compared to rationed demands is that no disequilibrium occurs in the case of mixed demands because the prices of commodities in fixed supply adjust to clear the market. Hence, these markets must clear at the shadow or virtual price. It follows that, in the mixed demand case, the total cost of achieving utility level \(u\) when \((p_A, x_B)\) are given is

\[
\frac{\partial C}{\partial \ell} = \frac{\partial C}{\partial p} x^i + \sum_{k=m+1}^{n} (\partial p^i / \partial p_k) x_k
\]

(6.1)

\[
\frac{\partial C}{\partial x} = \sum_{s=m+1}^{n} (\partial p^i / \partial x_s) x_s
\]

(6.2)

Differentiating the identities in (5) and using the results in (6) yields the following Slutsky equations:

\[
\frac{\partial x_i}{\partial p} = \frac{\partial C}{\partial p} x^i + \sum_{k=m+1}^{n} (\partial p^i / \partial p_k) x_k
\]

(7.1)

\[
\frac{\partial x_i}{\partial x} = \frac{\partial C}{\partial x} x^i + \sum_{s=m+1}^{n} (\partial p^i / \partial x_s) x_s
\]

(7.2)

\[
\frac{\partial p_i}{\partial p} = \frac{\partial C}{\partial p} p^i + \sum_{s=m+1}^{n} (\partial p^i / \partial x_s) p_s
\]

(7.3)

\[
\frac{\partial p_i}{\partial x} = \frac{\partial C}{\partial x} p^i
\]

(7.4)

Because symmetry restrictions are expressed in terms of the properties of compensated functions \(x^i(p_A, x_B, u)\) and \(p^i(p_A, x_B, u)\), whereas for estimation purposes it is ordinary mixed demand functions \(x_i(p_A, x_B, y)\) and \(p_i(p_A, x_B, y)\) that must be specified, Slutsky relations (7) allow symmetry restrictions to be incorporated into a flexible mixed demand system in a theoretically consistent fashion.

A Rotterdam Mixed Demand System

The maximization problem (1) from which mixed demands are derived illustrates that knowledge of both the direct and the indirect utility functions is required to derive mixed demand functions. Therefore, commonly used FFFs cannot be employed for empirical purposes because they typically do not have a closed form dual representation. For example, if one were to specify the direct utility function in terms of the translog form used by Christensen, Jorgenson, and Lau, no closed form dual function could consistently

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3 The compensation scheme used here is simpler and perhaps empirically more useful than the approach developed by Chavas, although it is not as general. For example, for \(C\) to be defined, it is clear that at least one good must belong to group A.
and simultaneously represent the indirect utility function. Nor does the alternative of specifying a FFF for mixed utility function \( V \) seem useful, as can be verified by applying the derivative property to the following identity relating the mixed cost function to the mixed utility function [and recalling (6)]:

\[
\tilde{V}(p_A, x_B, \bar{C}(p_A, x_B, u)) = u
\]

A better approach is to approximate mixed demands directly by a differential (Rotterdam) demand system and impose the theoretical restrictions using the results discussed in the previous section. Totally differentiating the mixed demands \( x_A(p_A, x_B, y) \) and \( p_A(p_A, x_B, y) \) obtains the following differential mixed demand system in absolute prices:

\[
\begin{align*}
\partial x_j / \partial p_l &= \theta_{jl} / \partial y, \\
\partial p_l / \partial x_j &= \theta_{lj} / \partial y,
\end{align*}
\]

(10.6) \( \theta_k = (\partial p_k / \partial y)(y / p_k), k \in B \)

As is standard in the specification of Rotterdam models, Marshallian elasticities can usefully be expressed in terms of compensated elasticities before parameterizing the demand system. To this end, the Slutsky relations can be rewritten in terms of elasticities as

\[
\begin{align*}
\eta_{ij} &= \eta_{ij} - \eta_i \left( w_j + \sum_{k=m+1}^n w_k \rho_{ij} \right) \\
\psi_{ik} &= \psi_{ik} - \eta_i \left( \sum_{s=m+1}^n w_k \theta_{ik} \right) \\
\rho_{ki} &= \rho_{ki} - \theta_k \left( w_i + \sum_{s=m+1}^n w_k \rho_{si} \right) \\
\theta_{ks} &= \theta_{ks} - \theta_k \left( \sum_{r=m+1}^n w_r \theta_{rs} \right)
\end{align*}
\]

where \( c \) superscripts denote elasticities obtained from compensated mixed demand functions.

Using the Slutsky relations (11) and choosing the parameterization \( \alpha_i = w_i \eta_{ij}, \beta_k = w_k \theta_{ik}, \alpha_{ij} = w_i \eta_{ij}, \beta_{ks} = w_k \theta_{ks}, \gamma_{ki} = -w_k \rho_{ki}, \) and \( \delta_{ik} = w_k \psi_{ik} \) yields

\[
\begin{align*}
\alpha_i d\ln x_i &= \alpha_i d\ln y + \sum_{j=1}^m \left[ \alpha_{ij} \left( \sum_{k=m+1}^n \gamma_{kj} \right) \right] d\ln p_j + \sum_{k=m+1}^n \left[ \delta_{ik} - \alpha_i \left( \sum_{s=m+1}^n \beta_{ks} \right) \right] d\ln x_k \\
\beta_k d\ln p_k &= \beta_k d\ln y + \sum_{j=1}^m \left[ -\gamma_{ij} + \beta_j \left( \sum_{s=m+1}^n \gamma_{sj} \right) \right] d\ln p_j + \sum_{s=m+1}^n \left[ \beta_{ks} - \beta_s \left( \sum_{r=m+1}^n \beta_{rs} \right) \right] d\ln x_i
\end{align*}
\]

where \( d\ln y = [d\ln y - \sum_{j=1}^m w_j d\ln p_j] \) represents the change in nominal income adjusted by the change in exogenous prices only. An equivalent way to express this income term is \( d\ln \bar{y} = [\Sigma_{i=1}^m w_i d\ln x_i + \Sigma_{j=m+1}^n w_j d\ln p_j], \) a form that allows adding-up to be satisfied exactly when, in the empirical application, the infinitesimal changes in (12) are approximated by finite changes.

Equations (12) represent the Rotterdam specification of the mixed demand system. It is linear in variables but nonlinear in parameters.
cause of the parameterization chosen, the homogeneity, symmetry, and adding-up properties can be set in terms of parametric restrictions. Homogeneity is satisfied when

\[
\sum_{j=1}^{m} \alpha_{ij} = 0
\]

\[
\sum_{k=1}^{m} \gamma_{ik} = -w_k
\]

the adding-up conditions are

\[
\sum_{i=1}^{m} \alpha_i + \sum_{k=m+1}^{n} \beta_k = 1
\]

\[
\sum_{i=1}^{m} \alpha_{ij} = 0
\]

\[
\sum_{i=1}^{m} \delta_{ik} = -w_k
\]

and symmetry requires

\[
\alpha_{ij} = \alpha_{ji}
\]

\[
\beta_{ik} = \beta_{ki}
\]

\[
\gamma_{ik} = \delta_{ki}
\]

Note that the adding-up and homogeneity conditions involve shares and thus, in general, can be satisfied only locally (at a point).

**Application to Canadian Meat Demand**

The mixed demand approach is particularly suited to analyzing Canadian meat demand. First, there is virtually free trade in beef and pork between the United States and Canada. Because Canada is a small country in the North American market, the assumption that beef and pork prices are exogenous to the Canadian market seems tenable. On the other hand, Canadian imports of poultry products are restricted by an import quota (Moschini and Meilke). The import quota insulates the domestic market, and the internal price formation mechanism heavily depends on the institutional setting. Chicken producers are organized in provincial Marketing Boards coordinated by the Canadian Chicken Marketing Agency. These producer organizations are endowed with the power to control output supply. Such 'supply management' policy is enforced by production quotas first allocated to each province and then to individual producers. Hence it seems that for chicken, equilibrium is characterized by exogenously determined supply, with price adjusting to clear the market.\(^{6}\)

Assuming the meats group is weakly separable from other commodities, a mixed demand system for beef, pork, and chicken is specified.\(^{7}\) Specifically, this is a conditional-demand system, and the results that we obtain should be interpreted accordingly. In particular, the elasticities that will be computed are conditional on the expenditure allocated to the meat group. To estimate the mixed demand system, quarterly data on consumption and prices (obtained from Agriculture Canada) for beef, pork, and chicken are used. These data are for 1980\(^{(1)}\) to 1990\(^{(1)}\), a period consistent with the policy setting described. Quantity data are per capita disappearance (in kilograms) of beef, pork, and chicken.\(^{8}\) Quantities were converted from carcass to retail weight by conversion factors supplied by Statistics Canada.\(^{9}\) Statistics Canada reports price indexes for these three commodities, but not nominal prices. Consumer price indexes (1981 base year) were converted to nominal prices using survey data from Family Food Expenditure Surveys, Statistics Canada, as follows. From weekly family expenditures and quantities consumed (all classes and provinces) prices were computed for the three commodities by dividing expenditures by quantities in years 1974, 1976, 1978, 1982, 1984, and 1986. These prices were regressed, through the origin, on the respective annual consumer price indexes. Raw moment \(R^2\) values were over 0.99 for all three equations and regression coefficients for beef, pork, and chicken were 0.052, 0.037, and 0.029, respectively. Estimated coefficients then were used to generate prices for the entire sample period.

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\(^{6}\) The Farm Products Marketing Act of 1972 allowed the creation of marketing boards with considerable supply-restricting powers (Van Kooten). The Canadian Chicken Marketing Agency was established in 1978 and chicken import quotas were introduced in 1979, so that supply management for the chicken industry became fully operational by the end of 1979.

\(^{7}\) Although this type of econometric utilization of weak separability is standard in demand analysis, it should be pointed out that simultaneous equation problems may arise in small demand systems such as ours, although it is not clear how serious this possible bias is (LaFrance).

\(^{8}\) We recomputed pork disappearance data because those supplied by Agriculture Canada, based on the methodology of Hewston and Rosien, do not seem to account for manufacturing and waste in a correct manner.

\(^{9}\) The conversion factor for beef was 0.74 from 1980 to 1985 and 0.73 from 1986 to 1990. The conversion factor for pork was 0.77 from 1980 to 1982 and 0.76 from 1983 to 1990.
Estimation Results

Implementing the Rotterdam model requires converting the differential terms in (12) to finite logarithmic changes. Because quarterly data are used, log differences are computed between the same quarter in consecutive years rather than between two contiguous quarters, and the shares used in multiplying each of the equations are averages for the same quarters. For example, in the beef equation, the approximation for \( d\log p_{bf,t} \) is \( (\log p_{bf,t} - \log p_{bf,t-4}) \) and the corresponding share is \( (w_{bf,t-4} + w_{bf,t})/2 \), where subscript \( bf \) indexes beef and \( r \) indexes time.

Symmetry and homogeneity restrictions are maintained. Because homogeneity and adding-up conditions entail budget shares, these restrictions are imposed at the mean point. The stochastic version of the model is obtained by adding to (12) error terms that are assumed multinormally distributed and contemporaneously correlated. Adding-up holds only at a point, so the resulting system is not singular. Similar to the estimation of systems based on the double-log form (Byron, Heien 1982, Huang and Haidacher), which also satisfy adding-up at a point only, all three equations are used.

The mixed differential model is estimated with an intercept and correction for first order autocorrelation using iterated nonlinear least squares as coded in SHAZAM 6.2. The curvature property is not imposed but was checked at the mean of the explanatory variables and found to hold.\(^{10} \)

To satisfy adding-up, intercepts of the three equations are constrained to add to zero. Also, the autocorrelation coefficient is constrained to be the same in all equations. Unreported results indicate no significant seasonality, a useful implication of the fourth-period differencing adopted.\(^{11} \)

Estimates of the mixed demand system are presented in table 1. Intercepts are negative in the beef and pork equations and positive in the chicken equation. They are jointly significantly different from zero. These intercepts can be interpreted as parametric approximations to \( w_i(d\ln x_i/\partial t) \) for \( i \in A \),\(^{12} \) and to \( w_i(d\ln p_i/\partial t) \) for \( i \in B \). Hence, dividing the intercept by the corresponding share yields an estimate of the rate of change of quantity demanded (for \( i \in A \)) or of the rate of change of price (for \( i \in B \)) not attributable to price and total expenditure effects. Equivalently, because \( d\ln x_i/\partial t = d\ln w_i/\partial t \) for \( i \in A \) and \( d\ln p_i/\partial t = d\ln w_i/\partial t \) for \( i \in B \), dividing the intercept by the corresponding share yields an estimate of the rate of change of the corresponding demand share.

Dividing estimated intercepts by mean shares yields \(-0.017 \) for beef, \(-0.011 \) for pork, and \(0.064 \) for chicken. The implication is that, over the sample period and for reasons not attributable to meat prices and meat expenditure, beef’s share of meat expenditure has been falling 1.7% per year (recall our fourth-period differencing), pork’s share has been falling 1.1% per year, and chicken’s share has been increasing 6.4% per year. This result may suggest a reduced consumers’ preference for red meat relative to white meat, a (still controversial) hypothesis put forward in many meat demand studies. Alternatively, the significance of intercepts may indicate model misspecification resulting from omitted variables correlated with trend, possibly from an inappropriate separability assumption or (somewhat less likely) from an inappropriate functional form.

Coefficients \( \alpha_{11} \) and \( \beta_{33} \) (weighted compensated mixed elasticities of beef and chicken, respectively) have the expected negative signs.

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\(^{10} \) As mentioned previously, curvature conditions require the matrices of partial derivatives \( [\partial x_i/\partial p_i] \) and \( [\partial p_i/\partial x_i] \) to be negative semi-definite. In our parameterization, this is equivalent to requiring that matrices \( [\alpha_i] \) and \( [\beta_i] \) be negative semi-definite.

\(^{11} \) A model with seasonal dummy variables was estimated and these dummies were found to be insignificant. A model with single-period differencing but with an AR (4) error process and dummy variables also yielded elasticities close to those of the model presented here.

\(^{12} \) As Theil (1975, vol. I, p. 187) puts it in the context of direct demands, this term measures the "... expectation of the quantity component of a budget share change under the condition that real income and relative prices remain unchanged."
Mixed Demand Systems

Table 2. Mixed Elasticities at the Mean

<table>
<thead>
<tr>
<th>p_{mf}</th>
<th>p_{mk}</th>
<th>x_{ck}</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compensated elasticities</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x_{mf}</td>
<td>-0.195</td>
<td>0.195</td>
<td>-0.303</td>
</tr>
<tr>
<td>(0.051)</td>
<td>(0.051)</td>
<td>(0.046)</td>
<td></td>
</tr>
<tr>
<td>x_{mk}</td>
<td>0.297</td>
<td>-0.297</td>
<td>-0.111</td>
</tr>
<tr>
<td>(0.078)</td>
<td>(0.078)</td>
<td>(0.070)</td>
<td></td>
</tr>
<tr>
<td>p_{ck}</td>
<td>0.806</td>
<td>0.194</td>
<td>-1.511</td>
</tr>
<tr>
<td>(0.123)</td>
<td>(0.123)</td>
<td>(0.286)</td>
<td></td>
</tr>
<tr>
<td>Marshallian elasticities</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x_{mf}</td>
<td>-0.885</td>
<td>-0.191</td>
<td>-0.002</td>
</tr>
<tr>
<td>(0.098)</td>
<td>(0.058)</td>
<td>(0.073)</td>
<td></td>
</tr>
<tr>
<td>x_{mk}</td>
<td>-0.287</td>
<td>-0.624</td>
<td>0.143</td>
</tr>
<tr>
<td>(0.132)</td>
<td>(0.088)</td>
<td>(0.080)</td>
<td></td>
</tr>
<tr>
<td>p_{ck}</td>
<td>0.194</td>
<td>-0.148</td>
<td>-1.245</td>
</tr>
<tr>
<td>(0.183)</td>
<td>(0.110)</td>
<td>(0.211)</td>
<td></td>
</tr>
</tbody>
</table>

Asymptotic standard errors are reported in parentheses.

Expenditure coefficients are positive. Estimated mixed compensated elasticities, obtained by dividing estimated coefficients by the relevant mean share, and their asymptotic standard errors, are reported in table 2. The ratios of the elasticities to their respective standard errors are asymptotically normally distributed. Beef and pork are net substitutes, and chicken is a substitute for both beef and pork. For instance, the mixed elasticity $x_{ck}$ of -0.111 shows that a 1% increase in chicken supply causes about 1/10% decrease in pork consumption.

Marshallian mixed elasticities, retrieved via the Slutsky relations, also are reported in table 2. Beef and pork are found to be gross complements. Chicken is a substitute for pork but a complement to beef. The own 'quantity' elasticity of chicken is greater than one in absolute value, indicating that a 1% rise in chicken supply would decrease chicken price by more than a percentage point. The mixed expenditure elasticities are all positive, as one would expect for normal goods, and close to unity. With beef and pork, they indicate the usual consumption change resulting from a change in total expenditure. With chicken, the expenditure elasticity indicates the change in how much consumers are willing to pay for the fixed chicken supply when meat expenditure increases by 1%.

To compare computed mixed elasticities with the more familiar direct elasticities in other studies, direct Marshallian elasticities and direct compensated elasticities can be retrieved from the mixed elasticities. Let $x_i^p(p_A, p_B, y)$ and $x_i^p(p_A, p_B, y)$ be direct Marshallian demand functions for the $(A, B)$ grouping of goods used here. Direct demand functions can be related to mixed demand functions by

$$x_i^p[p_A, p_B, A, B, y] = x_A(p_A, x_B, y)$$

Let $e_i^p$ denote direct Marshallian price elasticities, and in an obvious notation let $e_{AA}, e_{AB}, e_{BB}$ denote submatrices of these elasticities. Differentiating the identities in (16), we verify that Marshallian direct and mixed elasticities are related by

$$e_{AA} = [\eta - \psi \theta^{-1}]\rho, \quad e_{AB} = \psi \theta^{-1}, \quad e_{BB} = \theta^{-1},$$

where $\eta, \psi, \theta, \rho$ denote matrices of mixed elasticities defined in (10). Direct elasticities retrieved in this fashion are reported in table 3 (compensated elasticities are computed from the Marshallian elasticities using the Slutsky equation). Own-price elasticities of beef and chicken are -0.885 and -0.804, respectively, greater than that of pork (which is -0.641). Expenditure elasticities of beef and pork are close to unity, whereas that of chicken is 0.766.

For comparison, the absolute price version of a direct Rotterdam model also was estimated, with intercepts and correction for first-order autocorrelation. Reported in table 4 are compensated and Marshallian elasticities from this di-

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**Table 3. Direct Elasticities at the Mean, Retrieved from the Mixed System**

<table>
<thead>
<tr>
<th>p_{mf}</th>
<th>p_{mk}</th>
<th>y</th>
<th>p_{ch}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compensated elasticities</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x_{mf}</td>
<td>-0.357</td>
<td>0.156</td>
<td>0.201</td>
</tr>
<tr>
<td>(0.079)</td>
<td>(0.056)</td>
<td>(0.051)</td>
<td></td>
</tr>
<tr>
<td>x_{mk}</td>
<td>0.238</td>
<td>-0.311</td>
<td>0.074</td>
</tr>
<tr>
<td>(0.085)</td>
<td>(0.080)</td>
<td>(0.047)</td>
<td></td>
</tr>
<tr>
<td>x_{ch}</td>
<td>0.533</td>
<td>0.129</td>
<td>-0.662</td>
</tr>
<tr>
<td>(0.136)</td>
<td>(0.082)</td>
<td>(0.125)</td>
<td></td>
</tr>
<tr>
<td>Marshallian elasticities</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x_{mf}</td>
<td>-0.885</td>
<td>-0.191</td>
<td>0.002</td>
</tr>
<tr>
<td>(0.100)</td>
<td>(0.060)</td>
<td>(0.059)</td>
<td></td>
</tr>
<tr>
<td>x_{mk}</td>
<td>-0.264</td>
<td>-0.641</td>
<td>-0.115</td>
</tr>
<tr>
<td>(0.126)</td>
<td>(0.085)</td>
<td>(0.058)</td>
<td></td>
</tr>
<tr>
<td>x_{ch}</td>
<td>0.156</td>
<td>-0.119</td>
<td>-0.804</td>
</tr>
<tr>
<td>(0.156)</td>
<td>(0.087)</td>
<td>(0.136)</td>
<td></td>
</tr>
</tbody>
</table>

Asymptotic standard errors are reported in parentheses.

---

13 Substitutability in terms of the mixed compensated elasticities need not be equivalent to either $p$-substitutability in terms of the direct system, nor $q$-substitutability in terms of the inverse system.
Table 4. Direct Elasticities at the Mean from the Direct System

<table>
<thead>
<tr>
<th></th>
<th>$p_{mf}$</th>
<th>$p_{mp}$</th>
<th>$p_{mk}$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pb/PP</td>
<td>-0.267</td>
<td>0.174</td>
<td>0.093</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.058)</td>
<td>(0.047)</td>
<td>(0.028)</td>
<td></td>
</tr>
<tr>
<td>Pb/pk</td>
<td>0.265</td>
<td>-0.307</td>
<td>0.042</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.072)</td>
<td>(0.074)</td>
<td>(0.033)</td>
<td></td>
</tr>
<tr>
<td>Pk/pk</td>
<td>0.246</td>
<td>0.073</td>
<td>-0.319</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.079)</td>
<td>(0.058)</td>
<td>(0.028)</td>
<td></td>
</tr>
</tbody>
</table>

Compensated direct elasticities

<table>
<thead>
<tr>
<th></th>
<th>$x_{bf}$</th>
<th>$x_{bp}$</th>
<th>$x_{bk}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.837</td>
<td>-0.200</td>
<td>-0.121</td>
</tr>
<tr>
<td></td>
<td>(0.082)</td>
<td>(0.050)</td>
<td>(0.033)</td>
</tr>
<tr>
<td></td>
<td>-0.233</td>
<td>-0.635</td>
<td>-0.146</td>
</tr>
<tr>
<td></td>
<td>(0.111)</td>
<td>(0.080)</td>
<td>(0.043)</td>
</tr>
<tr>
<td></td>
<td>-0.027</td>
<td>-0.107</td>
<td>-0.422</td>
</tr>
<tr>
<td></td>
<td>(0.105)</td>
<td>(0.063)</td>
<td>(0.064)</td>
</tr>
</tbody>
</table>

Marshallian direct elasticities

Asymptotic standard errors are reported in parentheses.

rect Rotterdam model. Beef and pork own-price (compensated and Marshallian) and expenditure elasticities from the direct Rotterdam are somewhat similar to the direct elasticities retrieved from the mixed system. By contrast, the absolute value of the own-price elasticity of chicken in the direct demand system is much lower than that retrieved from the mixed system. This result is consistent with Thurman, and with Shonkwiler and Taylor, who show that least squares estimation of quantity-dependent demand equations underestimates demand elasticities when prices are endogenous. Hence, our analysis suggests that chicken demand in the Canadian market may be more elastic than reported in previous studies.

Concluding Remarks

In this paper we have developed a Rotterdam specification of a mixed demand system. Such mixed demands are applicable when some (but not all) goods are available in predetermined quantities, and are not rationed because their prices adjust to clear the market. The Rotterdam specification proposed here allows a flexible representation of the mixed demand system and in this context overcomes some problems associated with flexible functional forms. To make the Rotterdam specification operational, we developed a new approach to deriving Slutsky relations in a mixed demand context. The proposed mixed demand system was illustrated with an application to the Canadian meat market. The fact that Canada has virtually free trade in beef and pork, whereas chicken supply is restricted, means that a mixed demand approach is appealing in this case. Estimated elasticities from the mixed demand system are similar to those of a direct Rotterdam model, except that the own-price elasticity of chicken demand is greater in the mixed demand system.

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