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Testing Restrictions in a Flexible Dynamic Demand System: An Application to Consumers' Expenditure in Canada

GORDON ANDERSON
University of Southampton

and

RICHARD BLUNDELL
University of Manchester

Traditionally, restrictions on systems of demand equations have been tested using static models, whilst being estimated with time series data. This paper develops a vector time series model of expenditure shares in the context of a singular dynamic demand system. The model allows for non-symmetric and non-homogeneous short run behaviour. The homogeneity and symmetry restrictions are only examined in the long run structure. Results based on Canadian time series data are presented and reject the current practise of static modelling while restrictions suggested by economic theory are not rejected when imposed on the long run structure.

1. INTRODUCTION

The equilibrium theory of consumer demand and the development of static demand systems that can be applied to aggregate economic data has received considerable attention in the recent literature. A central feature of much of this work, see for example Deaton and Muellbauer (1980) and Barten (1969), has been to test the plausibility of certain restrictions implied by the constrained utility maximization hypothesis. As well as these restrictions, demand theory points to some interesting and testable simplifications to the structure of price and income responses, notably homotheticity and separability. However, in order to entertain conclusions based on estimation and testing within the confines of such static models, consumers must be assumed to fully adjust to price and income changes instantaneously.

In this paper it is suggested that consumers are unlikely to have adjusted to equilibrium in every time period. Habit persistence, adjustment costs, incorrect expectations and misinterpreted real price changes are among many possible reasons for such short run behaviour. Appropriate modelling of the dynamic adjustment of consumers' expenditure is considered essential before restrictions from demand theory are tested. That the results of such tests may depend on dynamic specification can be inferred from the recent work of Berndt, McCurdy and Rose (1980), where simply appending autoregressive disturbances to a static portfolio selection model is shown to significantly weaken the rejection of utility maximising restrictions. Static models are unlikely to provide reasonable maintained explanations of time series data. Hence a more general structure for demand systems is required that allows a test of the static model itself, as well as the theoretical restrictions and simplifications from demand theory.
The singular nature of demand systems has some interesting implications for the specification and estimation of a set of dynamic equations. It turns out that, in an estimable form, the underlying adjustment parameters of the dynamic system cannot be recovered from the estimated parameters without a priori information. However, the preference parameters relating to equilibrium behaviour, which are of particular interest in this study, are fully identified. Section 2 of this paper discusses these points and generates an estimable form for a general dynamic singular system. Due to the limited length of most economic time series on disaggregated consumers' expenditure, attention is focused on a first order system. Nested within this flexible general structure are some interesting and popular dynamic simplifications as well as the static model itself. In particular, the autoregressive, partial adjustment and habit persistence models can be derived, enabling such specifications to be tested against a more general alternative.

In the approach developed here, equilibrium demand theory is used to generate a representation of long run behaviour. A convenient reparameterisation, similar to that of Hendry and Von Ungern Sternberg (1981), is used to partition the dynamic model into its long and short run components. This considerably simplifies the recovery of the long run structure from the dynamic model, and since all nonlinear parameter restrictions are associated with the long run structure, it will be shown to have computational attractions as well. Of the many flexible demand systems available to describe long run behaviour, given the complications of estimating demand models in a dynamic setting, the Almost Ideal Demand System proposed by Deaton and Muellbauer (1980) is a particularly convenient choice. It is essentially linear in variables and does not assume homogeneity or symmetry although neatly allows the testing of these, as well as the homotheticity and homothetic separability restrictions. A complete description of the representation of long run preferences is given in Section 2.

Having discussed the modelling of short run dynamics in Section 3, the general dynamic model is subjected to annual data on five categories of non-durable Canadian expenditure in Section 4. Before testing the more usual homogeneity and symmetry restrictions an appropriate dynamic structure is chosen. If, as it turns out, the static system is rejected by the data, further tests and estimated elasticities based on such a model are no longer valid. To emphasize the importance of this point, a comparison is made between the implications for consumers' behaviour from both estimated static and dynamic models. Finally, in Section 5, some conclusions are drawn concerning the use of static systems as descriptions of consumers' behaviour over time.

2. THE REPRESENTATION OF LONG RUN PREFERENCES

To develop these ideas within the context of the specific model used in this paper, the details of the long run structure will be considered first. Letting \( w \) represent a \( n \times 1 \) vector of budget shares and \( z \) a \( k \times 1 \) vector of income and price variables, the long run structure may be written:

\[
w = f(z, \Theta)
\]

where \( \Theta \) are the underlying preference parameters. Using the Almost Ideal Demand model of Deaton and Muellbauer (1980) allows us to write (1) as:

\[
w = \Pi(\Theta)x
\]

where \( x \) is some \( l \times 1 \) vector of transformed income and price variables and \( \Pi \) is some constant matrix function of \( \Theta \).

In particular, a single equation from (2) representing the \( i \)-th budget share is given by:

\[
w_i = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln p_i + \beta_i \ln (m/P)
\]
\( p_j \) being the price of good \( j \), \( m \) the *per capita* expenditure on all \( n \) goods and \( P \) a suitable price index. The form of \( \ln P \), derivable from the underlying consumers’ expenditure function, is:

\[
\ln P = \alpha_0 + \sum_{k=1}^{n} \alpha_k \ln p_k + \frac{1}{2} \sum_{k=1}^{n} \gamma_{kk} \ln p_k \ln p_j.
\]  

(4)

Since the sum of shares across \( i \) is unity by definition, the parameters of (3) and (4) must satisfy the following adding up restrictions:

\[
\sum_{i=1}^{n} \alpha_i = 1, \quad \sum_{i=1}^{n} \gamma_{ij} = \sum_{i=1}^{n} \beta_i = 0.
\]  

(5)

Combining (3), (4) and (5) allows each expenditure share to be written as a linear equation with nonlinear parameter restrictions:

\[
w_i = (\alpha_i - \beta_i \alpha_0) + \sum_{k=1}^{n} (\gamma_{ik} - \beta_i \alpha_k) \ln p_k + \beta_i \ln m - \frac{1}{2} \gamma_{kk} \ln p_k \ln p_j + \gamma_{jk} \ln p_k \ln p_j - \frac{1}{2} \sum_{k=1}^{n-1} \sum_{i=k+1}^{n} (\gamma_{ki} + \gamma_{jk}) \ln p_k \ln p_j.
\]  

(6)

This form corresponds to the representation of long run preferences given in (2) where \( x \) contains an intercept, log price and income terms as well as the transformed log price terms. The vector of underlying preference parameters \( \Theta \) can be derived from nonlinear restrictions on \( \Pi \). Apart from the adding up restrictions (which cannot be tested), there are a series of nested restrictions that can be tested on the long run values of the preference parameters and these, along with their implications for demand elasticities, are given in Table I.

3. A FLEXIBLE DYNAMIC MODEL OF CONSUMER DEMAND

Given a time series of \( T \) observations on budget shares, price and *per capita* total expenditure, a general first order dynamic model may be written as:

\[
\Delta w_t = A^* \Delta x_t - B^*(w_{t-1} - \Pi(\Theta)x_{t-1}) + \varepsilon_t.
\]  

(7)

In (7) \( \Delta \) represents the first difference operator, \( A^* \) and \( B^* \) are appropriately dimensioned short run coefficient matrices and the vector of disturbances, \( \varepsilon_t \) is assumed to be characterised by a singular, independent and identical distribution over time. Letting \( B^* = (I-C_1), A^* = A_1 \) and \( \Pi(\Theta) = B^{*\perp}(A_1+A_2) \), it may be seen that (7) is observationally equivalent to a dynamic model of the form:

\[
w_t = A_1 x_t + A_2 x_{t-1} + C w_{t-1} + \varepsilon_t.
\]  

(8)

However, the convenience of using (7) over (8) is in the direct parameterisation in terms of the long run structure \( \Pi(\Theta) \) which is to be the focus of attention in this paper.

As they stand, (7) and (8) are not estimable since the explanatory variables appearing in each equation are perfectly collinear. This occurs because, apart from the constant term, which appears in both \( x_t \) and \( x_{t-1} \) the elements of \( w_t \) sum to unity for all \( t \). Letting superscript \( n \) define an operator that deletes the \( n \)-th row of any vector or matrix, an estimable version of (7) is given by:

\[
\Delta w_t = A \Delta \tilde{x}_t - B(w_{t-1} - \Pi(\Theta)x_{t-1}) + \varepsilon_t.
\]  

(9)

where \( \tilde{x}_t \) refers to \( x_t \) with the constant term excluded.

The \( n \times (n-1) \) elements of the coefficient matrix \( B \) in (9) are related to the \( n^2 \) elements of \( B^* \) by:

\[ b_{ij} = b_{ij}^* - b_{in}^* \quad \text{for all } i = 1, \ldots, n, \quad \text{and } j = 1, \ldots, n-1. \]

Without *a priori* restrictions, there is a loss of identification since the \( b_{ij}^* \) cannot be recovered from the estimated \( b_{ij} \). However, the adding up restrictions (5) imply that there is no loss of identification in the long run structure since all underlying preference
### TABLE I

*Restrictions on preferences and implied elasticities*

<table>
<thead>
<tr>
<th>Models</th>
<th>Income elasticity</th>
<th>Price elasticity ((e_{ij}))</th>
<th>Compensated price elasticity ((\tilde{e}_{ij}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unrestricted ((\sum \alpha_i = 1, \sum \gamma_{ij} = 0, \sum \beta_i = 0))</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Homogeneity ((\sum \gamma_{ij} = 0))</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Symmetry ((\gamma_{ij} = \gamma_{ij}))</td>
<td>(1 + \frac{\beta_i}{w_i} \left( \gamma_i + \beta_i (\alpha_i + \sum_k \gamma_{ki} \ln p_k) \right) - \delta_{ij})</td>
<td>(\frac{1}{w_i} (\gamma_i + \beta_i \ln (m/P) - w_i \delta_{ij} + w_i w_i))</td>
<td></td>
</tr>
<tr>
<td>Restricted Price Response ((\gamma_{ij} = 0 \text{ all } i, j))</td>
<td>(1 + \frac{\beta_i}{w_i} (-\beta_i \alpha_i) - \delta_{ij})</td>
<td></td>
<td>(\frac{1}{w_i} (\beta_i \beta_i \ln (m/p) - w_i \delta_{ij} + w_i w_i))</td>
</tr>
<tr>
<td>Homotheticity ((\beta_i = 0 \text{ all } i))</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Homothetic Separability ((\beta_i = \gamma_{ij} = 0 \text{ all } i \text{ and } j))</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note:**

1. \(\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & \text{otherwise.} \end{cases}\)
parameters $\Theta$ can be derived from restrictions on $\Pi^n$. Adding up restrictions on the complete model (7) or (8) require certain additional restrictions on the elements of $A$ and $B$ in (9) which have been outlined in Anderson (1980) and Anderson and Blundell (1982). These imply that the column sums of $A$ and $B$ in (9) are all zero.

Model (9) is a general first order dynamic model and as such may be too general for any particular data generating process resulting in a loss of precision in parameter estimation. Prior to performing the tests on $\Theta$ described in Table I, a sequence of tests may be pursued to find the most restrictive dynamic specification acceptable to the data. Two routes may be followed, each representing important models well established in the literature. The first (see, for example, Berndt and Savin (1975)) is to posit a static model, with all systematic responses to economic variables in period $t$ taking place in period $t$, but appending an autoregressive error process. The restrictions which this model implies will be most familiar in terms of (8) as $A_2 = CA_1$ and are characterized in Table II in terms of (9). It is interesting to note that this autoregressive model is the most general dynamic specification nested in (9) that requires equality between each long run and short run effect. This common root restriction is a strong constraint on consumer behaviour and one that it is important to test in any empirical study.

**TABLE II**

_Restrictions on dynamic behaviour_

<table>
<thead>
<tr>
<th>Maintained Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta w_t = A \Delta \tilde{x}<em>t - B (w^n</em>{t-1} - \Pi^n(\Theta) x_{t-1}) + \epsilon_t$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Autoregressive Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{ij} = \Pi_{ij+1}(\Theta)$ for $i = 1, \ldots, n-1; j = 1, \ldots, k-1$</td>
</tr>
<tr>
<td>where $a_{ij}$ and $\Pi_{ij}(\Theta)$ are respectively the $ij$-th elements of $A$ and $\Pi^n(\Theta)$.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Partial Adjustment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{ij} = \sum_k b_{ik} \Pi_{kj+1}(\Theta)$ for $i = 1, \ldots, n-1; j = 1, \ldots, k-1$</td>
</tr>
<tr>
<td>where $a_{ij}$ and $\Pi_{ij}(\Theta)$ are respectively the $ij$-th elements of $A$ and $\Pi^n(\Theta)$.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Static Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_{ii} = 1$ if $i = j$</td>
</tr>
<tr>
<td>$= 0$ if $i \neq j$</td>
</tr>
<tr>
<td>$b_{ij} = 1$ if $i = 1, \ldots, n-1$</td>
</tr>
<tr>
<td>$j = 1, \ldots, n-1$</td>
</tr>
<tr>
<td>where $b_{ij}$ is the $ij$-th element of $B$.</td>
</tr>
</tbody>
</table>

The second route is to posit an interrelated partial adjustment model (see, for example, Nadiri and Rosen (1969)) which for (8) implies $A_2 = 0$ and is described in Table II in terms of (9). An observationally equivalent form of this model is an interrelated habit persistence model. Here, preferences over budget shares evolve over time as a function of the previous budget shares of all communities, viz.:

$$\alpha_t = d + Dw_{t-1}.$$  

Such an approach may be thought of as a generalization of the more usual model (see, for example, Pollak and Wales (1969)).
Nested within both the autoregressive and partial adjustment models is the static model most commonly used in the literature (see Deaton and Muellbauer (1980)) and the restrictions that it implies for either model in terms of the structure (9) are also outlined in Table II. As discussed in Anderson and Blundell (1982), the first difference model is not nested within the stochastic structure of (9) and cannot, therefore, be tested using the usual likelihood ratio or other asymptotically equivalent statistics.

4. AN APPLICATION TO CONSUMERS' EXPENDITURE DECISIONS IN CANADA

The data used in this application are an annual time series on five categories of non-durable consumers' expenditure in Canada for the period 1947–1979 inclusive. Following Wales (1971), some care was taken in choosing the data so as to exclude durable goods, the full details of which are given in the Appendix. Briefly, the five groups are food, clothing, energy, transport and communications and finally, recreation. Full information maximum likelihood methods were used to estimate all models derived in Section 3 making use of the nonlinear routines in Snella (1978), White (1978) and Wymer (1973). As will be shown, there is sufficient variation in this five commodity Canadian sample to identify all the parameters of the unrestricted model developed in Section 3. From a consideration of equations (6) and (9), it can be seen that, with the thirty two observations remaining after allowing for lags, no equation can be made to fit perfectly. Nevertheless, for comparison with the study by Deaton and Muellbauer (1980) where such a problem occurred, some results from the trace minimization procedure adopted in that study are considered.

The methodology of the previous sections of this paper has been to generate a flexible model both in terms of aggregate consumers' preferences and dynamic adjustment. In small samples, the required flexibility naturally reduces the available degrees of freedom and may invalidate the use of asymptotic criteria, such as the likelihood ratio test statistic used in this paper. Recent experimental evidence with static demand systems (see Bera et al. (1981), Laitinen (1978) and Meisner (1979)) suggests that these procedures lead to over-rejection in finite samples. This propensity to over-reject may be severe but tends to be less so in small systems of the size considered here and diminishes with the use of the likelihood ratio rather than Wald type statistics. Various small sample corrections are available and have been used in empirical demand studies, for example Pudney (1981) and Simmons (1980). However, these adjustments are only approximate and their properties are unknown in dynamic systems where, even for linear within equation restrictions, no exact criteria exist. The impact on test conclusions of these adjustments can be large and will, therefore, be discussed throughout the empirical investigation presented here.

The results of the tests on dynamic structure are given in Table III which may be seen as the empirical counterpart of Table II in the previous section. The critical values (CV) and $\chi^2$ values refer to the asymptotic likelihood ratio criteria discussed above. To keep the overall significance level reasonable and to protect, to some extent, against the over-rejection discussed above, all tests were conducted at the 1% level. The important conclusion to be drawn from Table III is that, even at this low significance level, both the submodels are rejected using the asymptotic criteria. Despite the rejection of these two simpler dynamic representations, it is of interest to consider the results of testing the static model against either of these two popular alternatives. In both cases, Table III indicates a decisive rejection of the static model. For computational convenience, throughout these tests on dynamic structure, per capita total expenditure was deflated using the share weighted price index:

$$\ln P = \sum_{j=1}^{n} w_j \ln p_j$$

(10)
Tests on the dynamic structure

<table>
<thead>
<tr>
<th>Model</th>
<th>$\ln L$</th>
<th>D.F.</th>
<th>$\chi^2$</th>
<th>CV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maintained Model</td>
<td>686.2</td>
<td>24</td>
<td>48.2*</td>
<td>42.9</td>
</tr>
<tr>
<td>Autoregressive</td>
<td>662.1</td>
<td>16</td>
<td>132.6*</td>
<td>32.0</td>
</tr>
<tr>
<td>Partial Adjustment</td>
<td>651.5</td>
<td>16</td>
<td>111.4*</td>
<td>32.0</td>
</tr>
<tr>
<td>Static</td>
<td>595.8</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: *A* by the value of a $\chi^2$ test statistic indicates a rejection at the 1% level.

rather than index (4). Indeed, even when testing restrictions on the long run structure, this deflator was used in the construction of short run changes in real *per capita* total expenditure. As indicated in Deaton and Muellbauer (1980) and as can be seen from Table V, use of this index has little effect on the value of the log likelihood.

The result of applying the small sample corrections suggested in Pudney (1981) and Simmons (1980) is to leave the rejection of the static model unaffected but to allow the autoregressive model and even the partial adjustment model to move out of the critical region. Given the limited degrees of freedom (an average of 15 per equation), this may not seem surprising. Nevertheless, as shown in Section 3, the autoregressive model does impose strong restrictions on the relationship between long and short run behaviour in this framework. Clearly, the autoregressive specification would be inclined to impose stronger constraints when aggregate behaviour is restricted to obey the homogeneity and symmetry restrictions. That this is indeed the case for the Canadian sample is shown in Table IV which reproduces Table III under these restrictions. With resulting parameter space reduced and with the larger values of the test statistic, small sample adjustments of the type mentioned above tend to reject both dynamic simplifications at reasonable levels of significance.
Having chosen model (9) as a suitable maintained hypothesis, tests on economic theory may proceed. Not wishing to impose restrictions on short run behaviour, the $A$ coefficient matrix is left unrestricted throughout this test procedure. In order to reduce the computational burden of estimating (9), subject to the nonlinear restrictions required to recover and impose restrictions on $\Theta$ when using index (4), the $A$ coefficient matrix was concentrated out of the likelihood function. It is easy to see that this simply requires a set of auxiliary linear regressions. As mentioned above, to contrast these results with those using index (4) to deflate per capita expenditure, some models were re-estimated using index (10).

In Table V the sequence of tests performed, corresponding to those restrictions described in Table I are presented. The striking conclusion to be derived from Table V, which would be emphasized by small sample corrections, is that neither the homogeneity nor the symmetry restrictions are rejected by the data using the dynamic model. In addition, and again unaffected by small sample adjustments, there is sufficient precision in the estimated preference parameters to reject further restrictions on long run behaviour. In direct comparison, the same conclusion would not have been reached had the (incorrect) static model been chosen as the appropriate maintained hypothesis. Table V shows that the symmetry restrictions would have been rejected by the data, a
result unaffected by small sample adjustments of the type described above and one also concluded in the Deaton and Muellbauer (1980) study.

As suggested at the outset of this section, it is of interest to compare the broad conclusions derived above with those using the trace minimization procedure described in Deaton and Muellbauer (1980). Such a procedure imposes restrictions on the disturbance variance–covariance matrix and is equivalent to using restricted least squares on the stacked system of equations in (9). Table VI summarizes the results and shows that, although the procedure imposes restrictions that can be rejected, it has little impact on the results of Table V. As one would expect, the estimated parameters are also little affected.
The estimated coefficients from model (9) with the symmetry restrictions on $\Theta$ are presented in Table VII. To interpret these parameter values, the corresponding price and income elasticities are presented in Table VIII. For a comparison, the elasticities derived from the static symmetric model are also presented. The Slutsky substitution terms are related to the compensated elasticities of Table I by:

$$k_{ij} = \tilde{e}_{ij}w_i$$

and are symmetric if $\gamma_{ij} = \gamma_{ji}$. All the terms presented in Table VIII are evaluated at the predicted (long run) budget shares for the appropriate model at the base period 1971.

In contrast to the results of Deaton and Muellbauer for the U.K., none of the own price elasticities are positive. Indeed, all the diagonal elements of the Slutsky substitution matrix are negative, a necessary condition for concavity. However, apart from the one zero eigenvalue due to the adding up constraints, the dynamic model generates one non-negative eigenvalue for a majority of the data points. The corresponding static model generates two for most data points. The statistical rejection of concavity involves deriving the asymptotic distribution of the eigenvalues which is beyond the scope of this
### TABLE VII

**Parameter estimates***

<table>
<thead>
<tr>
<th>Commodity</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$a_4$</th>
<th>$a_5$</th>
<th>$a_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dynamic model</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Food</td>
<td>0.318</td>
<td>-0.234</td>
<td>0.071</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.051)</td>
<td>(0.029)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clothing</td>
<td>0.106</td>
<td>-0.102</td>
<td>-0.078</td>
<td>0.009</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.019)</td>
<td>(0.068)</td>
<td>(0.036)</td>
<td>(0.048)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Energy</td>
<td>0.097</td>
<td>0.062</td>
<td>-0.003</td>
<td>0.036</td>
<td>0.044</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.040)</td>
<td>(0.027)</td>
<td>(0.017)</td>
<td>(0.034)</td>
<td></td>
</tr>
<tr>
<td>Transport &amp; Comm.</td>
<td>0.176</td>
<td>0.100</td>
<td>0.010</td>
<td>-0.007</td>
<td>-0.012</td>
<td>0.068</td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td>(0.040)</td>
<td>(0.029)</td>
<td>(0.027)</td>
<td>(0.029)</td>
<td>(0.031)</td>
</tr>
<tr>
<td>Recreation</td>
<td>0.303</td>
<td>0.174</td>
<td>-0.000</td>
<td>0.040</td>
<td>-0.065</td>
<td>-0.059</td>
</tr>
<tr>
<td></td>
<td>(--)</td>
<td>(--)</td>
<td>(--)</td>
<td>(--)</td>
<td>(--)</td>
<td>(--)</td>
</tr>
<tr>
<td><strong>Static model</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Food</td>
<td>0.342</td>
<td>-0.096</td>
<td>0.095</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.025)</td>
<td>(0.032)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clothing</td>
<td>0.129</td>
<td>-0.102</td>
<td>-0.011</td>
<td>-0.008</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.031)</td>
<td>(0.019)</td>
<td>(0.026)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Energy</td>
<td>0.055</td>
<td>0.014</td>
<td>0.023</td>
<td>0.017</td>
<td>0.006</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.010)</td>
<td>(0.011)</td>
<td>(0.009)</td>
<td>(0.008)</td>
<td></td>
</tr>
<tr>
<td>Transport &amp; Comm.</td>
<td>0.147</td>
<td>0.036</td>
<td>0.007</td>
<td>-0.029</td>
<td>-0.030</td>
<td>0.032</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.016)</td>
<td>(0.012)</td>
<td>(0.007)</td>
<td>(0.018)</td>
<td></td>
</tr>
<tr>
<td>Recreation</td>
<td>0.327</td>
<td>0.148</td>
<td>-0.114</td>
<td>-0.031</td>
<td>-0.016</td>
<td>0.020</td>
</tr>
<tr>
<td></td>
<td>(--)</td>
<td>(--)</td>
<td>(--)</td>
<td>(--)</td>
<td>(--)</td>
<td>(--)</td>
</tr>
</tbody>
</table>

*Note: Asymptotic standard errors are given in parentheses and (--) indicates that the parameter was derived using the adding up restrictions (6).*
### TABLE VIII

**Elasticity and Slutsky substitution terms**

<table>
<thead>
<tr>
<th>(a) Dynamic Model Estimates</th>
<th>Income elasticity</th>
<th>Price elasticities</th>
<th>Slutsky substitution terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commodity i</td>
<td></td>
<td>$e_{i1}$</td>
<td>$e_{i2}$</td>
</tr>
<tr>
<td>Food</td>
<td>0.266</td>
<td>-0.542</td>
<td>-0.013</td>
</tr>
<tr>
<td>Clothing</td>
<td>0.041</td>
<td>-0.636</td>
<td>-0.814</td>
</tr>
<tr>
<td>Energy</td>
<td>1.636</td>
<td>-0.090</td>
<td>0.307</td>
</tr>
<tr>
<td>Trans. &amp; Comm.</td>
<td>1.567</td>
<td>-0.043</td>
<td>-0.138</td>
</tr>
<tr>
<td>Recreation</td>
<td>1.577</td>
<td>-0.175</td>
<td>-0.041</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(b) Static Model Estimates</th>
<th>Income elasticity</th>
<th>Price elasticities</th>
<th>Slutsky substitution terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commodity i</td>
<td></td>
<td>$e_{i1}$</td>
<td>$e_{i2}$</td>
</tr>
<tr>
<td>Food</td>
<td>0.720</td>
<td>-0.625</td>
<td>0.064</td>
</tr>
<tr>
<td>Clothing</td>
<td>0.209</td>
<td>0.018</td>
<td>-0.959</td>
</tr>
<tr>
<td>Energy</td>
<td>1.262</td>
<td>0.403</td>
<td>0.301</td>
</tr>
<tr>
<td>Trans. &amp; Comm.</td>
<td>1.248</td>
<td>0.010</td>
<td>-0.234</td>
</tr>
<tr>
<td>Recreation</td>
<td>1.451</td>
<td>-0.498</td>
<td>-0.055</td>
</tr>
</tbody>
</table>

*Note:* As for Table VII.
paper, hence it is not possible to draw statistical inferences concerning the regularity conditions relating to consumer behaviour.

The static model, as would be expected, generally biases the income elasticities toward unity (unit income elasticities occur in the homothetic case where budget shares are unaffected by income levels). This may be interpreted as accommodating some of the lack of adjustment in the short run. Similarly, own price elasticities are over-estimated in the static model, while cross price elasticities tend to be underestimated. However, note that the static symmetric specifications from which these elasticities are derived is easily rejected by the data and should not be entertained.

5. CONCLUSIONS

This paper has attempted to apply a reasonably flexible data determined dynamic model to the explanation of movements in the distribution of consumers' expenditure in Canada. It has been shown that static, simple autoregressive and partial adjustment models tend to be rejected by the data. In addition, it turns out that consumers are much more responsive to long run relative price changes than static models suggest and that symmetric responses to such changes are not rejected by the data. Static models are also seen to understate the changes consumers make to budget shares as real income changes over time.

The results have important implications for the modelling and prediction of consumers' behaviour. They suggest that it is not sufficient to simply adjust the error specification on the usual static systems. Neither does it appear satisfactory to choose a dynamic model representing particular underlying theories of short run adjustment such as the simple habit persistence or partial adjustment models. A more general dynamic specification is required. If this procedure is not followed, estimated income and relative price responses may be biased and resulting predictions of behaviour quite inaccurate. Given that the data indicate consumers do not make instantaneous adjustments, it is unreasonable to impose restrictions derived from equilibrium consumer behaviour in each period.

DATA APPENDIX

All data is published by Statistics Canada and available from the authors on request.

The Food category contains Food, Beverages and Tobacco; Clothing contains Clothing, Footwear and Laundry; Energy contains Electricity, Gas and Other Fuels; Transport and Communications contains Current Transport and Communications less New Cars and finally, Recreation contains Recreation, Education and Culture plus Personal Goods and Services less Durable Sport and Camping Equipment.

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