Effects of Federal Risk Management Programs on Optimal Acreage Allocation and Nitrogen Use in a Texas Cotton–Sorghum System

Sangtaek Seo, Paul D. Mitchell, and David J. Leatham

We analyze the effects of crop insurance and the Marketing Loan Program on optimal nitrogen use and acreage allocation for a case cotton–sorghum farm in Texas. A mathematical programming model is used to solve for the optimal nitrogen fertilizer rate, crop acreage allocation, coverage level, and price election factor, along with participation in the crop insurance and the Marketing Loan Program for both crops. Results show that depending on the crop and farmer risk aversion, these federal risk management programs increase or decrease optimal fertilizer rates −6% to 3%, increase optimal cotton acreage 94% to 144%, and decrease sorghum acres up to 50%.

Key Words: crop insurance, extensive margin, intensive margin, loan deficiency payments, revenue insurance

JEL Classifications: Q12, Q18

Federal risk management programs such as federal crop insurance and the Marketing Loan Program (MLP) have effects beyond directly improving farmer welfare. The income and risk changes that result from farmer participation in these and similar programs affect crop acreage allocations (the extensive margin) and the use of inputs on each crop (the intensive margin). The extensive and intensive margin effects are important because they can enhance or counteract the goals of other programs. These effects can induce farmers to increase or decrease acreage of more erosive or chemically intensive crops, or to use more or less chemicals on land already allocated to specific crops. For example, Goodwin and Smith find that about half of the reductions in soil erosion due to the Conservation Reserve Program (CRP) were offset by increases in erosion resulting from farmer responses to income support programs. Similarly, Babcock and Hennessy and Smith and Goodwin find that farmers purchasing crop insurance have incentives to reduce use of fertilizer and other

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chemicals, while Horowitz and Lichtenberg find that crop insurance increases the use of agricultural chemicals.

The extensive and intensive margin effects of federal risk management programs continue to be a pertinent issue because the availability and subsidization of federal risk management programs have increased in recent years. The Agricultural Risk Protection Act (ARPA) of 2000 resulted in increased premium subsidies and an expansion in the types of policies, the crops covered, and geographic availability. Total insured acres increased from 182 million in 1998 to 216 million in 2002, with total liability increasing from $28 billion to $37 billion (U.S. Department of Agriculture–Risk Management Agency [USDA–RMA] 2002a). Among the most popular insurance programs are Actual Production History (APH) yield insurance and Crop Revenue Coverage (CRC) revenue insurance, with liabilities in 2002 of $15 billion and $8 billion, respectively (USDA–RMA 2002a). The Farm Security and Rural Investment Act of 2002 continued the Marketing Loan Program (MLP), which provides loan deficiency payments as a form of price insurance that protects farmers from low prices, much as APH protects from low yields. Loan deficiency payments equaled $6 billion for the 2000 crop year (USDA–Farm Service Agency [USDA–FSA] 2002a).

Several studies have analyzed the effects of crop insurance and other federal risk management programs to quantify their intensive and/or extensive margin effects and interactions among different programs. Econometric, simulation, and mathematical programming approaches have been used, each with respective strengths and weaknesses.

Econometric studies concerning the effect of insurance and federal programs on input use and acreage allocations have the strength of using actual producer behavior. However, this strength is accompanied by a lack of detail in modeling crop insurance and/or input use and can generally only provide ex post policy analysis. For example, crop insurance participation is typically modeled as a dummy variable and input use is described as total per acre expenditures on all fertilizers and/or chemical inputs (Goodwin and Smith; Horowitz and Lichtenberg; Smith and Goodwin; Wu). Detail is missing concerning specific inputs and which insurance program is used (e.g., APH, CRC, Revenue Assurance, Group Risk Plan, Income Protection) and the chosen coverage level and price election for each. Hence, specific conclusions concerning specific programs and/or specific inputs usually cannot be developed, only generalizations concerning aggregate effects. Furthermore, most econometric studies examine the intensive margin or the extensive margin effects of crop insurance in isolation, though Wu is an exception. Finally, econometric studies are based on farmer responses to existing programs, making ex ante evaluation of new policies difficult if the policy departs substantially from existing policies.

Analyses using simulation or mathematical programming approaches overcome the problem of the lack of detail associated with an econometric approach by creating optimization models that endogenize the choice of insurance participation and/or input use. As a result, such approaches can analyze the effect of new policies types, coverage levels, or premium subsidies before data are available. For example, before data existed on farmer responses to newly available revenue insurance policies or decoupled transition payments, Babcock and Hennessy and Hennessy examined the effect of these programs on the optimal nitrogen fertilizer rate for a representative Iowa corn farmer. However, this added detail and ability for ex ante analysis generally comes at the cost of losing the ability to generalize beyond the specifics assumed.

Simulation approaches can use commonly accepted yield and price distributions (e.g., beta density for yields and lognormal density for prices [Goodwin and Ker]) and can impose desired correlations, i.e., negative correlation between crop yield and price due to downward sloping demand. However, because optimization requires combining a grid search with Monte Carlo integration (Babcock and Hennessy; Hennessy), endogenizing input use, the acreage allocations, crop insurance coverage levels, and price elections becomes cumbersome. As a result, simulation approaches typ-
ically do not simultaneously examine intensive and extensive margin effects, nor do they endogenize crop acreage allocations and insurance participation variables such as coverage levels and price elections (Babcock and Hennessy; Hennessy).

Mathematical programming can endogenize numerous variables (e.g., input use, acreage allocation, coverage levels, and price elections), but traditionally it has difficulty incorporating stochastic and correlated yields and prices. As a result, most mathematical programming analyses typically make restrictive distributional assumptions or use limited empirical data to specify stochastic crop yields and prices.

The use of quadrature to directly maximize expected utility requires specifying the joint distribution function for prices and yields (Kaylen, Preckel, and Loehman). However, specifying this joint distribution function is impossible for marginal densities commonly used for crop yields and prices with correlations among all yields and prices (e.g., beta density for yields and lognormal density for prices). To address this limitation, Kaylen, Loehman, and Preckel assume independence between crop yield and price and between the different crop prices, and assume perfect correlation between yields of the different crops, while Coble, Heifner, and Zuniga use a hyperbolic tangent transformation to approximate normality.

Some mathematical programming analyses capture the randomness and correlation in prices and yields by using historical yield and price data series as an empirical joint distribution. However, these data tend to be limited in length—for example, Turvey uses 21 years of data, while Stokes, Coble and Dismukes use 34 years. These stochastic programming analyses assume equal probability for each year of data, which poorly predicts the weight in the tails of these distributions, which is especially problematic for crop insurance studies (see Goodwin and Ker’s critique). Furthermore, these studies typically use county-average yield data, which, as Coble, Heifner, and Zuniga point out, underpredicts farm-level yield variability because the county data average over several farms. Most crop insurance policies pay indemnities based on farm-level yields, so farm yield data should be used or simulated.

Our contribution for empirical analysis of the intensive and extensive margin effects of federal risk management programs is to combine the mathematical programming optimization method of Lambert and McCarl with Richardson and Condra’s method of generating multiple correlated random variables. Lambert and McCarl developed a direct expected utility maximizing nonlinear programming to maximize expected utility when income is stochastic. However, their method of generating correlated random variables is not clearly explained (p. 850) and likely not usable for generating large sets of correlated random variables. Richardson and Condra’s method for drawing correlated random variables is clearly explained and well established. Though Richardson and Condra’s original application also linked mathematical programming with simulation, it did not include crop insurance and the representative farmer maximized expected returns, so risk aversion was not incorporated. Nevertheless, the method has much to offer, and its potential has gone unnoticed for analysis of the intensive and extensive margin effects of crop insurance. We are aware of no other crop insurance study combining mathematical programming and simulation as we describe here.

Our primary goal is to illustrate the combination of these two methods for analyzing the intensive and extensive margin effects of federal risk management programs. The method is more broadly applicable, but this application seemed especially pertinent because the issue remains policy relevant and several empirical studies already exist that could benefit from the method. First, we briefly review federal risk management programs, and then we describe the model’s objective function and constraints. Next, we explain the data and estimation of model parameters for a representative Texas cotton–sorghum farmer. Finally, we present and discuss our empirical results relative to previously published results.
Federal Risk Management Programs

A farmer with APH insurance coverage receives an indemnity if the harvested yield is less than the yield guarantee. Farmers choose a yield coverage level ranging from 50% to 75% (up to 85% in some counties) by 5% increments of their approved APH yield (a moving average of their harvested yields) and a price election factor ranging from 55% to 100% by 1% increments of the officially announced expected market price. A farmer with CRC insurance receives an indemnity if the guaranteed revenue exceeds actual revenue. The price for calculating revenue is derived from the daily settlement price of futures contracts for a given period for an appropriate month for the crop. Again, the farmer must choose a coverage level (50% to 85% by 5% increments) and either a 95% or 100% price election. Farmers receive a smaller indemnity with CRC than with APH when the realized market price used to calculate the APH indemnity exceeds the CRC base price or harvest price used to calculate CRC indemnities. Farmers participating in the MLP receive a loan deficiency payment (LDP) when the marketing loan rate exceeds the posted county price or the world market price depending on the crop. An LDP can be utilized when the eligible crop is still owned by the farmer at the time of harvest.

The specified model includes all eight possible combinations of APH crop insurance, CRC revenue insurance, and the MLP. In each case, the participation in insurance programs and/or the MLP is chosen separately for each crop among the available alternatives, so that the insurance policy type, the coverage level, and price election factor can differ for each crop. The eight combinations (and their abbreviations) are: no program, the MLP only, APH crop insurance only, APH crop insurance with the MLP (APH + MLP), CRC revenue insurance only, CRC revenue insurance with the MLP (CRC + MLP), both APH crop insurance and CRC revenue insurance available (APH + CRC), and both APH crop insurance and CRC revenue insurance available with the MLP (APH + CRC + MLP).

Conceptual Framework

The modeled representative farmer earns income by allocating total acreage A and a purchased input x to crops j = 1 to J. The farmer can also purchase crop yield or revenue insurance and choose to participate in the MLP. Thus, the farmer also chooses the price election factor \((PEF_{ij})\) and coverage level \((CVG_{ij})\) for each insurance policy \(i = 1\) to \(I\) and crop \(j = 1\) to \(J\). The farmer can purchase only one type of insurance for each crop and if a crop is insured, all planted acres of that crop are insured, all with the same price election and coverage level. However, the farmer can purchase different types of insurance for different crops. These restrictions are in accordance with current federal crop insurance programs. Finally, a realization of the random variables is a state of nature and is indexed by \(k\).

For the most general case when all risk management programs are available, per acre income with crop insurance program \(i\) and crop \(j\) in state of nature \(k\) is:

\[
\pi_{jk} = p_{jk}y_{jk}(x_j) - c_j - rx_j + d_jLDP_{jk}(p_{jk}) + \sum_i I_{ijk}(PEF_{ij}, CVG_{ij}) - M_{ij}(PEF_{ij}, CVG_{ij}),
\]

where \(p_{jk}\) is the realized crop price, \(y_{jk}\) is the realized crop yield as a function of the input level \(x_j\), \(c_j\) is the nonrandom variable cost, and \(r\) is the nonrandom price of the input \(x\). \(LDP_{jk}\) is the realized loan deficiency payment that depends on the realized crop price \(p_{jk}\) and \(d_j\) is an indicator variable for participation in the MLP (\(d_j = 1\) if the farmer chooses to participate, 0 otherwise). \(I_{ijk}\) is the realized insurance indemnity and \(M_{ij}\) is the nonrandom insurance premium for policy \(i\), which both depend on the chosen price election factor \((PEF_{ij})\) and coverage level \((CVG_{ij})\). Because only one type of insurance can be purchased for any crop \(j\), at most \(PEF_{ij} > 0\) and \(CVG_{ij} > 0\) for only one policy \(i\) for each crop \(j\). Hence, when summing indemnities and premiums over the index \(i\), at most only one policy will have a positive premium and (possibly) indemnity.
For crop \( j \), realized income in state \( k \) is \( A_j \pi_{jk} \), where \( A_j \) is acreage planted to crop \( j \). Total realized income in state \( k \) is the sum of realized income over all crops: \( \pi_k = \Sigma_j A_j \pi_{jk} \). Random income from crop production is \( \pi_j \), which depends on random prices \( p_1 \) to \( p_j \) and random yields \( y_1 \) to \( y_j \) with the joint distribution \( F(\cdot) \). A specific realization of this random income is denoted \( \pi_{tk} \). Similarly, random per acre income from crop \( j \) is \( \pi_j \) and is as reported in Equation (1), except that all random variables have no realized state of nature index \( k \).

The representative farmer maximizes the expected utility of income, choosing the acreage allocation \( A_j \), input use \( x_j \), and participation in the MLP \( j \), the price election factor \( PEF_{ij} \) and coverage level \( CVG_{ij} \) for all \( i \) and \( j \), and insurance program \( i \):

\[
\max_{A_j,x_j,PEF_{ij},CVG_{ij},d_j} \int u(\pi) \, dF \times (p_1, p_2, \ldots, p_j, y_1, y_2, \ldots, y_j),
\]

where \( u(\cdot) \) is the farmer's utility function \((u' > 0, u'' < 0)\). Constraints include an acreage allocation constraint \((A \geq \Sigma_j A_j)\) and technical constraints on the insurance programs (e.g., one policy per crop, and a \( PEF \) and a \( CVG \) from available levels). Solving this optimization problem, we get the optimal acreage allocation and input use for each crop \((A_j \) and \( x_j \) for all \( j \)), as well as the optimal participation in risk management programs \((PEF_{ij}, CVG_{ij} \) for all \( i \) and \( j \), and \( d_j \) for all \( j \)).

Direct expected utility maximizing nonlinear programming (DEMP) solves this optimization problem using realizations of the random variables (Lambert and McCarl). Specifically, for a given utility function, DEMP solves problem (2) as:

\[
\max_{A_j,x_j,PEF_{ij},CVG_{ij},d_j} \sum_k u(\pi_{tk}),
\]

where \( \pi_{tk} = \Sigma_j A_j \pi_{jk} \) and \( \pi_{jk} \) is defined by Equation (1). Our solution for the empirical issue of where to obtain realizations of prices and yields is explained in the empirical section.

For DEMP, the APH and CRC insurance indemnities for any state \( k \) and crop \( j \) are

\[
(4a) \quad I_{APH,jk} = PEF_{APH,j} p_j^f \max(CVG_{APH,j} \bar{y}_j - y_{jk}, 0),
\]

\[
(4b) \quad I_{CRC,jk} = \max(PEF_{CRC,j} \max(p^f_j, p_j^*), CVG_{CRC,j} \bar{y}_j - (p_{jk} y_{jk}), 0),
\]

where \( \bar{y}_j \) is the average yield used by both APH and CRC, \( p_j^f \) is the expected price used to calculate the APH indemnity, and \( p_j^* \) and \( p_{jk} \) are the futures price before planting (base price) and the futures price before harvest (harvest price) used to calculate CRC indemnities.

The nonrandom insurance premium for each crop depends on the coverage level and the price election factor. The actual (subsidized) premium the representative farmer would pay can be determined (USDA–RMA 2002c). The expected net indemnity is the expected difference between the indemnity and the premium. Because the premium is nonrandom, the expected net indemnity is the expected indemnity minus the actual premium. Because the integration required to calculate the expected indemnity is analytically intractable, Monte Carlo integration is used to numerically estimate the expected indemnity (Greene, pp. 181–183). Hence, the expected indemnity for each policy is the average indemnity over all states \( k \): \( \Sigma_k I_{ijk}(PEF_{ij}, CVG_{ij}); \)

The per acre loan deficiency payment \((LDP)\) for any crop \( j \) in state \( k \) is

\[
(5) \quad LDP_{jk} = \max(MLR_j - p_{jk}, 0)y_{jk},
\]

where \( MLR_j \) is the marketing loan rate set for crop \( j \). The marketing loan rate guarantees a minimum price and so this program essentially serves as price insurance without a premium.

This model specification is completely static. Monte Carlo draws for each state of nature \( k \) do not represent different years. Rather, because maximizing expected utility of income with the specified joint price and yield distribution is analytically intractable, the Monte Carlo draws simulate the joint distribution for numerical maximization of expected utility. As a result, the model incorporates
no dynamic effects, such as the effect of current input and acreage choices on the mean and variance of crop yields which then change future insurance premiums.

Once optimal solutions have been determined, the intensive margin effect of each risk management program for a crop is the difference in the optimal use of the input $x_i$ when the program is available versus when it is not. Similarly, the extensive margin effect is the change in optimal acreage $A_j$ when the program is available versus when it is not. Determining the intensive and extensive margin effects of these federal risk management programs requires finding the solution to problem (2) for the eight possible combinations of program availability. However, once the details of each program are accurately specified, analytical solutions generally become intractable, so that we use DEMP to solve problem (3) for a representative farmer and use sensitivity analysis to generalize from this specific case.

**Empirical Model**

Following other analyses, we use a negative-exponential utility function (Babcock and Hennessy; Kaylen, Loehman, and Preckel; Lambert and McCarl). As a result, wealth effects (including those from premiums) do not affect production decisions, and so all other income is ignored. Hence, the utility function for problem (3) is $u(\pi_t) = 1 - \exp(-R\pi_t)$, where $R > 0$ is the coefficient of absolute risk aversion. Values for $R$ were chosen so the farmer’s risk premium was a reasonable percentage of the income standard deviation (Babcock, Choi, and Feinerman), which also satisfies the upper bound suggested by McCarl and Bessler.

For empirical analysis, we develop a representative farm for San Patricio County, Texas, an important cotton–sorghum area. Texas accounted for 41% and 33% of total U.S. planted acres of cotton and sorghum, respectively, in 2002, and San Patricio County accounted for 2.2% and 2.9% of total cotton and sorghum acres planted in Texas in 2002 (U.S. Department of Agriculture–National Agriculture Statistics Service [USDA–NASS]). Using USDA–NASS agricultural census data as a guide, the representative farm allocates 1,700 acres between cotton and sorghum. In San Patricio County, both APH and CRC are available for both cotton and sorghum, with available APH and CRC coverage levels ranging 50% to 85% for cotton and 50% to 75% for sorghum, both by 5% increments. The available APH price election factor ranges from 55% to 100% by 1% increments, but with CRC the price election factor is either 95% or 100% (USDA–RMA 2002c). The marketing loan rate for this region in 2002 was $0.52/lb for cotton and $2.17/bu for sorghum (USDA–FSA 2002b).

**Prices, Yields, and Correlations**

Table 1 reports the price and yield parameters used for the empirical analysis; the source of these parameters is summarized here. Yield trends in USDA–NASS county-average yields for cotton and sorghum from 1980 to 2001 were estimated using least squares. Because no statistically significant yield trend was found at the 5% level of significance, the average of county average yields from 1997 to 2000 is used for the mean yield for each crop. Because field-level yield variability is greater than the variability of county average yield, the empirical analysis uses a yield standard deviation 1.5 times greater than that for the 1980 to 2001 county average yields, so each yield coefficient of variation is comparable to results for farm level yields from crop insurance studies (Coble, Heffner, and Zuniga).

To be consistent with the yield assumptions, we used the four-year state average price from 1997 to 2000 for the mean price, with price standard deviations estimated using prices from 1980 to 2001. USDA–RMA (2002b) reports APH price guarantees and CRC base prices (futures price before planting) for 2002. The base price is used for the CRC harvest price for both crops because it is a commonly used estimate of the harvest price at planting time. The 2002 marketing loan rate (i.e., the MLP guaranteed price floor) for this area was $0.52/lb for cotton and $2.17/bu for sorghum (USDA–FSA 2002b). Crop budgets report the
Table 1. Parameter Values Used for Empirical Analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Cotton</th>
<th>Sorghum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield mean</td>
<td>677.5 lbs/acre</td>
<td>70.0 bu/acre</td>
</tr>
<tr>
<td>Yield standard deviation</td>
<td>252.5 lbs/acre</td>
<td>18.8 bu/acre</td>
</tr>
<tr>
<td>Price mean</td>
<td>$0.51/lbs</td>
<td>$1.98/bu</td>
</tr>
<tr>
<td>Price standard deviation</td>
<td>$0.10/lbs</td>
<td>$0.42/bu</td>
</tr>
<tr>
<td>APH price guarantee</td>
<td>$0.50/lbs</td>
<td>$1.85/bu</td>
</tr>
<tr>
<td>CRC base price</td>
<td>$0.42/lbs</td>
<td>$2.18/bu</td>
</tr>
<tr>
<td>Marketing loan rate</td>
<td>$0.52/lbs</td>
<td>$2.17/bu</td>
</tr>
<tr>
<td>Nitrogen price</td>
<td>$0.20/lbs</td>
<td>$0.20/lbs</td>
</tr>
<tr>
<td>Base nitrogen rate</td>
<td>75.0 lbs/acre</td>
<td>60.0 lbs/acre</td>
</tr>
<tr>
<td>Variable cost of production</td>
<td>$316.40/acre</td>
<td>$116.70/acre</td>
</tr>
</tbody>
</table>

Correlation Matrix

<table>
<thead>
<tr>
<th></th>
<th>Cotton</th>
<th>Sorghum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cotton</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yield</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Price</td>
<td>-0.30</td>
<td>1</td>
</tr>
<tr>
<td>Sorghum</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yield</td>
<td>0.58</td>
<td>-0.36</td>
</tr>
<tr>
<td>Price</td>
<td>-0.39</td>
<td>0.43</td>
</tr>
</tbody>
</table>

*Sources reported in text.

The price of nitrogen, base nitrogen application rates, and the variable costs of production for both crops (Texas Cooperative Extension). Following these budgets, cotton seed proportionally increases cotton revenue by 12%.

USDA–NASS county average yield and state price data from 1980 to 2001 were used to estimate the price–yield correlation matrix. Because county data normally have higher correlation between price and yield than farm-level data, own price–yield correlations were reduced by one fourth to obtain values comparable to those reported by Coble, Heifner and Zuniga. Imposing appropriate own and cross price and yield correlations is necessary because, for example, a negative own price–yield correlation incorporates at the farm level the moderating effects of market prices on acreage allocations, while a positive crop yield correlation prevents overestimation of the benefit from crop diversity due to common yield shocks.

Cotton has a larger yield coefficient of variation than sorghum (37.3% versus 26.8%) and a similar price coefficient of variation as sorghum (21.2% versus 20.4%). As a result, cotton is generally considered the riskier crop: without any risk management programs, cotton income has a larger mean and standard deviation—$60.00/acre and $142.90/acre, respectively, versus $29.30/acre and $40.60/acre for sorghum.

**Crop Production Function**

Random crop yield has a beta distribution with mean and variance that depend on the nitrogen fertilizer rate. The beta distribution is commonly used for crop insurance analyses (Goodwin and Ker review several examples). The beta density function for yield \( y \) is

\[
b(y) = \frac{(y - A)^{\nu - 1}(B - y)^{\nu - 1}\Gamma(\nu + \gamma)}{(B - A)^{\nu + \gamma - 1}\Gamma(\nu)\Gamma(\gamma)}.
\]

\( A \) is the minimum, \( B \) is the maximum, \( \nu \) and \( \gamma \) are shape parameters, and \( \Gamma(\cdot) \) is the gamma function (Evans, Hastings, and Steffey). Following Mitchell, Gray, and Steffey, we determine the conditional beta density for crop yields by first specifying the mean and variance of yield as functions of the fertilizer rate, and then deriving the implied functions for the parameters \( \nu \) and \( \gamma \).

When crop yield has a beta density, mean
yield is \( \mu_y = (B - A)\nu/(\nu + \gamma) \) and the yield variance is \( \sigma^2_y = (B - A)^2\nu\gamma/[(\nu + \gamma)^2(\nu + \gamma + 1)] \). Solving these equations for \( \nu \) and \( \gamma \) gives:

\[
\nu = \frac{(\mu_y - A)^2(B - A) - \sigma^2_y(\mu_y - A)}{\sigma^2_x(B - A)},
\]

\[
\gamma = \frac{(\mu_y - A)(B - A)^2 - \sigma^2_y(B - A)}{\sigma^2_x(B - A)}.
\]

Using a conditional beta density for crop yield requires specifying or estimating the mean \( \mu_y \) and the variance \( \sigma^2_y \) as functions of the nitrogen fertilizer rate, and then substituting these functions into Equations (7) and (8) to obtain equations for \( \nu \) and \( \gamma \) that depend on the fertilizer rate.

Functions for the dependence of the mean and variance of cotton yield on the nitrogen application rate were estimated using unpublished data from experiments conducted from 1999 to 2002 in San Patricio, Calhoun, and Wharton counties in the Texas Coastal Bend cotton growing region (Mcfarland). Nitrogen fertilizer rates were experimentally varied from 0 to 150 lbs/acre and cotton lint yields measured for each plot for a total of 124 observations. A beta density with a minimum of 0 lbs/acre and a maximum of 2000 lbs/acre was estimated for cotton yield using maximum likelihood. Polynomial terms in the fertilizer rate were added successively for both the mean and the variance until a likelihood ratio test identified the best model. The final model used quadratic equations for the mean (\( \mu_y \)) and variance (\( \sigma^2_y \)) of cotton yield as a function of the nitrogen rate (\( x_c \)): \( \mu_y(x_c) = 853 + 2.92x_c - 0.0111x_c^2 \) and \( \sigma^2_y(x_c) = 37,600 - 569x_c + 6.72x_c^2 \) (with respective standard errors 33.93, 1.134, 0.007854, 8.562, 305.7, and 2.411). Most estimates were significant at the 1% level. Only the mean’s quadratic term was not significant at the 1% level (its \( p \) value is 0.156), but is retained to ensure a concave production function.

For the estimated model, the expected profit-maximizing nitrogen application rate and associated mean and variance of yield did not match those derived from crop budgets and observed county data as reported in Table 1. Hence, estimated coefficients were calibrated so that optimal risk neutral fertilizer rate and the mean and variance of yield at this rate would match the values in Table 1 from observed data. To maintain the original shape, we calibrated using an intercept shift \( I_m \) to move the mean vertically and a nitrogen rate shift \( N_m \) to move the mean horizontally, i.e., \( \mu_c(x_c) = 8.53 - I_m + 2.92(x_c - N_m) - 0.0111(x_c - N_m)^2 \). Calibration for the variance only used an intercept shift \( I_s \) to shift the variance vertically. Identified values were \( I_m = 364.1, N_m = -38.9, \) and \( I_s = -31,045 \), implying the following equations for the mean and variance of cotton yields as a function of the nitrogen application rate:

\[
\mu_c = 586 + 2.06x_c - 0.0111x_c^2,
\]

\[
\sigma^2_c = 68600 - 569x_c + 6.72x_c^2.
\]

Similar experimental data were not available for sorghum, so published estimates from Preckel, Loehman, and Kaylen were used. After converting coefficients from the reported pounds per acre to bushels per acres assuming 56 lbs/bu for sorghum, sorghum yield coefficients were calibrated just as for cotton. Identified values were \( I_m = -18.7, N_m = 2.7, \) and \( I_s = -289 \), so that calibrated equations for the mean (\( \mu_y \)) and variance (\( \sigma^2_y \)) of sorghum yield as a function of the nitrogen rate (\( x_g \)) are:

\[
\mu_g = 55.8 + 0.373x_g - 0.00227x_g^2,
\]

\[
\sigma^2_g = 363 - 1.29x_g + 0.0271x_g^2 - 0.000140x_g^2.
\]

Crop rotation has a positive effect on crop yields because of a variety of factors (Hague and Overstreet; Matocha et al.). To capture this effect, we assigned acres consecutively planted to the same crop a 5% lower mean yield. Thus, if the farm modeled here plants 1,000 acres of cotton from the total of 1,700 available acres, 850 acres of cotton have a mean yield determined as reported by Equation (9), while the remaining 150 acres of cotton have a 5% lower mean yield.
Table 2. Optimal Farmer Choices without the Marketing Loan Program (MLP)

<table>
<thead>
<tr>
<th>Government Programb</th>
<th>Moderately Risk Aversea</th>
<th>Highly Risk Aversea</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cotton</td>
<td>Sorghum</td>
</tr>
<tr>
<td>No program</td>
<td>65.4</td>
<td>58.8</td>
</tr>
<tr>
<td>APH only</td>
<td>71.2</td>
<td>58.5</td>
</tr>
<tr>
<td>CRC only</td>
<td>67.4</td>
<td>50.1</td>
</tr>
<tr>
<td>APH and CRCc</td>
<td>71.6</td>
<td>50.7</td>
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<td></td>
<td>Optimal Nitrogen Fertilizer Rate (lbs/acre)</td>
<td></td>
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<tr>
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<td>1,111</td>
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<td>APH only</td>
<td>850</td>
<td>850</td>
</tr>
<tr>
<td>CRC only</td>
<td>748</td>
<td>952</td>
</tr>
<tr>
<td>APH and CRCc</td>
<td>850</td>
<td>850</td>
</tr>
<tr>
<td></td>
<td>Optimal Acreage Allocation (acre)</td>
<td></td>
</tr>
<tr>
<td>Government Programb</td>
<td>Optimal Insurance Coverage Level (%)</td>
<td></td>
</tr>
<tr>
<td>No program</td>
<td>—</td>
<td>CAT\textsuperscript{d}</td>
</tr>
<tr>
<td>APH only</td>
<td>75</td>
<td>70</td>
</tr>
<tr>
<td>CRC only</td>
<td>60</td>
<td>70</td>
</tr>
<tr>
<td>APH and CRCc</td>
<td>75</td>
<td>70</td>
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</table>

\textsuperscript{a} Coefficients of absolute risk aversion are $4.0 \times 10^{-6}$ and $7.0 \times 10^{-6}$ for moderately and highly risk averse, respectively.
\textsuperscript{b} Abbreviations are Actual Production History (APH) yield insurance and Crop Revenue Coverage (CRC) revenue insurance.
\textsuperscript{c} APH for cotton and CRC for sorghum is optimal when both insurance programs are available.
\textsuperscript{d} CAT denotes catastrophic insurance with 55% price election and no premium.

Model Implementation

The General Algebraic Modeling System (GAMS) solved the mathematical programming model using the nonlinear program (NLP) solver or the simple branch and bound (SBB) solver to maximize the expected utility of random profit with randomly drawn correlated prices and yields determining realized profit for each draw. The optimal fertilizer rate is determined for each crop as an integer variable for fertilizer rates in 0.1 lb/acre increments. Output was examined to ensure that the optimal fertilizer rate was never on the boundary. GAMS was linked to Excel using the GDXXRW program distributed with GAMS to draw correlated prices and yields with the means and variances implied by the crop-specific fertilizer rates. GAMS sends the means and variances of yields (as a function of the fertilizer rate) and prices to Excel, which generates appropriately correlated yields and prices using the method of Richardson and Condra. This method begins by transforming a matrix of uncorrelated uniform random variables using the square root of the correlation matrix to generate appropriately correlated uniform random variables. Next, Excel's beta or normal inverse cumulative distribution function transforms these uniform random variables into yields with a beta distribution or prices with a lognormal distribution. Experimentation indicated that 5,000 random draws were needed for model results to stabilize.

Empirical Results and Discussion

Tables 2 and 3 report the optimal fertilizer application rates, acreage allocations, and insurance coverage levels when the current subsidized insurance is available. Table 2 reports results without the MLP, and Table 3 reports results with the MLP. Results are not reported for the price election factor \textit{PEF} because the optimum in all cases is the maximum available—55% with catastrophic APH and 100% for all other policies and coverage levels.

Table 2 shows that APH and CRC insurance both have a positive effect on the optimal nitrogen fertilizer rate for cotton and a negative effect for sorghum. Depending on the farmer's level of risk aversion, the optimal rate
Table 3. Optimal Farmer Choices with the Marketing Loan Program (MLP)

<table>
<thead>
<tr>
<th></th>
<th>Cotton</th>
<th>Sorghum</th>
<th>Cotton</th>
<th>Sorghum</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Moderately Risk Averse</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Government Program</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MLP only</td>
<td>64.1</td>
<td>61.6</td>
<td>63.2</td>
<td>60.7</td>
</tr>
<tr>
<td>APH and MLP</td>
<td>70.3</td>
<td>61.5</td>
<td>63.0</td>
<td>57.1</td>
</tr>
<tr>
<td>CRC and MLP</td>
<td>67.3</td>
<td>55.9</td>
<td>65.5</td>
<td>50.2</td>
</tr>
<tr>
<td>APH+CRC+MLP</td>
<td>67.5</td>
<td>56.2</td>
<td>63.0</td>
<td>55.2</td>
</tr>
<tr>
<td><strong>Highly Risk Averse</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Government Program</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MLP only</td>
<td>743</td>
<td>957</td>
<td>363</td>
<td>1,337</td>
</tr>
<tr>
<td>APH and MLP</td>
<td>850</td>
<td>850</td>
<td>850</td>
<td>850</td>
</tr>
<tr>
<td>CRC and MLP</td>
<td>850</td>
<td>850</td>
<td>513</td>
<td>1,187</td>
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<tr>
<td>APH+CRC+MLP</td>
<td>1,143</td>
<td>557</td>
<td>850</td>
<td>850</td>
</tr>
</tbody>
</table>

\* Coefficients of absolute risk aversion are $4.0 \times 10^{-6}$ and $7.0 \times 10^{-6}$ for moderately and highly risk averse, respectively.
\* Abbreviations are Actual Production History (APH) yield insurance, Crop Revenue Coverage (CRC) revenue insurance, and Marketing Loan Program (MLP) price supports.
\* APH for cotton and CRC for sorghum are optimal when both insurance programs are available.
\* CAT denotes catastrophic insurance with 55% price election and no premium.

for cotton increases up to 6 lbs/acre (9%) and decreases at most 18 lbs/acre (31%) for sorghum. Crop insurance has a large effect on the optimal acreage allocation. When both APH and CRC are available, cotton acres increase 44% to 120%, with an appropriate decrease in sorghum acres, and the optimal purchase is APH for cotton and CRC for sorghum, with optimal coverage levels the same as when each policy is available alone.

Comparing Tables 2 and 3 indicates the effect of the MLP. With the MLP, optimal nitrogen rates decrease up to 4% for cotton and increase up to 13% for sorghum, depending on risk aversion, implying that the MLP and crop insurance counteract each other. As a result, when both crop insurance and the MLP are available, the net effect on fertilizer rates is ambiguous, depending on which effect dominates. Thus, when the MLP and both insurance policies are available, at a low level of risk aversion, the optimal nitrogen rate for cotton increases 3% relative to the no-program case, but decreases 3% at a high level of risk aversion. For sorghum, the optimal rate decreases 4% to 6% for sorghum at both levels of risk aversion.

With the MLP, optimal cotton acres increase up to 35% depending on the program and farmer risk aversion, with an accompanying decrease in sorghum acres. The only exception is the difference between the no-program and the MLP-only cases for a highly risk-averse farmer, for which optimal sorghum acres increase about 62%. This case differs because for the no program case, it is optimal for a highly risk-averse farmer not to plant all available acres, but once the MLP is available, it becomes optimal to plant all 1,700 acres, with a net decrease in cotton acres. When the MLP and both insurance policies are available, optimal cotton acres increase around 94% to 144% relative to the no-program case. Lastly, the MLP has no effect on insurance participation and only in a few instances does it slightly affect optimal coverage levels.

This shift in the optimal acreage allocation from sorghum to cotton matches NASS county acreage data for San Patricio County, TX. From 1997 to 1999, on average there were
about 1.5 times more sorghum acres than cotton acres planted. However, in 2001 and 2002, there were almost 1.5 times more cotton acres than sorghum acres planted, after changes in the crop insurance program were implemented as a result of the passage of ARPA in 2000.

The results in Tables 2 and 3 also show that as farmer risk aversion increases, the optimal nitrogen rate decreases for all alternatives regardless of the crop. This result occurs because nitrogen is used as a variance (risk) increasing input in this study. In addition, optimal cotton acreage decreases and optimal sorghum acreage increases because cotton is the riskier crop. For the range of risk-aversion levels explored, the optimal insurance coverage level did not change for cotton, but increased for sorghum. To understand this result, Table 4 reports the expected net indemnity (expected indemnity minus the premium) for each case.

Table 4 shows that the optimal coverage level is slightly higher than the coverage level with the largest net indemnity because the added risk benefit that higher coverage provides exceeds the small decrease in the expected net indemnity. For sorghum APH and CRC, and for cotton CRC, the net indemnities are negative, i.e., premiums exceed expected losses for this farmer. Table 4 shows why APH is optimal for cotton and CRC for sorghum when both policies are available. The APH net indemnity is noticeably larger for cotton APH, while for sorghum, the case is reversed. These results are consistent with the actual farmer behavior in San Patricio County. In 2002, 98.6% of farmers in the county buying crop insurance for cotton bought APH and 62.3% of those buying crop insurance for sorghum bought CRC (USDA–RMA 2002d).

The magnitude and direction of intensive and extensive margin effects vary according to the crops and regions as a result of differing inputs and crops on the variability of income. Our positive effect of crop insurance on the optimal nitrogen fertilizer rate for cotton and the negative effect for sorghum are consistent with the ambiguous results in the literature. Horowitz and Lichtenberg, in their econometric study, find that crop insurance increases fertilizer use for corn in the Midwest. However, Smith and Goodwin, in their econometric study of wheat farmers in Kansas, find that crop insurance decreases fertilizer use, as do Babcock and Hennessy in their simulation-based analysis of corn in Iowa. Our results are generally consistent with the results of Wu’s econometric analysis of Nebraska corn–soybean farmers because he finds that crop insurance increases fertilizer use and acreage for the riskier crop (corn). Similarly, Chavas and Holt find that price supports similar to the current MLP create moderate acreage increases in the supported crop (corn) and that cross-commodity risk reductions are important to consider, much as we find. Turvey focused only on acreage effects and found that the Canadian crop insurance program increases optimal acreage devoted to riskier crops, just as we find for the U.S. insurance program.

Table 5 reports farmer certainty equivalents for the optimal choices reported in Tables 2 and 3. From the farmer’s perspective, having all three federal risk management programs
Table 5. Certainty Equivalent and Mean and Standard Deviation of Profit ($1,000s) with Optimal Farmer Choices

<table>
<thead>
<tr>
<th>Government Program</th>
<th>No Program</th>
<th>APH only</th>
<th>CRC only</th>
<th>APH and CRC</th>
<th>MLP only</th>
<th>APH+MLP</th>
<th>CRC+MLP</th>
<th>APH+CRC+MLP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Certainty Equivalent</td>
<td>32.6</td>
<td>20.6</td>
<td>36.8</td>
<td>20.6</td>
<td>20.6</td>
<td>20.6</td>
<td>20.6</td>
<td>20.6</td>
</tr>
<tr>
<td>SD Profit</td>
<td>36.5</td>
<td>20.6</td>
<td>36.8</td>
<td>20.6</td>
<td>20.6</td>
<td>20.6</td>
<td>20.6</td>
<td>20.6</td>
</tr>
<tr>
<td>Mean Profit</td>
<td>106.5</td>
<td>114.5</td>
<td>112.8</td>
<td>110.9</td>
<td>120.8</td>
<td>120.8</td>
<td>120.8</td>
<td>120.8</td>
</tr>
</tbody>
</table>

Available is preferred: APH + CRC + MLP has the highest certainty equivalent regardless of the risk-aversion level. Relative to the no-program case, together these programs increase the farmer's certainty equivalent around 180% to 260% depending on the level of risk aversion, with about two-thirds of this increase due to the MLP and about one-third due to crop insurance. The MLP has a larger effect on farmer welfare because it is essentially free price insurance that in this example used marketing loan rates above mean prices. Though farmers pay for crop insurance, the cotton APH net indemnity is positive, so it also increases income. Also, the optimal farmer response for all scenarios examined is to change fertilizer use and crop acreage to increase both the mean and standard deviation of income, implying that these risk management programs encourage farmers to bear more risk.

Conclusion

To examine the effects of federal risk management programs on optimal nitrogen fertilizer use and land allocation to crops, we combined the mathematical programming optimization method of Lambert and McCarl with Richardson and Condra's method of generating multiple correlated random variables. Several empirical studies have developed related models, but ours is the first to combine both methods to simultaneously analyze the intensive and extensive margin effects of these federal programs by endogenizing crop insurance and program participation, insurance coverage levels, and price election factors, as well as the acreage allocation and input use. To empirically illustrate the method, we developed a model of a typical cotton–sorghum farm in San Patricio County, TX.

We find that overall, current crop insurance and price support programs generally have a small negative effect on optimal nitrogen fertilizer rates and substantially increase optimal cotton acreage, with a smaller negative effect on sorghum acreage. These results depend on the variance effects of nitrogen fertilizer and cotton acreage in our model. Other intensive and extensive margin responses would be op-
tional for other specifications of the stochastic revenue functions, but our results are generally consistent with results from other empirical studies.

Optimal participation in available federal risk management programs includes using the MLP for both crops and purchasing APH insurance for cotton and CRC for sorghum, which are consistent with observed behavior in the region. Optimal coverage levels are 70% or 75%, and the optimal price election factor is the maximum available. The expected net indemnity largely explains these optimal insurance participation choices. Together, these federal risk management programs increase farmer certainty equivalents about 180% to 260%, of which about one third is from crop insurance and two thirds from the MLP. The MLP has a larger effect because it is free to farmers.

The intensive and extensive margin effects of these and other federal programs have environmental effects that are being increasingly scrutinized because they can enhance or counteract the goals of other programs (Goodwin and Smith; Skees). Assuming that environmental effects from these programs are positively related to nitrogen fertilizer use, both types of risk management programs imply negative environmental effects. The small intensive margin effect that reduces optimal fertilizer use will be dominated by the large extensive margin effect that both programs have on cotton acreage, the more nitrogen-intensive crop. The extensive margin effect of both types of programs is sufficiently large that it should probably be included in any comprehensive analysis of the environmental effects of federal policies.

Several caveats apply to these results. Our results concerning the intensive margin effect depend on the crop production function. However, the effect of fertilizer on the variance of crop yield and risk in general is debated (Babcock and Hennessy, Goodwin and Smith, Horowitz and Lichtenberg). Our analysis does not incorporate dynamic effects, nor does it fully account for the extensive margin effect because total crop acreage is not endogenized, or for effects from other risk management tools, such as futures and options (Coble, Heifner, and Zuniga). In addition, our results depend on the utility function used—other utility functions would give different magnitudes and possibly different directions for these effects (e.g., see Hennessy). Lastly, our analysis of the intensive margin effect might change if multiple inputs were modeled (e.g., pesticides separate from fertilizer), because inputs can be stochastic substitutes or complements (Pope and Kramer). Nevertheless, despite these and other limits for our empirical analysis of the intensive and extensive margin effects, we still achieve our goal of illustrating this useful empirical method of endogenizing several variables while maintaining correlations among multiple random variables.

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—. "Summary of Business Data/Reports" (2002d).