CHAPTER I: INTRODUCTION

This book is intended to both serve as a reference guide and a text for a course on Applied Mathematical Programming. The material presented will concentrate upon conceptual issues, problem formulation, computerized problem solution, and results interpretation. Solution algorithms will be treated only to the extent necessary to interpret solutions and overview events that may occur during the solution process.

1.1 Mathematical Programming Approach

Mathematical programming refers to a set of procedures dealing with the analysis of optimization problems. Optimization problems are generally those in which a decision maker wishes to optimize some measure(s) of satisfaction by selecting values for a set of variables. We will discuss the set of mathematical programs where the variable values are constrained by conditions external to the problem at hand (for example, constraints on the maximum amount of resources available and/or the minimum amount of certain items which need to be on hand) and sign restrictions on the variables. The general mathematical programming problem we will treat is:

Optimize	$F(X)$		
Subject To (s.t.)	$G(X)$	€	S_1
	X	€	S_2

Here X is a vector of decision variables. The level of X is chosen so that an objective is optimized. The objective is expressed algebraically as F(X). The function F(X) is commonly called the objective function and tells how alternative choices of X effect the decision maker satisfaction in terms of the objective. This objective function will be maximized or minimized. However, in setting X, a set of constraints must be obeyed requiring that the X's behave in some manner. These constraints are reflected in the above formulation by the requirements that: a) G(X) must belong to S_1 and b) the variables individually must fall into S_2.

A number of applications have been cast into mathematical programming terms. Some examples of practical applications are

1. A firm wishes to minimize the cost of feeding cattle so sets up an LP problem. In this problem the objective is to minimize the cost of feeding expressed as the cost per lb of each ingredient times the amount of feed used summed over all feed stuff possibilities. The variables are the amount of each feedstuff used. However, in choosing the quantity of feedstuffs the diet must be structured so it meets the nutritional requirements of the animals. Thus for example constraints are needed insuring the calorie and protein content summed across all the feedstuffs used is greater than or equal to the animal requirement.
2. A firm wishes to learn how to manage its production facilities given that it may choose to either produce a good or buy it from another manufacturer and resell it. Specifically suppose as firm is in the business of electricity sale and can either generate it or buy it from a distant plant to meet customer needs. In such a case the model built would minimize the cost of generating or purchasing plus delivering energy given constraints on productive capacity, cost volume relationships, transmission capacity, demand and other factors. The variables would be quantity generated by facility, quantity purchased by supplier and quantity moved across the transmission lines.
3. A firm may wish to determine how to cut up a set of incoming logs to maximize profits. In such case the firm would introduce variables for the way to process the logs and the sale of final products. Constraints would be imposed on the quantity of logs by type, log handing

facilities and product demand.

As the examples above illustrate, the mathematical programming problem encompasses many different types of problems some of which will be discussed in this book. In particular, if F(X) and G(X) are linear and the X's are individually non-negative, then the problem becomes a linear programming problem. If the X 0 S_2 restriction requires some X's to take on integer values, then this is an integer programming problem. If G(X) is linear, F(X) quadratic, and the S_2 restrictions are simply non-negativity restrictions, then we have a quadratic programming problem. Finally, if F(X) and G(X) are general nonlinear functions with S_2 being nonnegativity conditions, the problem is a nonlinear programming problem.

1.2 Practical Problem Analysis

Problem analysis is by nature an interactive process in which an analyst perceives (or is told about) a problem; conceptualizes an approach; tries out the approach; revises the approach to better fit the problem (alternatively terminates the investigation or tries a new approach) implements the approach; interprets the results; and terminates the inquiry, or transfers the approach to operational personnel. This book will explicitly or implicitly deal with these topics under the assumptions that the problem analysis technique is mathematical programming.

Mathematical programming problem analysts generally have comparative advantage in knowledge of the problem, not in algorithm development procedures. Consequently, the problem analyst should be thoroughly informed on the topics of problem formulation, results interpretation, and model use but in large part can treat the solution processes as a "black box." Here we will concentrate more on use issues and algorithmic treatment will be left to other texts.

1.3 Mathematical Programming in Use

Mathematical programming is most often thought of as a technique which decision makers can use to develop optimal values of the decision variables. However, there are a considerable number of other potential usages of mathematical programming. Furthermore, as we will argue below, numerical usage for identification of specific decisions is probably the least common usage in terms of relative frequency.

Three sets of usages of mathematical programming that we regard as common are: 1) problem insight construction; 2) numerical usages which involve finding model solutions; and 3) solution algorithm development and investigation. We will discuss each of these in turn.

1.3.1 Generating Problem Insight

Mathematical programming forces one to state a problem carefully. One must define: a) decision variables; b) constraints; c) the objective function; d) linkages between variables and constraints that reflects complementary, supplementary and competitive relationships among variables; and e) consistent data. The decision maker is forced to understand the problem interacting with the situation thoroughly, discovering relevant decision variables and constraining factors. Frequently, the resultant knowledge outweighs the value of any solutions and is probably the number one benefit of most mathematical programming exercises.

A second insight generating usage of mathematical programming involves analytical investigation of problems. While it is not generally acknowledged that mathematical programming is used, it provides the underlying basis for a large body of microeconomic theory. Often one sets up, for example, a utility function to be maximized subject to a budget constraint, then uses mathematical programming results for the characterization of optimal values. In turn, it is common to derive theoretical conclusions and state the assumptions under which those conclusions are valid. This is probably the second most common usage of mathematical programming and again is a nonnumerical use.

1.3.2 Numerical Mathematical Programming

Numerical usages fall into four subclasses: 1) prescription of solutions; 2) prediction of consequences; 3) demonstration of sensitivity; and 4) solution of systems of equations.

The most commonly thought of application of mathematical programming involves the prescriptive or normative question: Exactly what decision should be made given a particular specification of objectives, variables, and constraints? This is most often perceived as the usage of mathematical programming, but is probably the least common usage over the universe of models. In order to understand this assertion, one simply has to address the question: "Do you think that many decision makers yield decision making power to a model?" Very few circumstances entail this kind of trust. Most often, models are used for decision guidance or to predict the consequences of actions. One should adopt the philosophical position that models are an abstraction of reality and that an abstraction will yield a solution suggesting a practical solution, not always one that should be implemented.

The second numerical mathematical programming usage involves prediction. Here the model is assumed to be an adequate depiction of the entity being represented and is used to predict in a conditional normative setting. Typically, this occurs in a business setting where the model is used to predict the consequences of environmental alterations (caused by investments, acquisition of resources, weather changes, market price conditions, etc.). Similarly, models are commonly used in government policy settings to predict the consequences of policy changes. Models have been used, for example, to analyze the implications for social benefits of a change in ambient air quality. Predictive use is probably the most common numerical usage of mathematical programming.

The third and next most common numerical usage of mathematical programming is sensitivity demonstration. Many research inquiries are of this nature where no one ever tries to implement the solutions, and no one ever uses the solutions for predictions. Rather, the model is used to demonstrate what might happen if certain factors are changed. Here the model is usually specified with a "realistic" data set, then is used to demonstrate the implications of alternative input parameter and constraint specifications.

The final numerical use is as a technical device in empirical problems. Mathematical programs can be used to develop such things as solutions to large systems of equations, equation fitting such that the estimated parameters minimize absolute deviations, or exhibit in all positive or all negative error terms. In this case, the ability of modern day solvers to treat problems with thousands of variables and constraints may be called to use. For example, a large USDA econometric model was solved for a time using a mathematical programming solver.

1.3.3 Algorithmic Development

Much of the mathematical programming related effort involves solution algorithm development. Formally, this is not a usage, but an enormous amount of work is done here as is evidenced by the many textbooks treating this topic. In such a setting the mathematical programming model is used as a vehicle for solution technique development. Work is also done on new formulation techniques and their ability to appropriately capture applied problems.

1.4 Book Plan

Mathematical programming in application consists, to a large degree, of applied linear programming. This book will not neglect that. Chapters II-X will cover linear solution procedures, duality, modeling, and computational issues. Discussion will then move onto nonlinear programming covering the general case, then price endogenous programming, risk programs, and integer programming.