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# CHAPTER VII: MORE LINEAR PROGRAMMING MODELING

In this chapter we continue our concentration on LP modeling. However, we lessen our concentration on GAMS[1] and duality. This presentation is organized around common LP problems. The first problem involves product assembly where component parts are assembled into final products. This is followed by problems which cover: a) raw products disassembled into component parts; b) simultaneous raw product disassembly and component assembly processes; c) optimal operation sequencing; d) commodity storage; e) input output analysis; and f) block diagonal problems.

## 7.1 Assembly Problem

An important LP formulation involves the assembly or blending problem. This problem deals with maximizing profit when assembling final products from component parts. The problem resembles the feed formulation problem where mixed feeds are assembled from raw commodities; however, the assumption of known component mixtures is made. This problem appears in Dano, who presents a brief literature review.

The problem formulation involves k component parts which can be purchased at a fixed price. The decision maker is assumed to maximize the value of the final products assembled less the cost of c components. Each of the final products uses component parts via a known formula. Also, fixed resources constrain the production of final products and the purchase of component parts. The formulation is

$$
\begin{array}{llllll}
\text{Max} & \sum_j c_j X_j & - & \sum_k d_k Q_k & & \\
& \sum_j a_{kj} X_j & - & w_k Q_k & \leq h_k & \text{for all } k \\
& \sum_j e_{ij} X_j & + & \sum_k f_{ik} Q_k & \leq b_i & \text{for all } i \\
& X_j & & & \geq g_j & \text{for all } j \\
& X_j, & & Q_k & \geq 0 & \text{for all } k, j
\end{array}
$$

where: j is the final product index; $c_j$ is the return per unit of final product j assembled; $X_j$ is the

[1] The GAMS formulations are included on the attached disk in the CH7 subdirectory.

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number of units of final product j assembled; k is the component part index; $d_k$ is the cost per unit of component part k; $Q_k$ is the quantity of component part k purchased; $a_{kj}$ is the quantity of component part k used in assembling one unit of product j; $w_k$ is the number of units of the component part received when $Q_k$ is purchased; I is the index on resource limits; $e_{ij}$ is the use of limited resource I in assembling one unit of product j; $f_{ik}$ is the use of the $i^{th}$ limited resource when acquiring one unit of $Q_K$; $b_i$ is the amount of limited resource I available; $g_j$ is the amount of $j^{th}$ product which must be sold; and $h_k$ is the firm's inventory of ingredient k.

In this formulation the objective function maximizes the return summed over all the final products produced less the cost of the component parts purchased. The first constraint equation is a supply-demand balance and constrains the usage of the component parts to be less than or equal to inventory plus purchases. The second constraint limits the resources used in manufacturing final products and purchasing component parts to the exogenous resource endowment. The last constraint imposes a minimum sales requirement on final product production. All of the variables are assumed to be nonnegative. This problem contains production variables which produce the $j^{th}$ product ($X_j$) and purchase variables ($Q_k$).

The dual problem is not very much different from those before, thus, suppose we only look at the dual constraint associated with $Q_k$. That constraint

$$- w_k U_k + \sum_i f_{ik} Z_i \geq - d_k$$

where $U_k$ is the return to one unit of component part k; and $Z_i$ is the return to one more unit of limited resource I.

This constraint is more easily interpreted if it is rewritten as follows

$$\sum_i f_{ik} Z_i + d_k \geq w_k U_k$$

or, equivalently,

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$$\frac{\sum_i f_{ik} Z_i + d_k}{w_k} \geq U_k$$

This inequality says that the internal value of a component part unit is less than or equal to its purchase price plus the cost of the resources used in its acquisition. Therefore, the internal value of a component part can be greater than the amount paid externally.

**7.1.1 Example**

The assembly problem example involves PC compatible computer assembly by Computer Excess (CE). CE is assumed to assemble one of six different computer types: XT, AT, 80386-25, 80386-33, 80486-SX, and 80486-33. Each different type of computer requires a specific set of component parts. The parts considered are 360K floppy disks, 1.2 Meg floppy disks, 1.44 Meg floppy disks, hard disks, monochrome graphics setups, color graphics setups, plain cases, and fancy cases. The component part requirements to assemble each type of computer are given in Table 7.1. The table also contains component parts' prices, as well as sales, inventory and resource (labor and shelf space) requirements. The resource endowment for labor is 550 hours while there are 240 units of system space and 590 units of shelf space. The problem formulation is given in Table 7.2 while the solution is given in Table 7.3. The GAMS implementation of this formulation is called ASSEMBLE.

There are no particularly unique features of the empirical formulation or solution, so interpretation is left to the reader.

**7.1.2 Comments**

The assembly problem is related to the feed formulation problem. Namely, the assembly problem assumes that known least cost mixes have been established, and that one wishes to obtain a maximum profit combination of these mixes. There are numerous assumptions in this problem. For example we assume all prices are constant and the quantity of fixed resources is constant. One could extend the model to relax such assumptions.

## 7.2 Disassembly Problems

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Another common LP formulation involves raw product disassembly. This problem is common in agricultural processing where animals are purchased, slaughtered and cut into parts (steak, hamburger, etc.) which are sold. The problem is also common in the forest products and petroleum industries, where the trim, cutting stock and cracking problems have arisen. In the disassembly problem, a maximum profit scheme for cutting up raw products is devised. The primal formulation involves the maximization of the component parts revenue less the raw product purchase costs, subject to restrictions that relate the amount of component parts to the amount of raw products disassembled. The basic formulation is

$$
\begin{array}{lrcrcll}
\text{Max} & -\sum_j c_j X_j & + & \sum_k d_k Q_k & & & \\
 & -\sum_j a_{kj} X_j & + & Q_k & \leq & 0 & \text{for all } k \\
 & \sum_j e_{rj} X_j & + & \sum_k f_{rk} Q_k & \leq & b_r & \text{for all } r \\
 & X_j & & & \leq & g_j & \text{for all } j \\
 & & & Q_k & \leq & h_k & \text{for all } k \\
 & & & Q_k & \geq & M_k & \text{for all } k \\
 & X_j, & & Q_k & \geq & 0 & \text{for all } k, j
\end{array}
$$

where j indexes the raw products disassembled; k indexes the component parts sold; r indexes resource availability limits; $c_j$ is the cost of purchasing one unit of raw product j; $X_j$ is the number of units of raw product j purchased; $d_k$ is the selling price of component part k; $Q_k$ is the quantity of component part k sold; $a_{kj}$ is the yield of component part k from raw product j; $e_{rj}$ is the use of resource limit r when disassembling raw product j; $f_{rk}$ is the amount of resource limit r used by the sale of one unit of component part k; $b_r$ is the maximum amount of raw product limit r available; $g_j$ is the maximum amount of component part I available; and $h_k$ is the maximum quantity of component k that can be sold, while $M_k$ is the minimum amount of the component k that can be sold.

The objective function maximizes operating profit, which is the sum over all final products sold ($Q_K$) of the total revenue earned by sales less the costs of all purchased inputs. The first constraint is a

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product balance limiting the quantity sold to be no greater than the quantity supplied when the raw product is disassembled. The next constraint is a resource limitation constraint on raw product disassembly and product sale. This is followed by an upper bound on disassembly as well as upper and lower bounds on sales.

The $X_j$ are production variables indicating the amount of the $j^{th}$ raw product which is disassembled into the component parts (the items produced) while using the inputs $e_{jr}$. The $Q_k$ are sales variables indicating the quantity of the $k^{th}$ product which is sold.

### 7.2.1 Example

The disassembly problem example involves operations at Jerimiah's Junk Yard. The firm is assumed to disassemble up to four different types of cars : Escorts, 626's, T-birds, and Caddy's. Each different type of car yields a unique mix of component parts. The parts considered are metal, seats, chrome, doors and junk. The component part yields from each type of car are given in Table 7.4 as are data on car purchase price, weight, disassembly cost, availability, junk yard capacity, labor requirements, component part minimum and maximum sales possibilities, parts space use, labor use, and sales price. The resource endowment for labor is 700 hours while there is 42 units of junk yard capacity and 60 units of parts space. We also extend the basic problem by requiring parts to be transformed to other usages if their maximum sales possibilities have been exceeded. Under such a case, chrome is transformed to metal on a pound per pound basis, while seats become junk on a pound per pound basis, and doors become 70% metal and 30% junk. The problem formulation is given in Table 7.5 while the solution is given in Table 7.6. The GAMS implementation of this formulation is called DISSASSE.

Note the empirical formulation follows the summation notation formulation excepting for the addition of the parts transformation activities and the equality restrictions on the parts balance rows. This is reflected in the solution where, for example, the excess seats are junked making more seats worth the junk disposal cost.

### 7.2.2 Comments

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It is difficult to find exact examples of the disassembly problem in literature. This formulation is a rather obvious application of LP which, while having been studied a number of times, is not formally recognized.

A number of observations are possible. First, raw products are assumed to be separable into the individual component parts. While this assumption is not restrictive here, it is more important in the assembly-disassembly problem as discussed below. Second, the product demand curve reflects an finitely inelastic demand at the minimum, then an infinitely elastic portion at the price up until the maximum is met, then zero demand. This is very common in LP, although inventories or minimum requirements would yield slightly different setups. Third, this is an example of the joint product formulation where multiple products are created by the acquisition of the raw products.

### 7.3 Assembly-Disassembly

A unification and extension of the above two models involves the assembly-disassembly problem. In that problem, one purchases raw products, disassembles them and reassembles the component parts into finished products. This type of problem would be most applicable for vertically

$$
\begin{array}{llllllll}
\text{Max} & -\sum_j c_j X_j & +\sum_k d_k Q_k & +\sum_i s_i T_i & -\sum_i p_i Z_i & & & \\
& -\sum_j a_{ij} X_j & +\sum_k b_{ik} Q_k & +T_i & -Z_i & \leq & 0 & \text{for all i} \\
& \sum_j e_{rj} X_j & +\sum_k f_{rk} Q_k & +\sum_i g_{ri} T_i & +\sum_i h_{ri} Z_i & \leq & \alpha_r & \text{for all r} \\
& X_j & & & & \leq & \beta_j & \text{for all j} \\
& & Q_k & & & \leq & \delta_k & \text{for all k} \\
& & & T_i & & \leq & \theta_i & \text{for all i} \\
& & & & Z_i & \leq & \lambda_i & \text{for all i} \\
& X_j & Q_k & T_i & Z_i & \geq & 0 & \text{for all i, j, k}
\end{array}
$$

integrated processing facilities. One example is meat packing, where animals are purchased, disassembled into parts, and then reassembled into such composite products as sausage, ham, processed meats, etc. A similar example would be given by a furniture manufacturer that bought raw logs, cut them up, then used the sawn lumber in a furniture manufacturing business. The basic formulation is

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where: j indexes raw product; k indexes manufactured product; i indexes component parts; r indexes resource limits; $c_j$ is the cost per unit of raw material; $X_j$ is the number of units of raw product purchased; $d_k$ is the return per unit of manufactured product k; $Q_k$ is the number of units of manufactured product k which are assembled; $s_i$ is the return the firm realizes from selling one unit of component part i; $T_i$ is the number of units of component parts i which are sold; $p_i$ is the per unit purchase price for acquiring component part i; $Z_i$ is the number of units of component part i purchased; $a_{ij}$ is the yield of component part i from one unit of raw product j; $b_{ik}$ is the use of component part i to produce one unit of manufactured product k; $e_{rj}$ is the use purchasing one unit of the $X_j$ raw product makes of the $r^{th}$ resource; $f_{rk}$ is the use manufacturing one unit of $Q_k$ product makes of the $r^{th}$ resource; $g_{ri}$ is the use that one unit of $T_i$ makes of the $r^{th}$ resource; $h_{ri}$ is the use one unit of $Z_i$ makes of the $r^{th}$ resource; and $\forall_r$ is the availability of the $r^{th}$ resource. In addition each of the four types of variables are bounded above and nonnegative.

This model covers the disassembly of raw products ($X_j$), the assembly of final products ($Q_k$), the sale of component parts ($T_i$), and the purchase of component parts ($Z_i$). The objective function maximizes the revenue from final products and component parts sold less the costs of the raw products and component parts purchased. The first constraint is a supply-demand balance, and balances the use of component parts through their assembly into final products and direct sale, with the supply of component parts from either the disassembly operation or purchases. The remaining equations impose resource limitation constraints and upper bounds. The problem contains production ($X_j$, $Q_k$), sale ($T_i$) and purchase variables ($Z_i$).

### 7.3.1 Example

Charles Chicken Plucking and Sales Company purchases chickens, cuts them and repacks them into chicken meat packages. All chickens are available for $1.00, have the same weight and breakdown identically in terms of wings, legs, etc. Charles can, however, cut up the chickens in several different manners. Chickens may be cut into parts, meat, quarters, halves, or breast, thigh, and leg cuts. From these Charles gets wings, legs, thighs, backs, breasts, necks, gizzards, meat, breast quarters, leg quarters

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and halves. Chickens yield 1 lb. of meat, 80 percent of which is in the leg-thigh and breast region. The cutting patterns and yields, labor requirements, and sale prices are shown in Table 7.7.

Charles sells parts individually or sells packs which contain: a) a cut-up chicken with all parts from one chicken except gizzards; b) 4 breast quarters; c) 4 leg quarters; d) 2 chicken halves; and e) two legs and two thighs. Charles also sells gizzard packs with 10 gizzards in them. The individual parts sell at prices shown in Table 7.7.

Production capacity allows 1,000 chickens to be cut up per day. The company may purchase wing, leg, or thighs from other suppliers; however, no more than 20 units of each part are available. The price of purchasing is $0.02 above the market sales price. The firm has 3,000 units of labor available.

To formulate this problem four classes of variables must be defined: 1) chicken disassembly variables depicting the number of chickens cut via the patterns for - Parts ($X_p$), Halves ($X_h$), Quarters ($X_q$), Meat ($X_m$), and Leg-Breast-Thigh ($X_L$); 2) assembly variables for the packs A - E ($X_a$ - $X_e$) and the gizzard pack ($X_g$); 3) raw-product sales variables; and 4) raw product purchase variables. These variables are set subject to constraints on part supply, chicken availability, labor and purchase limits. The formulation is shown in Table 7.8. The GAMS implementation is called ASSDISSM.

There are several features of this formulation which merit explanation. First, negative coefficients in the supply and demand rows depict supplies of component parts from either purchase or raw product disassembly, while the positive coefficients depict usages. Examining the thigh row, thighs can be obtained if one cuts a chicken into parts, or into the leg-breast-thigh pattern. Thighs may also be acquired through external purchase. Demand for these thighs comes from the sale of packs of A and E, as well as the direct sale of thighs. Second, the last three constraints give the resource limit constraints on purchases. However, these constraints limit a single variable by placing an upper bound on its value. Constraints which limit the maximum value of a single variable are called upper bound constraints.

The solution to this problem is shown in Table 7.9. This solution leads to an objective function value of $1362.7 where 1000 chickens are bought, cut by the Leg-Breast Thigh Pattern and sold as 1010

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units of pack E. In addition, 2000 breasts, 1000 necks, 200 lbs. meat, 20 thighs and 20 legs purchased. The resultant marginal value of products of the resources are given by the shadow prices.

**7.3.2 Comments**

There are several important assumptions embodied in the Charles Chicken problem. One of these involves separability -- that the parts can be disassembled and assembled freely. While this assumption appears obvious, let us illustrate an example wherein this assumption is violated. Suppose that one wishes to solve a blending problem mixing two grades of grain (A,B) from two batches ($G_1$, $G_2$). Suppose that moisture and foreign matter are the component parts, and the relevant parameters are given in Table 7.10. Further, suppose there are 20 units of each of two batches of grain available, and that grade A grain sells for $3.00 per unit, and grade B grain for $2.00 per unit.

Now suppose this problem is formulated as an assembly-disassembly problem. This formulation

$$
\begin{array}{lrcrcrcrcr}
\text{Max} & 3A & + & 2B & & & & & & \\
 & -A & - & 2B & + & 2G_1 & + & G_2 & \leq & 0 \\
 & -A & - & 2B & + & G_1 & + & 2G_2 & \leq & 0 \\
 & A & + & B & - & G_1 & - & G_2 & = & 0 \\
 & & & & & G_1 & & & \leq & 20 \\
 & & & & & & & G_2 & \leq & 20 \\
 & A, & & B, & & G_1, & & G_2 & \geq & 0
\end{array}
$$

is where A is the quantity of grade A grain made, B the quantity of grade B grain made, $G_1$ the quantity of batch 1 grain used, and $G_2$ the quantity of batch 2 grain used (see file GRAIN1). The model balances the maximum moisture, foreign matter, and weight with the amount in each grain. The resultant solution gives an objective function value equal to 100. The variable values and shadow prices are presented in Table 7.11.

There is a problem with this solution. It is impossible, given the data above, to make a mix 20 units each of grade A and grade B grain. The requirement for maximum moisture and foreign matter is 1

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percent in grade A grain, so neither of the grain batches could be used to produce grade A, as they both exceed the maximum on this requirement. The solution above, however, implies that we could make 20 units of grade A grain and 20 units of grade B grain. The model uses excess moisture from grain batch 1 in grade B grain, while the excess foreign material from grain batch 2 is also put in the grade B grain. This is clearly impossible, as moisture and foreign matter are not separable (Ladd and Martin make this mistake as pointed out in Westgren and Schrader). Thus, this situation violates the separability assumption; the items in a row cannot be used freely in either of the two blends. The proper formulation of the blending problem is

$$
\begin{array}{llcrcrcrcrcrcr}
\text{Max} & 3A & + & 2B & & & & & & & & & & \\
\text{s.t.} & -A & & & + & 2G_{11} & + & G_{21} & & & & & \leq & 0 \\
 & -A & & & + & G_{11} & + & 2G_{21} & & & & & \leq & 0 \\
 & A & & & - & G_{11} & - & G_{21} & & & & & = & 0 \\
 & & - & 2B & & & & & + & 2G_{12} & + & G_{22} & \leq & 0 \\
 & & - & 2B & & & & & + & G_{12} & + & 2G_{22} & \leq & 0 \\
 & & & B & & & & & - & G_{12} & - & G_{22} & = & 0 \\
 & & & & & G_{11} & & & + & G_{12} & & & \leq & 20 \\
 & & & & & & & G_{21} & & & + & G_{22} & \leq & 20 \\
 & A, & & B, & & G_{11}, & & G_{21}, & & G_{12}, & & G_{22} & \geq & 0
\end{array}
$$

The optimal solution gives an objective function value of 80 (see file GRAIN2). The optimal value of the variables and equation information is shown in Table 7.12.

Here, note that all the grain goes into grade B, and the objective function is smaller. Separability is an important assumption, and one must be careful to insure that it holds in any assembly-disassembly problem.

## 7.4 Sequencing Problems

Often, production entails multiple intermediate processes and requires that each process be completed before the next one is started. Furthermore, the intermediate processes often compete for

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resources. A farming example of this situation involves the requirement that plowing be done before planting and that planting be done before harvest. However, plowing and planting may go on at the same time on different tracts of land. Thus, plowing and planting could draw from the same labor and machinery pools. Similarly, plowing could be done in the fall during the harvesting period, thus harvesting and fall plowing compete for the same resources.

A problem which explicitly handles such a situation involves sequencing. Sequencing models insure that the predecessor processes are completed before the successor processes can begin. In LP models the sequencing considerations generally take one of two forms. Sequencing may be controlled within a variable or between variables using constraints. Sequencing within a variable is done whenever the occurrence of one event implies that another event occurs a fixed time afterward, or whenever the timing of events influences their economic returns. In this case both the predecessor and successor tasks are embedded in a variable. Sequencing between variables is done whenever the successor process follows the predecessor process an indefinite amount of time later, but the economic return to the successor is not a function of when the predecessor was done. For example, one may plant a crop and not care when it was plowed; on the other hand, if one harvests a crop and the yield of the crop depends on both the planting and harvesting dates, then this would require sequencing within a variable. Both cases will be illustrated.

Sequencing considerations are hard to write algebraically and the result is often confusing to students. Thus, we will alter our presentation style and use an example before the algebraic formulation. Suppose a firm produces an output using two tasks. Further suppose that production process occurs somewhere in a three week period, and that the successor or predecessor can be done in any of those three weeks. However, the predecessor task must be completed before the successor task. Two cases can arise. First, the successor/predecessor date could jointly determine economic returns and/or resource use. Second, the yield, returns, etc., could be independent of timing as long as the predecessor occurs first.

Let us consider the latter case first. Suppose we denote $X_1$, $X_2$ and $X_3$ as the amount of the

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predecessor done in weeks 1, 2, and 3, and $Y_1$, $Y_2$ and $Y_3$ as the amount of the successor done in weeks 1, 2, and 3. In week 1 the level of the successor activity ($Y_1$) must be less than or equal to the amount of the predecessor activity ($X_1$) completed. Algebraically, this implies

$$Y_1 \leq X_1 \text{ or } -X_1 + Y_1 \leq 0$$

In week 2, the amount of the predecessor activity which could be completed by then is $X_1$+$X_2$. The total amount of the successor activity at that time would be that used this week ($Y_2$), plus that used last week $Y_1$. The sequencing requirement is that $Y_2$ must be less than or equal to $X_1$ plus $X_2$ minus what was used in Period 1 or, algebraically

$$Y_2 \leq X_1 + X_2 - Y_1 \text{ or } -X_1 - X_2 + Y_1 + Y_2 \leq 0$$

The intersection of this constraint with that above allows no more of the successor activity to be nonzero in the first week than is present at that time. However, more of the predecessor may be produced than used in the first week; with the extra carried over into later weeks for usage.

In the third week, the amount of the successor activity ($Y_3$) is less than or equal to the amount of predecessor activity that could be supplied up to that period ($X_1 + X_2 + X_3$) less that used in periods 1 and 2 ($Y_1 + Y_2$).

$$Y_3 \leq X_1 + X_2 + X_3 - Y_1 - Y_2 \text{ or } -X_1 - X_2 - X_3 + Y_1 + Y_2 + Y_3 \leq 0$$

Algebraically, the action of this set of constraints is such that $X_1$ could equal 500, while $Y_1$=100, $Y_2$=100 and $Y_3$=300, indicating that the predecessor activity was completely undertaken in the first week but the successor activity slowly used up the inventory during the life of the model. However, the successor cannot get ahead of the predecessor. A complete tableau of this setup is given by

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| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Week1 | $-X_1$ | | | $+ Y_1$ | | | $\le$ | 0 |
| Week2 | $-X_1$ | $- X_2$ | | $+ Y_1$ | $+ Y_2$ | | $\le$ | 0 |
| Week3 | $-X_1$ | $- X_2$ | $- X_3$ | $+ Y_1$ | $+ Y_2$ | $+ Y_3$ | $\le$ | 0 |
| Week1 | $aX_1$ | | | $+ dY_1$ | | | $\le$ | $T_1$ |
| Week2 | | $bX_2$ | | | $+ eY_2$ | | $\le$ | $T_2$ |
| Week3 | | | $cX_3$ | | | $+ fY_3$ | $\le$ | $T_3$ |

The first three constraints are discussed above. The others are resource limitations. Week 1 resources are used by $X_1$ and/or $Y_1$; Week 2 resources by $X_2$ and/or $Y_2$; and Week 3 by $X_3$ and/or $Y_3$. The sequencing constraints insure that successor activities will not be undertaken until the predecessors are complete.

The sequencing restraints may also be explained in terms of the variables. Suppose $X_1$ and $Y_1$ are variables for acres of land. $X_1$ supplies land for use in time periods 1, 2 and/or 3. $Y_1$ requires land for use in period 1 and precludes use in periods 2 and 3. Variable $X_2$ supplies land in periods 2 and 3 while $Y_2$ uses land in period 2 and precludes use in period 3.

The above formulations assume that returns and resource usage are independent of activity timing. This may not always be true. Returns to the successor activities may depend on the timing of the preceding activities. Such a formulation involves changing the variable definitions so that the week of the predecessor and successor define the variable. In the above example, this yields six variables - the first involving the successor and predecessor both carried out in week 1; the second, the predecessor in week 1 and the successor in week 2, etc. A tableau of this situation is

| Predecessor date | Wk 1 | Wk 1 | Wk 1 | Wk 2 | Wk 2 | Wk 3 | | |
|---|---|---|---|---|---|---|---|---|
| Successor date | Wk 1 | Wk 2 | Wk 3 | Wk 2 | Wk 3 | Wk 3 | | |
| Wk 1 | $aZ_{11}$ | $+ bZ_{12}$ | $+ dZ_{13}$ | | | | <= | $T_1$ |
| Wk 2 | | $cZ_{12}$ | | $+ fZ_{22}$ | $+ gZ_{23}$ | | <= | $T_2$ |
| Wk 3 | | | $eZ_{13}$ | | $+ hZ_{23}$ | $+ iZ_{33}$ | <= | $T_3$ |

The variables represent the amount of predecessor and successor activities undertaken during specific times (ie. $Z_{13}$ involves the predecessor in week 1 and the successor in week 3). The constraints restrict resource usage by week. Resource use or objective function coefficients would differ from acti

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vity to activity, indicating that features of the model are dependent on the sequencing.

Given this background, we may now introduce a general formulation. Suppose we have three phases, X, Y and Z, each of which must be completed in sequence. Further, we will allow a set of alternatives for each of X, Y, and Z. A general summation model embodying sequencing considerations is

$$
\begin{array}{llllllll}
\text{Max} & -\sum_j \sum_{t_1} c_j X_{jt_1} & - \sum_k \sum_{t_2} d_k Y_{kt_2} & + \sum_s \sum_{t_3} e_s Z_{st_3} & & & & \\
\text{s.t.} & -\sum_j \sum_{t_1 \prec t} X_{jt_1} & + \sum_k \sum_{t_2 \prec t} Y_{kt_2} & & \leq & 0 & & \text{for } t \in t_2 \\
& & -\sum_k \sum_{t_2 \prec t} Y_{kt_2} & + \sum_s \sum_{t_3 \prec t} Z_{st_3} & \leq & 0 & & \text{for } t \in t_3 \\
& +\sum_j a_j X_{jt} & \sum_k b_j Y_{kt} & + \sum_s f_s Z_{st} & \leq & g_{mt} & & \text{for all } m, t \\
& X_{jt}, & Y_{kt_2}, & Z_{st_2} & \geq & 0 & & \text{for all } j, k, s, t_1, t_2, t_3
\end{array}
$$

The variables are $X_{jt_1}$, the j[th] alternative for the completion of task X in time period $t_1$; $Y_{kt_2}$, the k[th] alternative for the completion of task Y in time period $t_2$; and $Z_{st_2}$ the s[th] alternative for the completion of task Z in time period $t_3$. The first two constraints depict sequencing as in the first example where the predecessor activities are summed as long as they precede the period over which the constraint is defined (denoted by $t_1$□ t). These constraints are defined for the time periods in which the successor activities begin. The third constraint depicts resource availability.

A formal definition of notation is: t designates time periods; j indexes the technologies by which the first task can be done; k indexes the technologies by which the second task can be done; s indexes the technologies by which the third task can be done; $t_1$ gives the periods in which the first task can be done; $t_2$ gives the periods of the second task; $t_3$ gives the periods of the third task; $c_j$ is the cost of a unit of first task j; $X_{jt_1}$ is the number of units of first task j performed in time period $t_1$; $d_k$ is the cost of a unit of second task k; $Y_{kt_2}$ is the number of units of second task k performed in period $t_2$; $e_s$ is the revenue of a unit of third task version s; $Z_{st_3}$ is the number of units of third task version s performed in period $t_3$; $a_{jm}$ is the number of units of resource m used by the j[th] alternative for X; $b_{km}$ is the number of units of

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resource m used by the $k^{th}$ alternative for Y, $f_{sm}$ is the number of units of resource m used by the $s^{th}$ alternative for Z; $g_{mt}$ is the endowment of resource m in period t.

There are no particularly new features to this formulation in terms of types of constraints and/or variable. All the variables are some form or another of a production variable. The first two constraints are supply/demand balances on the intermediate products passed between the predecessor and successor variables, and the last constraint is a resource limitation constraint.

Due to the complexity of the above formulation, two examples will be given, one straight forward but of limited realism, the other more complex.

**7.4.1 Example 1**

Suppose that a farmer plows, discs, plants, cultivates, and harvests land and that yield does not depend on activity timing. Assume plowing is done in April, May and June. The principal resources are land and plowing labor. Plowing labor usage is 0.2 hours of labor per acre with a cost of $100 per acre. Discing follows plowing. Suppose that discing is done in May, June and July, requiring 0.3 hours of labor per acre while costing $20 per acre. Planting is done in May, June and July, requiring 0.3 hours of labor per acre and costing $25 per acre. Cultivation is done in each of the three months following planting, and harvesting must be done in the fourth month. Cultivation requires 0.1 hours of labor per acre with no added cost, while harvesting uses 0.5 hours of labor per acre plus a cost of $75 per acre. Further, suppose that the crop yield is worth $500 an acre. Also, the farm's resource endowment is 600 acres of land and 160 hours of labor in each of the months of April through November.

A LP formulation of this situation is given in Table 7.13 and the GAMS implementation of this formulation is called SEQUEN. The activities X represent the plowing possibilities and are defined for April, May and June. The activities Y represent the discing possibilities and are defined for May, June and July. The activities Z represent the planting-harvesting possibilities and are defined by beginning month - May, June, and July. The second three constraints give the link between plowing and discing. Note these constraints are defined for the periods in which discing can be started - May, June and July.

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The next three constraints are the link between discing and planting and are again defined for May, June and July. The next eight constraints are labor constraints which are defined for each of the months during which farm activity can be done. The last constraint is the land constraint.

The planting variable Z encompasses the sequencing between planting, cultivating and harvesting since those activities are done in fixed sequence. This variable's coefficients require resources committed in May be accompanied by a commitment in June, July and August for cultivating and in September for harvesting. The objective function for these activities is calculated as margin between the cost of planting and harvesting and the value of the crop sold. Thus, the return is $400 per acre. The solution to this model is given in Table 7.14.

This solution depicts 600 acres of plowing in April followed by 407.41 acres of discing in May and 192.59 acres in July. Finally, 125.93 acres are planted in May, 281.48 in June and 192.59 in July. The sequencing constraints allow the predecessor to occur initially faster than they are used by the successor, although the successor eventually catches up. Labor has no shadow prices. Land is the only binding resource constraint.

### 7.4.2 Example 2

The second example is more complex and does not closely follow the summation notation. However, it does contain the sequencing considerations reflected above. This example again reflects a farm planning situation and illustrates what needs to be done when planting and harvesting date influence yield. Assume that a farm grows two crops. The crops are plowed in March through June. Plowing is done the same way regardless of the crop, and the plowing rate is four acres per hour. In addition, the farmer needs one hour of maintenance for each 20 acres of plowing, and the plowing cost is $5.00 an acre. Both crops are then disced. Discing can be done in April-June for crop 1 and March-June for crop 2. The farmer can disc five acres an hour of either crop. Crop 2 is always disced exactly one month preceding planting. Discing of crop 1 can be done any time before planting. Discing costs $3.00 per acre.

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Both crops are planted in April-June. The farmer figures it takes 0.22 hours of labor to do one acre. The planting cost for crop 1 is $40 per acre, and the cost for crop 2 is $20 per acre. Both crops are cultivated exactly one month after planting. The farmer can cultivate 10 acres in one hour, the cost for cultivation is $10 per acre.

The yield achieved depends on the crop planting and harvest dates and is given in Table 7.15. Harvesting takes 0.7 hours per acre for crop 1 and 0.6 hours for crop 2. Harvesting costs are $10 per acre.

The farm has 1500 acres and 300 hours of labor available in each month. Labor can be hired for $10 an hour. The objective of the model is to maximize profits. Crop 1 sells for $3.00 per unit, Crop 2 for $8.70 per unit.

A formulation of this problem is given in Table 7.16. The GAMS implementation of this formulation is called FARM2. The tableau is formed with plowing variables defined in March through June. These variables use land and supply plowed land for the subsequent discing activities. The discing operation is modeled in two different ways, depending on crop. For crop 1 there are explicit discing variables; for crop 2, the discing activities are embedded into the planting activity because of the one month sequencing requirement. Crop 1 planting variables are defined for April, May and June. We also add the harvest time to the variable definition. Cultivating resource usages are also included. Thus, the variable for a crop planted in April and harvested in September includes resource use for April planting, May cultivating, and September harvest. Plowing resource usage is computed as 1/(acres per hour) + 1/(maintenance time per acre) = 0.25 + 0.05 or 0.3 hours for the total. Similarly, the discing time is 1/(acres per hour) = 0.20

The crop 2 planting and harvesting activities reflect a slightly different setup. Here, discing is also included. Thus, the resources use accounts for discing, planting, cultivation, and harvest. Also note that the yields are entered in the bottom of the tableau and are sold through the selling activities.

The resultant solution to this problem is given in Table 7.17 and indicates that $449,570 is made from the farm with 775 acres planted to crop 1 and remainder planted to crop 2. Labor is hired only in

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October.  The value of land is $292.50 per acre; labor is worth $10 an hour in March, April, and October, which means the labor is fully exhausted up until the point at which it will need to be hired.  Labor is in slack in June and July and worth $3 in May and September.

### 7.4.3 Comments

The above sequencing problem is a special case of a much larger class of sequencing problems. The particular problem we present has the predecessor followed by only one type of successor task. There have been many differently setup sequencing problems.  For example the PERT/CPM project scheduling problems allow multiple following activities to occur (see Bradley, Hax and Magnanti).  Job shop scheduling problems are related (Bradley, Hax and Magnanti).  Most of the scheduling problems, however, are integer programming problems.  In addition, numerous sequencing problems have been formulated and solved which involve sequencing between years.  Such problems are examined in the dynamics chapter.

The second example illustrates an important general point.  That is, in applied modeling, we often should not reflect annual resource constraints (i.e. labor for the year) but, rather, period by period resource constraints within a year.  Often resources in some periods of the year are not perfect substitutes with resources in other periods of the year (Heady and Candler).  Farming provides an obvious example where labor during the winter is not substitutable with labor during the harvest or planting periods.  Similarly, labor during the planting periods  is not usually substitutable with labor during the harvest period.

## 7.5 Storage Problems

Problems can involve storage where a product resource or input can be retained between time periods.  LP has been used to analyze such problems virtually since its inception, (see Dantzig [1963] and Gass (1985) for a historical perspective).  This section reviews a general formulation of a relatively simple storage problem.  Ordinarily, a storage problem would not be solved alone but, rather would be a model subcomponent.

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Assume a decision maker is planning over a time horizon involving T periods and has a single item in inventory which can be stored or sold in period. Let us assume that the decision maker incurs a storage cost in carrying the item from one time period to the next. The decision problem involves maximizing the value of the sales less storage costs subject to storage capacity. Thus, the problem is to determine the optimum sale and holding policy. We will also include constraints on maximum storage capacity and the maximum/minimum amount which can be sold in any time period. Further, it will be assumed that the inventory is not increased at any time during the time horizon. Thus, the storage constraint is only active in the first time period. The formulation of the problem is

$$
\begin{array}{llllllll}
\text{Max} & \sum_t c_t X_t & - & \sum_{\substack{t \\ t\neq T}} cs_t H_t & & & & \\
\text{s.t.} & X_1 & & & + H_1 & \leq & s_0 & \\
 & X_t & - & H_{t-1} & + H_t & \leq & 0 & \text{for all } 1 < t < T \\
 & X_T & - & H_{T-1} & & \leq & 0 & \\
 & X_t & & & & \leq & U_t & \text{for all } t \\
 & X_t & & & & \geq & L_t & \text{for all } t \\
 & & & & H_1 & \leq & SI & \\
 & X_t, & & & H_t & \geq & 0 & \text{for all } t
\end{array}
$$

$X_t$ stands for the amount sold in the $t^{th}$ period; $H_t$ stands for the amount held over from time period t to time period t+1; $c_t$ stands for returns from sales of the item in time period t; $cs_t$ stands for the cost of storing from period t into period t+1; $s_0$ the amount of inventory available in the first time period; $U_t$ is the upper limit on the sales possibility in time t; $L_t$ is the lower limit on the sales possibility in time period t; S is the total storage capacity limit which only is binding on the amount stored in the first time period during which the greatest amount would be stored.

The first equation in the model is the objective function. It involves summation across all the periods of the revenues from the sales of the good less the costs of storage of the good. We only include storage from the time periods 1 through T-1, assuming that everything must be sold in the last time

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period. The first constraint limits the quantity sold in the first period plus the quantity stored into the second period to be less than or equal to the initial inventory available. The next constraints are active in all time periods excepting 1 and T. This limits the amount sold in each period plus the amount stored into the next period to not exceed the amount held over from the period before. The third constraint gives the inventory condition for the last time period requiring that sales not exceed inventory carried over from the time period before. This constraint precludes outgoing storage because we are assuming everything has to be sold by the last period. Similarly, the first constraint does not include incoming storage except as an exogenous quantity on the right hand side. The next two constraints impose upper and lower limits on the amount that can be sold during any time period. The last constraint imposes an upper limit on storage in the first period. Additional constraints on storage capacity are not needed for the subsequent periods as stored amount cannot increase.

This problem does not contain new types of constraints and variables. However, it does use a new form of the transformation variable. The H variables transform the time utility of an item by moving it from one time period to another at a cost. The X variables are again sales activities. The constraints are a mixture of resource limitations and supply demand balances.

Insight can be gained into the model solution properties considering the dual. The dual constraint associated with the variable $X_t$ states that the value of the item in time period t must be greater than or equal to the revenue from selling it less the costs of the upper bound plus the costs of the lower bound. Assuming there are no bounds, this constraint then would require that the value of grain be no less than the price at which it could be sold. The dual constraints associated with the storage activities insure a relationship between the marginal values of grain in adjacent periods where they state that the value of grain in time period t+1 must be less than or equal to its value in time period t plus the cost of storing it between the periods. This occurs at optimality since, if the sale price of grain in time period t+1 is greater than or equal to the sale price of grain in time t by more than the storage costs, then it would be economical to store. The only exception comes in relation to period 1, where storage is limited. If

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storage is binding there may be a larger wedge between the first two shadow prices than the cost of storage.

### 7.5.1 Example

Suppose a farmer has 100 bushels of a crop available to sell over four time periods. The farmer expects the price in the first time period to be $2.30; in the second, $2.50; in the third, $2.70; and in the fourth, $2.90. The farmer expects the cost of holding grain from time period 1 to time period 2 to be $.10 a bushel; from time period 2 to time period 3, $.20 a bushel; and from time period 3 to time period 4, $.30 a bushel. The farmer cannot sell any more than 50 bushels in any one time period. Also, the farmer must (for cash flow reasons) sell at least 15 bushels in the first time period and at least 5 in the second. Finally, the farmer has no more than 75 bushels of storage capacity available. A formulation of this is presented in Table 7.18. The GAMS implementation of this formulation is called STORE.

The solution to this problem is given in Table 7.19. The solution has 25 bushels of grain sold in time period 1; these are sold because there is not enough capacity to store into the second time period as reflected by the $.10 shadow price on the overall storage constraint. Then, 50 bushels of grain are sold in the second time period limited by the upper limit on the ability to sell in the time period. Subsequently, 25 bushels are sold in the third time period. The inventory pattern shows 75 bushels carried from the first to the second period with 25 carried from the second to the third. The first four shadow prices show the marginal values of grain in each time period. The fifth equation exhibits slack of 25 bushels indicating 25 more bushels could be sold in the first time period.

### 7.5.2 Comments

Several assumptions could be relaxed in this formulation. One could allow inventory to be replenished. Storage costs could be made a function of volume and/or one could allow acquisition of storage capacity. The model is commonly used in conjunction with a planning problem to develop an overall aggregate plan (as discussed in Holt et al.), wherein production and storage decisions are jointly determined. More examples of storage and dynamic carry-over will be presented in the dynamic

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modeling chapter.

Another comment involves the nature of the dual relationships that a transformation variable imposes. Transformation variables bound the maximum difference between the shadow prices. This is an absolute property in any LP solution containing such variables. This is exhibited in our solution where the storage variables cause the shadow prices to differ by no more than the storage cost.

### 7.6 Input Output Analysis

Another application of linear programming involves the use of Leontiefs' (1951) input output (IO) formulation. However, IO formulations are not used alone in an LP context as there is specialized software for IO models. Rather IO's are used in conjunction within larger LP models (i.e. see Penn et al.).

Leontief formulated the IO model and it has been utilized in a number of different contexts since then (Miernyk). The IO model fundamentally deals with the development of the economy wide implications of changes in the export level and/or production practices within the economy.

The fundamental data in setting up an IO model involves three things. First, there is an identification of the sectors. These sectors are divided into endogenous and exogenous sectors. The endogenous sectors are integral parts of the economy which purchase and sell items to other sectors within the economy. The exogenous sectors are those from which imports are made or those to which goods and services flow or export. Given identification of these, one develops the transaction matrix which tells how the endogenous sectors within the economy dispose of the revenue that they receive for production. These endogenous sectors completely dispose of the revenue that they receive across the endogenous and exogenous sectors. Thus, the model data accounts for the full distribution of earnings including that to exogenous sectors and retained earnings. The third data item is the final demand vector. This tells the value of goods that flow outside the local economy.

Suppose we identify four components of an economy. There are endogenous sectors: 1) manufacturing, 2) services, and 3) agriculture with an exogenous sector for other uses of funds which

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includes imports and retained earnings. The transaction matrix gives the distribution of revenue by sector. For example, in the manufacturing sector, that matrix would tell how much was spent: a) within the manufacturing sector on manufactured goods from other producers; b) on the service sector for purchasing services used in the manufacturing process; c) on agricultural goods; and d) on acquisitions from or distributions to exogenous parties. In turn, this would also be done for the service and households sectors. The household sector data gives the household consumption function showing how the gross household income is distributed across purchases of manufactured goods, services and other household activities as well as flows into the exogenous category which accounts for savings and purchases from outside the economy. Finally, final demand gives the money flow from outside the economy into the manufacturing, service, and agricultural sectors.

After formation of the transactions matrix one then turns to forming the technical coefficients matrix. This tells the proportion of revenue spent by sector. The parameters of the technical coefficient matrix ($a_{ij}$) given the transaction matrix $t_{ij}$ equal:

$$a_{ij} = t_{ij} / \sum_{K} t_{Kj}$$

The denominator term is total revenue to the $j^{th}$ sector. All the coefficients will be less than or equal to one indicating the proportion spent within each sector.

In turn then, given this technical coefficients matrix one can form the fundamental equation of input output analysis which states that

$$X = Y + AX$$

where Y is final demand, A is the technical coefficients matrix, and X is a vector giving the amount of total activity in the economy. This indicates that the total economic activity is equal to the amount of final demand plus the value of the intermediate products that are necessary in order to produce that final demand. In turn, this equation can be manipulated to say

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$$X - AX = Y$$

which indicates the total production minus the intermediate use needed to support that total production equals exports or

$$(I - A)X = Y.$$

Now I - A in this case is a square matrix with one positive entry for each of the sectors of the economy. Providing I - A is invertible then the solution is

$$X = (I - A)^{-1} Y$$

This is the computational procedure in conventional Input Output analysis. However, in a linear programming context one utilizes the fundamental equation and an objective function to obtain input output solutions. We use the objective function recommended by Brink and McCarl (1977) which maximizes the sums of the activity across all sectors. The resulting formulation is

$$\begin{array}{llcll} \text{Max} & \sum_j X_j & & & \\ \text{s.t.} & \sum_j (I_{ij} - A_{ij}) X_j & \leq & Y_i & \text{for all } i \\ & X_j & \geq & 0 & \text{for all } j \end{array}$$

where X, A and Y are as defined before and $I_{ij}$ is an identity matrix (possessing entries of 1 if I = j and zero otherwise). This particular formulation should lead to all the X's being nonzero with the dual variables equaling the total amount of activity induced in the sector from a change in the right hand side (final demand). This duality property can be argued as follows: if all the X's are nonzero then the LP basis inverse should equal $(I - A)^{-1}$ and since all the objective function coefficients of the basic variables are ones, then the dual variables will equal

$$\sum_i (I - A)^{-1}_{ij}$$

or should equal the total induced activity inside the model by one unit of additional exports the classic output multiplier (Miernyk).

### 7.6.1 Example

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Consider an example with four sectors: manufacturing, agriculture, finance, households and imports. Suppose that the relevant transaction matrix is given in Table 7.20 as is the final demand vector. Under these circumstances the technical coefficient matrix is Table 7.21 and one sets up the linear programming model with the first three sectors as endogenous as in Table 7.22. The GAMS implementation of this formulation is called INOUT. The solution is given in Table 7.23. This solution shows that sectoral activity is $250 for manufacturing, $122 for agriculture, $75 for finance and $230 for services, but that this activity is used to supply a lesser amount of final demand. Namely, there is $250 of production by manufacturing to directly deliver $75 of final demand. Across all sectors, $677 of total production is needed to satisfy the $145 of final demand. Thus, there is $532 worth of intermediate production. Also, notice that the values of the shadow prices on the manufacturing balance row is 4.615. This implies that in order to meet one dollar worth of demand, $ 4.615 worth of total manufactured products need to be created.

### 7.6.2 Comments

Input-Output models have been used in many contexts (See Miernyk for a review).

## 7.7 Block Diagonal

Many of the early LP problems involved a "block diagonal" structure (see Dantzig (1963) for a historical perspective). Models depicting production in several different locations and/or time periods exhibited such a structure. The name arose since the models contained blocks of constraints and activities which did not overlap with other sets of constraints and other activities. The blocks arise when individual production units utilize immobile resources. The problem also depicts some usage of unifying resources at the overall firm level. The primal formulation of the block diagonal problem is given by

$$\text{Max} \quad \sum_k c_k X_k \quad + \quad \sum_j \sum_L d_{jL} Y_{jL}$$

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$$
\begin{array}{llllll}
\text{s.t.} & \sum_k a_{ik} X_k & + \sum_j \sum_L g_{ijL} Y_{jL} & <= & b_i & \text{for all I} \\
 & & \sum_j e_{jLM} Y_{jL} & <= & f_{LM} & \text{for all L and M} \\
 & X_k \quad , & Y_{jL} & >= & 0 & \text{for all k, j and L}
\end{array}
$$

where: k indexes variables for the overall firm; j indexes variables for the separate entities; L indexes entities; I indexes overall resources; M indexes separate entity resources; $c_k$ is the per unit return of overall firm variable k; $X_k$ is the number of units produced of overall firm variable k; $d_{jL}$ is the per unit return of separate entity L's $j^{th}$ production variable; $Y_{jL}$ is the number of units produced of separate entity L's $j^{th}$ production variable; $a_{ik}$ is the use of overall firm resource I by $X_k$; $g_{ijL}$ is the use of overall firm resource I by $Y_{jL}$; $b_i$ is the endowment of overall firm resource I; $e_{jLM}$ is the use of the $m^{th}$ resource at location L by $Y_{jL}$; and $f_{LM}$ is the endowment of resource M at location L.

The decision variables $X_k$ reflect actions at the unifying level of the problem, while the $Y_{jL}$ reflect actions at the sublevels of the problem. Generally, the sublevels arise because of spatial, temporal or functional separations. The first constraint is the overall unifying set of constraints. The second set of constraints deal with each sub-unit. The problem maximizes profit summed over the global and sub-unit activities subject to an overall linking constraint and individual sub-unit constraints. Note that the second set of constraints do not involve sums across L, thus only one sub-unit is involved in any constraint. Thus, the second set of constraints is independent across the sub-units. This particular problem is called block diagonal because of the structure of the second constraint. An overview of this problem is given below

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$$
\begin{array}{lcccccccccc}
cX & + & d_1Y_1 & + & d_2Y_2 & . & . & . & + & d_nY_n & & \max \\
AX & + & g_1Y_1 & + & g_2Y_2 & . & . & . & + & g_nY_n & \le & b \\
 & & & & e_1Y_1 & & & & & & \le & f_1 \\
 & & & & & e_2Y_2 & & & & & \le & f_2 \\
 & & & & & & . & & & & & . \\
 & & & & & & & . & & & & . \\
 & & & & & & & & . & & & . \\
 & & & & & & & & & e_nY_n & \le & f_n
\end{array}
$$

All the cells in this matrix which do not have entries or dots are 0's and the name "block diagonal" comes from the diagonal blocks representing each of the subcomponent resource constraints and activities. This formulation also illustrates the phenomena of sparsity. Linear programming often possesses large blocks of zero coefficients. This particular one is famous for it. Sparsity is commonly exploited in LP algorithms.

**7.7.1 Example**

The block diagonal problem will be illustrated using the data from the resource allocation example above, adding features regarding multiple plants and the making of tables. Suppose the firm from the resource allocation example has expanded and now possesses 3 plants. At these plants they fabricate and sell chairs and tables which are sold individually or together as dinette sets. Plant 1 makes only tables, plant 2 only makes chairs, and plant 3 can make chairs and tables. The costs and input usages for the chairs and tables are the same across all three plants, and the chair data is in the above example. Making functional tables requires three hours of labor and one unit of top capacity. It takes five hours of labor to make fancy tables and one unit of top capacity. Functional tables involve direct costs of $80 per unit and fancy tables $100 per unit. Chairs can be sold either at their point of production or can be transported to an overall assembly point which is located at plant 1. Chairs cost $5 to transport to the assembly point from plant 2 and $7 from plant 3. Tables cost $20 to ship in from plant 3 to the assembly point. Functional tables sell for $200 when sold alone and fancy tables sell for $300. The firm can sell functional sets containing four chairs and one table for $600 and fancy sets containing six chairs and one

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table for $1100.   Resource endowments in plant 1 are 175 units of labor and 50 units of top capacity. Plant 2 has 140 units of small lathe capacity, 90 units of large lathe capacity, 120 units of chair bottom capacity, and 125 units of labor.  Plant 3 has 130 units of small lathe capacity, 100 units of large lathe capacity, 110 units of chair bottom capacity, 210 units of labor, and 40 units of top capacity.

This problem's objective is to maximize net returns to the total firm operation subject to the overall firm linking considerations and the various constraints arising at the plants.  The resultant formulation is in Table 7.24.  The GAMS implementation is called BLOCKDIA.  Note the block diagonal structure involving the resources at plants 1, 2, and 3.  Also note the linking considerations on tables and chairs.

The solution to this problem is given in Table 7.25.  The variables $Y_{10}$ and $Y_{11}$ are the table fabrication activities for plant 1.  Variables $Y_{20}$ - $Y_{29}$ are the transportation and fabrication activities for plant 2, and the variables $Y_{30}$ - $Y_{35}$ are the transportation and fabrication activities for plant 3.  The objective function value is $36,206.9.  The firm manufactures tables and chairs in plants across the various locations as the reader may determine by investigating the solution.

**7.7.2 Comments**

This type of formulation appears in many places in the literature, usually linking other formulations.  In the example, we have used elements of the transportation, resource allocation, and assembly problems.  Generally, many LP problems look like this where multiple formulations are combined together in the analysis of a particular problem.  Other classes of block diagonal structures appear in the integer programming and dynamic model sections.  Dynamic models can be made block diagonal by incorporating the transfer activities from one period to the next into the overall structure. However, dynamic models generally contain a second type of block angular structure where there is some overlap; for example, production in period t is carried into period t+1 using storage.

The final comment involves the solution to this problem.  Dantzig and Wolfe observed that one could solve the block diagonal problem as a set of independent problems providing one had an estimate of

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the shadow prices on the overall linking constraints. This led to the Dantzig-Wolfe decomposition algorithm which formally exploits problem structure.

## 7.8 Concluding Comments

A number of LP formulations have been presented in this and the preceding chapter. These have all been simplified and are not easily applicable to any actual case. However, we hope the material increases the reader's familiarity with common usages of LP and shows how formulations may be combined in the analysis of empirical problems. There are a number of other additional comments which arise out of the above.

First, we hope the reader gained appreciation for empirical modeling issues. Data are never directly available rather, they must be calculated. We illustrate this in our examples where, for example, the coefficients required calculation through economic engineering or deductive accounting. Readers should also gain an appreciation for sparsity, as many LP problems contain few non-zero coefficients. This is exploited in the modern implementations of the simplex method where only nonzeros are stored, and fancy re-inversion schemes are utilized to solve the problems exploiting matrix sparsity (Orchard and Hays; Murtaugh).

In addition, we hope the reader has developed an appreciation for the implications of the particular primal variable structures on the shadow prices as arise through duality. Generally, the dual restraints require that the marginal cost of any variable be greater than or equal to the marginal revenue arising under that variable. Specific forms of this for any problem can be discovered by examining the dual. Often, such an exercise clears up confusion regarding shadow price values.

Another important point involves the role of summation notation. In many of the examples we were able to generate summation notation representations which later exactly translated into empirical models. This provides an important way of thinking about the problem and getting its structure right. Subsequently, one can easily check the properness of the structure and the properties of the dual variables. We believe this is important for modelers, as they can utilize summation representations to generate small

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example problems which are typical of larger structures.  This allows one to try out structures in small empirical problems and also provides guidelines from which computer implementations may be written. It also provides an important way of thinking about and analyzing the overall structure of the problem without concentrating a great deal on the particular empirical numbers involved.

Finally, we would like to mention that the coverage above is by no means comprehensive.  We will present many LP problems in the subsequent chapters.  There are also LP problems such as the input/output problem (Dorfman, Samuelson and Solow), the trim problem (Eisemann and Golden), and the caterer problem (Jacobs), along with many others which will not be included in this book.  The reader should see:  a) the literature cited in Riley and Gass, Day and Sparling, and Assad and Golden; b) the presentations and literature cited in such texts as Hillier and Lieberman; Gass (1985); Wagner (1969), Williams (1985); Bradley, Hax, and Magnanti; or Salkin and Saha, along with many others; and c) the many articles appearing in such journals as Management Science, Operations Research, Decision Sciences, Mathematical Programming,  American Journal of Agricultural Economics, Canadian Journal of Agricultural Economics, Western Journal of Agricultural Economics, North Central Journal of Agricultural Economics, and Southern Journal of Agricultural Economics.

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**Table 7.1. Data for Computer Excess Example**

Components Required to Assemble a System

| | XT | AT | 386SX | 38633 | 486SX | 48633 |
|---|---|---|---|---|---|---|
| 360FLOPPY | 1 | 1 | | | | |
| 12MFLOPPY | 1 | 1 | 2 | 1 | 1 | 1 |
| 144MFLOPPY | | | | 1 | 1 | 1 |
| HARDDISK | | 1 | 1 | 1 | 1 | 1 |
| MONO | 1 | 1 | 1 | | | |
| COLORVGA | | | | 1 | 1 | 1 |
| PLAINCASE | 1 | 1 | 1 | | | |
| FANCYCASE | | | | 1 | 1 | 1 |

Components Parts Acquisition Information

| Name | Cost | Inventory | Labor | Shelf Space |
|---|---|---|---|---|
| 360KFLOPPY | 35 | 20 | 0.01 | 0.01 |
| 12MFLOPPY | 49 | 29 | 0.01 | 0.01 |
| 144MFLOPPY | 52 | 32 | 0.01 | 0.01 |
| HARDDISK | 245 | 45 | 0.03 | 0.03 |
| MONO | 102 | 15 | 0.07 | 1.50 |
| COLORVGA | 302 | 45 | 0.10 | 2.00 |
| PLAINCASE | 41 | 11 | 0.15 | 1.70 |
| FANCYCASE | 80 | 12 | 0.12 | 1.70 |

Final Products Assembly and Sales Information

| Name | Sales Price | Minimum Sales | Assembly Cost | Labor | Space |
|---|---|---|---|---|---|
| XT | 689 | 1 | 59 | 2.00 | 1 |
| AT | 992 | 3 | 102 | 2.05 | 1 |
| 386SX | 1200 | 2 | 100 | 2.21 | 1 |
| 38633 | 1400 | 4 | 300 | 2.24 | 1 |
| 486SX | 1500 | 2 | 400 | 2.18 | 1 |
| 48633 | 1800 | 2 | 700 | 2.12 | 1 |

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**Table 7.2. Tableau of Computer Excess Example**

| | Assembly | | | | | | Buy | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | XT | AT | 386SX | 38633 | 486SX | 48633 | 360k | 12M | 144M | HARD | MONO | CVGA | PLAIN | FANCY | RHS |
| OBJECTIVE | 630 | 890 | 1100 | 1100 | 1100 | 1100 | -35 | -49 | -52 | -245 | -102 | -302 | -41 | -80 | Max |
| 360KFLOPPY | 1 | 1 | | | | | -1 | | | | | | | | ≤ 20 |
| 12MFLOPPY | 1 | 1 | 2 | 1 | 1 | 1 | | -1 | | | | | | | ≤ 29 |
| 144MFLOPPY | | | | 1 | 1 | 1 | | | -1 | | | | | | ≤ 32 |
| HARDDISK | | 1 | 1 | 1 | 1 | 1 | | | | -1 | | | | | ≤ 45 |
| MONO | 1 | 1 | 1 | | | | | | | | -1 | | | | ≤ 15 |
| COLORVGA | | | | 1 | 1 | 1 | | | | | | -1 | | | ≤ 45 |
| PLAINCASE | 1 | 1 | 1 | | | | | | | | | | -1 | | ≤ 11 |
| FANCYCASE | | | | 1 | 1 | 1 | | | | | | | | -1 | ≤ 12 |
| LABOR | 2 | 2.05 | 2.21 | 2.24 | 2.18 | 2.12 | 0.01 | 0.01 | 0.01 | 0.03 | 0.07 | 0.1 | 0.15 | 0.12 | ≤ 550 |
| SHELFSPACE | | | | | | | 0.01 | 0.01 | 0.01 | 0.03 | 1.5 | 2.0 | 1.7 | 1.7 | ≤ 590 |
| SYSTEMSPC | 1 | 1 | 1 | 1 | 1 | 1 | | | | | | | | | ≤ 240 |
| Lower Bound | 1 | 3 | 2 | 4 | 2 | 2 | | | | | | | | | |

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**Table 7.3. Solution for Computer Excess Example**

Objective 155330.097

| Variable | Value | Reduced Cost | Constraint | Slack | Shadow Price |
|---|---|---|---|---|---|
| XT | 1.0 | -168.4 | 360KFLOPPY | 16 | 0 |
| AT | 3.0 | -159.1 | 12MFLOPPY | 0 | 50.9 |
| 386SX | 172.9 | 0.0 | 144MFLOPPY | 0 | 53.9 |
| 38633 | 41.0 | 0.0 | HARDDISK | 0 | 250.7 |
| 486SX | 2.0 | 0.0 | MONO | 0 | 385.4 |
| 48633 | 2.0 | 0.0 | COLORVGA | 0 | 343.4 |
| 360KFLOPPY | 0.000 | -36.9 | PLAINCASE | 0 | 362.2 |
| 12MFLOPPY | 365.772 | 0.0 | FANCYCASE | 0 | 401.2 |
| 144MFLOPPY | 13.000 | 0.0 | LABOR | 10.09 | 0 |
| HARDDISK | 175.886 | 0.0 | SHELFSPACE | 0 | 188.9 |
| MONO | 161.886 | 0.0 | SYSTEMSPC | 18.11 | 0 |
| COLORVGA | 0.000 | -336.5 | | | |
| PLAINCASE | 165.886 | 0.0 | | | |
| FANCYCASE | 33.0 | 0.0 | | | |

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**Table 7.4. Data for Jerimiah Junk Yard Example**

| Car Data | ESCORTS | 626S | TBIRDS | CADDIES |
|---|---|---|---|---|
| PURCHASE PRICE | 85 | 90 | 115 | 140 |
| WEIGHT | 2300 | 2200 | 3200 | 3900 |
| DISASSEMBLY COST | 100 | 120 | 150 | 170 |
| AVAILABILITY | 13 | 12 | 20 | 10 |

Resource Use to Breakdown Cars

| | | | | |
|---|---|---|---|---|
| CAPACITY | 1 | 1 | 1.2 | 1.4 |
| LABOR | 10 | 12 | 15 | 20 |

Proportional Breakdown of Cars into Component Parts

| | ESCORTS | 626S | TBIRDS | CADDIES |
|---|---|---|---|---|
| METAL | .60 | .55 | .60 | .62 |
| SEATS | .10 | .10 | .06 | .04 |
| CHROME | .05 | .05 | .09 | .14 |
| DOORS | .08 | .10 | .10 | .07 |
| JUNK | .17 | .20 | .15 | .13 |

| Part Data | MINIMUM | MAXIMUM | PRICE | PARTSPACE | LABOR |
|---|---|---|---|---|---|
| METAL | 0 | | 0.15 | 0 | 0.0010 |
| SEATS | 4000 | 6000 | 0.90 | 0.003 | 0.0015 |
| CHROME | 70 | | 0.70 | 0.0014 | 0.0020 |
| DOORS | 2 | 5000 | 1.00 | 0.0016 | 0.0025 |
| JUNK | | | -0.05 | 0 | 0.0001 |

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**Table 7.5. Tableau of Jerimiah Junk Yard Example**

| | ESCORTS | 626S | TBIRDS | CADDIES | METAL | SEATS | CHROME | DOORS | JUNK | CONVERT SEATS | CONVERT CHROME | CONVERT DOORS | RHS MIN |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| OBJECTIVE | -185 | -210 | -265 | -310 | 0.15 | 0.90 | 0.70 | 1.00 | -0.05 | | | | |
| METAL | -1380 | -1210 | -1920 | -2418 | 1 | | | | | | -1 | -0.7 | = 0 |
| SEATS | -230 | -220 | -192 | -156 | | 1 | | | | 1 | | | = 0 |
| CHROME | -115 | -110 | -288 | -546 | | | 1 | | | | 1 | | = 0 |
| DOORS | -184 | -220 | -320 | -273 | | | | 1 | | | | 1 | = 0 |
| JUNK | -391 | -440 | -480 | -507 | | | | | 1 | -1 | | -0.3 | = 0 |
| CAPACITY | 1 | 1 | 1.2 | 1.4 | | | | | | | | | ≤ 42 |
| LABOR | 10 | 12 | 15 | 20 | .001 | .0015 | .0020 | .0025 | .0001 | | | | ≤ 700 |
| PARTSPACE | | | | | | .003 | .0014 | .0016 | | | | | ≤ 60 |
| LOWER BOUND | | | | | | 4000 | 70 | 2 | | | | | |
| UPPER BOUND | 13 | 12 | 20 | 10 | 100000 | 6000 | 10000 | 5000 | | | | | |

Note the requirements for the component parts constraints are set to an equality since junk has a negative price and the other .

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**Table 7.6. Solution for Jerimiah Junk Yard Example**

Objective = 18337.2

| Variable | Value | Reduced Cost | Constraint | Slack | Shadow Price |
|---|---|---|---|---|---|
| ESCORTS | 4.00 | 0 | Parts | | |
| 626S | 0 | -49.960 | METAL | 0 | 0.150 |
| TBIRDS | 20 | 31.688 | SEATS | 0 | -0.050 |
| CADDIES | 10 | 91.356 | CHROME | 0 | 0.150 |
| | | | DOORS | 0 | 0.090 |
| Sell | | | JUNK | 0 | -0.050 |
| METAL | 73186.2 | 0 | CAPACITY | 0 | 24.760 |
| SEATS | 6000 | 0.95 | LABOR | 43.512 | 0 |
| CHROME | 10000 | 0.550 | PARTSPACE | 20 | 0 |
| DOORS | 5000 | 0.910 | | | |
| JUNK | 18013.8 | 0 | | | |
| Convert | | | | | |
| SEATS | 320 | 0 | | | |
| CHROME | 1680 | 0 | | | |
| DOORS | 4866 | 0 | | | |

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## Table 7.7. Data for Chicken Example Yields from Cutting

| | Parts | Halves | Quarters | Meat | Leg-Breast-Thigh |
|---|---|---|---|---|---|
| Wings | 2 | | | | |
| Legs | 2 | | | | 2 |
| Thighs | 2 | | | | 2 |
| Back | 1 | | | | |
| Breasts | 1 | | | | 2 |
| Necks | 1 | | | | 1 |
| Gizzards | 1 | 1 | 1 | 1 | |
| Meat | | 0.05 | 0.07 | 1 | 0.2 |
| Breast Quarter | | | 2 | | |
| Leg Quarter | | | 2 | | |
| Halves | | 2 | | | |

**Selling Price and Labor Use for Chicken Packs**

| Pack | Labor | Price |
|---|---|---|
| A | 2 | $2.05 |
| B | 1.3 | 2.00 |
| C | 1.2 | 1.45 |
| D | 1.1 | 1.95 |
| E | 1.25 | 1.25 |
| Gizzard | 1.0 | 0.90 |

**Individual Selling Prices for Parts**

| Part | Price | Part | Price |
|---|---|---|---|
| Wings | 0.10 | Gizzards | 0.07 |
| Legs | 0.20 | Meat | 2.00/lb. |
| Thighs | 0.25 | Breast Quarters | 0.45 |
| Backs | 0.12 | Leg Quarter | 0.40 |
| Breasts | 0.33 | Halves | 0.90 |
| Necks | 0.05 | | |

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## Table 7.8. Primal Formulation of Charles Chicken Company Problem

| | Disassemble | | | | | Assemble | | | | | | Sell | | | | | | | | | | | Buy | | | RHS | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $X_p$ | $X_h$ | $X_q$ | $X_m$ | $X_L$ | $X_a$ | $X_b$ | $X_c$ | $X_d$ | $X_e$ | $X_g$ | Wings | Leg | Thigh | Back | Breast | Neck | Gizzard | Meat | Breast Qtr | Leg Qtr | Halves | Wings | Legs | Thighs | | |
| Object | -1 | -1 | -1 | -1 | -1 | 2.05 | 2.00 | 1.45 | 1.95 | 1.25 | .90 | .10 | .20 | .25 | .12 | .33 | .05 | .07 | 2.0 | .45 | .40 | .90 | -.12 | -.22 | -.27 | Max | |
| Wings | -2 | | | | | 2 | | | | | | 1 | | | | | | | | | | | -1 | | | ≤ | 0 |
| Legs | -2 | | | | -2 | 2 | | | | 2 | | | 1 | | | | | | | | | | | -1 | | ≤ | 0 |
| Thighs | -2 | | | | -2 | 2 | | | | 2 | | | | 1 | | | | | | | | | | | -1 | ≤ | 0 |
| Backs | -1 | | | | | 1 | | | | | | | | | 1 | | | | | | | | | | | ≤ | 0 |
| Breasts | -1 | | | | -2 | 1 | | | | | | | | | | 1 | | | | | | | | | | ≤ | 0 |
| Necks | -1 | | | | -1 | 1 | | | | | | | | | | | 1 | | | | | | | | | ≤ | 0 |
| Gizzards | -1 | -1 | -1 | -1 | | | | | | | 10 | | | | | | | 1 | | | | | | | | ≤ | 0 |
| Meat | | -.05 | -.07 | -1 | -.2 | | | | | | | | | | | | | | 1 | | | | | | | ≤ | 0 |
| Breast Qtr. | | | -2 | | | | 4 | | | | | | | | | | | | | 1 | | | | | | ≤ | 0 |
| Leg Qtr. | | | -2 | | | | | 4 | | | | | | | | | | | | | 1 | | | | | ≤ | 0 |
| Halves | | -2 | | | | | | | 2 | | | | | | | | | | | | | 1 | | | | ≤ | 0 |
| Chickens | 1 | 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | ≤ | 1000 |
| Labor | | | | | | 2 | 1.3 | 1.2 | 1.1 | 1.25 | 1 | | | | | | | | | | | | | | | ≤ | 3000 |
| Wing | | | | | | | | | | | | | | | | | | | | | | | 1 | | | ≤ | 20 |
| Leg | | | | | | | | | | | | | | | | | | | | | | | | 1 | | ≤ | 20 |
| Thigh | | | | | | | | | | | | | | | | | | | | | | | | | 1 | ≤ | 20 |

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**Table 7.9. Solution to the Charles Chicken Co. Problem**

| Objective function = 1362.7 | | | | | |
|---|---|---|---|---|---|
| Variable | Value | Reduced Cost | Equation | Slack | Shadow Price |
| $X_p$ | 0 | -0.22 | Wings | 0 | 0.120 |
| $X_h$ | 0 | 0 | Legs | 0 | 0.355 |
| $X_q$ | 0 | -0.33 | Thighs | 0 | 0.270 |
| $X_m$ | 0 | -0.27 | Backs | 0 | 0.180 |
| $X_L$ | 1000 | 0 | Breasts | 0 | 0.330 |
| $X_a$ | 0 | 0 | Necks | 0 | 0.050 |
| $X_b$ | 0 | 0 | Gizzards | 0 | 0.090 |
| $X_c$ | 0 | -0.15 | Meat | 0 | 2.000 |
| $X_d$ | 0 | -0.22 | Breast Qtr. | 0 | 0.500 |
| $X_e$ | 1010 | 0 | Leg Qtr. | 0 | 0.400 |
| Gizzards | 0 | 0 | Halves | 0 | 1.085 |
| Wings | 0 | -0.02 | Chickens | 0 | 1.36 |
| Legs | 0 | -0.02 | Labor | 1737.5 | 0 |
| Thighs | 0 | -0.155 | | | |
| Backs | 0 | -0.06 | | | |
| Breasts | 2000 | 0 | | | |
| Necks | 1000 | 0 | | | |
| Gizzards | 0 | -0.02 | | | |
| Meat | 200 | 0 | | | |
| Breast Qtr. | 0 | -0.05 | | | |
| Leg Qtr. | 0 | 0 | | | |
| Halves | 0 | -0.185 | | | |
| Wings | 0 | 0 | | | |
| Legs | 20 | 0 | | | |
| Thighs | 20 | 0.135 | | | |

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**Table 7.10. Data for the Grain Blending Example**

| | Grade | | Characteristics | |
|---|---|---|---|---|
| | Maximums | | Grain Batch 1 | Grain Batch 2 |
| | A | B | | |
| Moisture | 1 | 2 | 2 | 1 |
| Foreign Matter | 1 | 2 | 1 | 2 |

**Table 7.11. Solution of the First Formulation of the Grain Blending Problem**

Objective = 100

| Variable | Value | Reduced Cost | Equation | Slack | Shadow Price |
|---|---|---|---|---|---|
| A | 20 | 0 | Moisture | 0 | 1 |
| B | 20 | 0 | Foreign Matter | 0 | 0 |
| $G_1$ | 20 | 2 | Weight | 0 | 4 |
| $G_2$ | 20 | 3 | | | |

**Table 7.12. Optimal Solution to the Correct Formulation of the Grain Blending Problem**

Objective = 80

| Variable | Value | Reduced Cost | Equation | Slack | Shadow Price |
|---|---|---|---|---|---|
| A | 0 | 0 | 1 | 0 | 1 |
| B | 40 | 0 | 2 | 0 | 1 |
| $G_{11}$ | 0 | 0 | 3 | 0 | 5 |
| $G_{12}$ | 20 | 0 | 4 | 20 | 0 |
| $G_{21}$ | 0 | 0 | 5 | 20 | 0 |
| $G_{22}$ | 20 | 0 | 6 | 0 | 2 |
| | | | 7 | 0 | 2 |
| | | | 8 | 0 | 2 |

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**Table 7.13. LP Formulation of Sequencing Example 1**

| | | Plow - X | | | Disc - Y | | | Plant etc. - Z | | | RHS | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | April | May | June | May | June | July | May | June | July | | |
| Obj | | -100 | -100 | -100 | -20 | -20 | -20 | 400 | 400 | 400 | max | |
| X - Y | May | -1 | -1 | | 1 | | | | | | <= | 0 |
| link | June | -1 | -1 | -1 | 1 | 1 | | | | | <= | 0 |
| | July | -1 | -1 | -1 | 1 | 1 | 1 | | | | <= | 0 |
| Y - Z | May | | | | -1 | | | 1 | | | <= | 0 |
| link | June | | | | -1 | -1 | | 1 | 1 | | <= | 0 |
| | July | | | | -1 | -1 | -1 | 1 | 1 | 1 | <= | 0 |
| Labor | April | 0.2 | | | | | | | | | <= | 160 |
| | May | | 0.2 | | 0.3 | | | 0.3 | | | <= | 160 |
| | June | | | 0.2 | | 0.3 | | 0.1 | 0.3 | | <= | 160 |
| | July | | | | | | 0.3 | 0.1 | 0.1 | 0.3 | <= | 160 |
| | Aug. | | | | | | | 0.1 | 0.1 | 0.1 | <= | 160 |
| | Sept. | | | | | | | 0.5 | 0.1 | 0.1 | <= | 160 |
| | Oct. | | | | | | | | 0.5 | 0.1 | <= | 160 |
| | Nov. | | | | | | | | | 0.5 | <= | 160 |
| Land | | 1 | 1 | 1 | | | | | | | <= | 600 |

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**Table 7.14. Solution to Sequencing Example 1**

| Variable | | Value | Reduced Cost | | Equation | Slack | Shadow Price |
|---|---|---|---|---|---|---|---|
| Objective function = 168,000 | | | | | | | |
| Plow | April | 600 | 0 | Plow-Disc | May | -192.59 | 0 |
| | May | 0 | 0 (alt) | | June | 200.00 | 0 |
| | June | 0 | 0 (alt) | | July | 0 | 380 |
| Disc | May | 407.41 | 0 | Disc-Plant | May | 88.89 | 0 |
| | June | 0 | 0 | | June | 0 | 0 |
| | July | 192.59 | 0 | | July | 0 | 400 |
| Plant | May | 125.93 | 0 | Labor | April | 97.78 | 0 |
| | June | 281.48 | 0 | | May | 0 | 0 |
| | July | 192.59 | 0 | | June | 0 | 0 |
| | | | | | July | 0 | 0 |
| | | | | | Aug. | 100 | 0 |
| | | | | | Sept. | 11.11 | 0 |
| | | | | | Oct. | 51.11 | 0 |
| | | | | | Nov. | 60 | 0 |
| | | | | Land | | 0 | 280 |

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**Table 7.15. Yields for Crops 1 and 2 by Crop Planting and Harvest Dates**

| | Planting Date | | | | | |
|---|---|---|---|---|---|---|
| Harvest Date | Crop 1 | | | Crop 2 | | |
| | April | May | June | April | May | June |
| September | 110 | 105 | 90 | 38 | 40 | 35 |
| October | 125 | 120 | 118 | 35 | 38 | 40 |

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# Table 7.16.
# Formulation of Problem for Sequencing Example 2_

| Plow in Monyh | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Rows | | | | | | | | | | | | | | | | Mar | April | May | Mar | April | May | | | | | | | | | | | | |
| | | Mar | Apr | May | Jun | Apr | May | Jun | Apr | May | Jun | Apr | May | Jun | Apr | May | Jun | Apr | May | Jun | Mar | Apr | May | Jun | Sep | Oct | Nov | Crop 1 | Crop 2 | | |
| | | | | | | | | | Sep | Sep | Sep | Oct | Oct | Oct | Sep | Sep | Sep | Oct | Oct | Oct | | | | | | | | | | | |
| Objective | | | -5 | -5 | -3 | -3 | -3 | -60 | -60 | -60 | -60 | -60 | -60 | -43 | -43 | -43 | -43 | -43 | -43 | -10 | -10 | -10 | -10 | -10 | -10 | -10 | 3 | 8.7 | Max | | |
| Land Balance | | 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | <= | 1500 |
| | Mar | -1 | | | | | | | | | | | | | 1 | | | 1 | | | | | | | | | | | | <= | 0 |
| Plowed | Apr | -1 | -1 | | | 1 | | | | | | | | | 1 | 1 | | 1 | 1 | | | | | | | | | | | <= | 0 |
| Land | May | -1 | -1 | -1 | | 1 | 1 | | | | | | | | 1 | 1 | 1 | 1 | 1 | 1 | | | | | | | | | | <= | 0 |
| Balance | Jun | -1 | -1 | -1 | -1 | 1 | 1 | 1 | | | | | | | 1 | 1 | 1 | 1 | 1 | 1 | | | | | | | | | | <= | 0 |
| Disced | Apr | | | | | -1 | | | 1 | | | 1 | | | | | | | | | | | | | | | | | | <= | 0 |
| Land | May | | | | | -1 | -1 | | 1 | 1 | | 1 | 1 | | | | | | | | | | | | | | | | | <= | 0 |
| Balance | Jun | | | | | -1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | <= | 0 |
| | Mar | 0.3 | | | | | | | | | | | | | 0.2 | | | 0.2 | | | -1 | | | | | | | | | <= | 300 |
| | Apr | | 0.3 | | | 0.2 | | | 0.22 | | | 0.22 | | | 0.22 | 0.2 | | 0.22 | 0.2 | | | -1 | | | | | | | | <= | 300 |
| Labor | May | | | 0.3 | | | 0.2 | | 0.1 | 0.22 | | 0.1 | 0.22 | | 0.1 | 0.22 | 0.2 | 0.1 | 0.22 | 0.2 | | | -1 | | | | | | | <= | 300 |
| Avail- | Jun | | | | 0.3 | | | 0.2 | | 0.1 | 0.22 | | 0.1 | 0.22 | | 0.1 | 0.22 | | 0.1 | 0.22 | | | | -1 | | | | | | <= | 300 |
| ability | Jul | | | | | | | | | | 0.1 | | | 0.1 | | | 0.1 | | | 0.1 | | | | | -1 | | | | | <= | 300 |
| | Sep | | | | | | | | 0.7 | 0.7 | 0.7 | | | | 0.6 | 0.6 | 0.6 | | | | | | | | | -1 | | | | <= | 300 |
| | Oct | | | | | | | | | | | 0.7 | 0.7 | 0.7 | | | | 0.6 | 0.6 | 0.6 | | | | | | | -1 | | | <= | 300 |
| Yield | Crop 1 | | | | | | | | -110 | -105 | -90 | -125 | -120 | -118 | | | | | | | | | | | | | | 1 | | <= | 0 |
| | Crop 2 | | | | | | | | | | | | | | -38 | -40 | -35 | -35 | -38 | -40 | | | | | | | | | 1 | <= | 0 |

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**Table 7.17. Solution for Sequencing Example 2**

Objective function = 449,570

| Variable | | Value | Reduced Cost | Equation | | Slack | Shadow Price |
|---|---|---|---|---|---|---|---|
| Acreage Plowed in: | March | 1275 | 0 | Land | | 0 | 292.5 |
| | April | 0 | 0 | Plowd Land: | March | 1275 | 0 |
| | May | 225 | 0 | | April | 0 | 2.10 |
| | June | 0 | 0 | | May | 0 | 14.4 |
| Acre Disc for Crop 1 in: | April | 775 | 0 | | June | 0 | 284.0 |
| | May | 0 | 0 | Discd Land: | April | 0 | 13.16 |
| | June | 0 | 0 | | May | 0 | 5.34 |
| Acre Crp 1 plant/harvest | Sept./April | 0 | -40.15 | | June | 0 | 287.0 |
| | Sept./May | 0 | -49.81 | Labor: | March | 0 | 10 |
| | Sept./June | 0 | -92.65 | | April | 0 | 10 |
| | Oct./April | 775 | 0 | | May | 0 | 3 |
| | Oct./May | 0 | -9.66 | | June | 200.5 | 0 |
| | Oct./June | 0 | -13.5 | | July | 277.5 | 0 |
| Acreage of Crop 2 planted/harvested in: | Sept./April | 0 | -19.24 | | Sept. | 0 | 3.067 |
| | Sept./May | 500 | 0 | | Oct. | 0 | 10 |
| | Sept./June | 0 | -39.34 | Yield: | Crop 1 | 0 | 3 |
| | Oct./April | 0 | -49.5 | | Crop 2 | 0 | 8.7 |
| | Oct./May | 0 | -21.56 | | | | |
| | Oct./June | 225 | 0 | | | | |
| Labor hired in: | March | 82.5 | 0 | | | | |
| | April | 125.5 | 0 | | | | |
| | May | 0 | -7 | | | | |
| | June | 0 | -10 | | | | |
| | July | 0 | -10 | | | | |
| | Sept. | 0 | -6.93 | | | | |
| | Oct. | 377.5 | 0 | | | | |
| Crop 1 Sales | | 96875 | 0 | | | | |
| Crop 2 Sales | | 29000 | 0 | | | | |



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| **Table 7.18. Formulation of Storage Example** | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Objective | | Sell | | | | Store | | | | |
| | | $2.3X_1$ + | $2.5X_2$ + | $2.7X_3$ + | $2.9X_4$ - | $.1h_1$ - | $.2h_2$ - | $.3h_3$ | | |
| Grain Invent-ory | 1 | $X_1$ | | | + | $h_1$ | | | <= | 100 |
| | 2 | | $X_2$ | | - | $h_1$ + | $h_2$ | | <= | |
| | 3 | | | $X_3$ | | - | $h_2$ + | $h_3$ | <= | 0 |
| | 4 | | | | $X_4$ | | - | $h_3$ | <= | 0 |
| Max Sales | 1 | $X_1$ | | | | | | | <= | 50 |
| | 2 | | $X_2$ | | | | | | <= | 50 |
| | 3 | | | $X_3$ | | | | | <= | 50 |
| | 4 | | | | $X_4$ | | | | <= | 50 |
| Min Sales | 1 | $X_1$ | | | | | | | >= | 15 |
| | 2 | | $X_2$ | | | | | | >= | 5 |
| Max Store | | | | | | $h_1$ | | | <= | 75 |


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**Table 7.19. Primal Solution to the Storage Problem Example**

Objective = 237.5

| Variable | Value | Reduced Cost | Constraint | Slack | Shadow Price |
|---|---|---|---|---|---|
| $X_1$ | 25 | 0 | Pd1 Inventory | 0 | 2.3 |
| $X_2$ | 50 | 0 | Pd2 Inventory | 0 | 2.5 |
| $X_3$ | 25 | 0 | Pd3 Inventory | 0 | 2.7 |
| $X_4$ | 0 | 0 | Pd4 Inventory | 0 | 2.9 |
| $h_1$ | 75 | 0 | Max sale Pd1 | 25 | 0 |
| $h_2$ | 25 | 0 | Max sale Pd2 | 0 | 0 |
| $h_3$ | 0 | -0.1 | Max sale Pd3 | 25 | 0 |
| | | | Max sale Pd4 | 50 | 0 |
| | | | Capacity | 0 | 0.1 |
| | | | Min sale Pd1 | 10 | 0 |
| | | | Min sale Pd2 | 45 | 0 |
| | | | Min sale Pd3 | 25 | 0 |
| | | | Min sale Pd4 | 0 | 0 |



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**Table 7.20. Input Output Example Data**

| | Transactions Matrix | | | |
|---|---|---|---|---|
| | Manufacturing | Agriculture | Finance | Services |
| Manufacturing | 50 | 40 | 10 | 75 |
| Agriculture | 20 | 10 | 2 | 40 |
| Finance | 25 | 8 | 12 | 20 |
| Services | 100 | 40 | 40 | 40 |
| Exogenous | 55 | 24 | 11 | 55 |

**Final Demand Data**

| Sector | Final Demand for Sectors |
|---|---|
| Manufacturing | 75 |
| Agriculture | 50 |
| Finance | 10 |
| Services | 10 |

**Table 7.21. Technical Coefficient Matrix for Input Output**

| | Manufacturing | Agriculture | Finance | Services |
|---|---|---|---|---|
| Manufacturing | 0.200 | 0.328 | 0.133 | 0.326 |
| Agriculture | 0.080 | 0.082 | 0.027 | 0.174 |
| Finance | 0.100 | 0.066 | 0.160 | 0.087 |
| Services | 0.400 | 0.328 | 0.533 | 0.174 |
| Exogenous | 0.220 | 0.197 | 0.147 | 0.239 |



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**Table 7.22. LP Formulation of Input Output Example**

| | Manufacturing | Agriculture | Finance | Services | |
|---|---|---|---|---|---|
| Maximize | 1 | 1 | 1 | 1 | |
| Manufacturing | 0.8 | -0.33 | -0.13 | -0.33 | <= 75 |
| Agriculture | -0.08 | 0.92 | -0.03 | -0.17 | <= 50 |
| Finance | -0.1 | -0.07 | 0.84 | -0.09 | <= 10 |
| Services | -0.4 | -0.33 | -0.53 | 0.83 | <= 10 |

**Table 7.23. Solution for Input Output Example**

Objective = 677

| Variable | Value | Reduced Cost | Constraint | Slack | Shadow Price |
|---|---|---|---|---|---|
| Manufacturing | 250 | 0 | Manufacturing | 0 | 4.615 |
| Agriculture | 122 | 0 | Agriculture | 0 | 4.716 |
| Finance | 75 | 0 | Finance | 0 | 4.960 |
| Services | 230 | 0 | Services | 0 | 4.547 |



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# Table 7.24. Matrix Formulation of Block Diagonal Problem

| | | PLANT 1 | | | | | | PLANT 2 | | | | | | | | | | PLANT 3 | | | | | | | | | | | | | | | | RHS | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Sell Sets | | Make Table | | Sell Table | | Transport Chair | | Sell Chair | | Make Functional Chairs | | | Make Fancy Chairs | | | Transport Table | | Transport Chair | | Sell Table | | Sell Chair | | Make Table | | Make Functional Chairs | | | Make Fancy Chairs | | | | |
| | | FC | FY | FC | FY | | | FC | FY | FC | FY | Norm | MxSm | MxLg | Norm | MxSm | MxLg | FC | FY | FC | FY | FC | FY | FC | FY | FC | FY | Norm | MxSm | MxLg | Norm | MxSm | MxLg | | |
| Objective | | 600 | 1100 | -80 | -100 | 200 | 300 | -5 | -5 | 82 | 105 | -15 | -16 | -15.7 | -25 | -26.5 | -26.6 | -20 | -20 | -7 | -7 | 200 | 300 | 82 | 105 | -80 | -100 | -15 | -16 | -15.7 | -25 | -26.5 | -26.5 | Max | |
| PLANT 1 | Table FC | 1 | | -1 | | 1 | | | | | | | | | | | | -1 | | | | | | | | | | | | | | | | <= | 0 |
| | Inventory FY | | 1 | | -1 | | 1 | | | | | | | | | | | | -1 | | | | | | | | | | | | | | | <= | 0 |
| | Chair FC | 4 | | | | | | -1 | | | | | | | | | | | | -1 | | | | | | | | | | | | | | <= | 0 |
| | Inventory FY | | 6 | | | | | | -1 | | | | | | | | | | | | -1 | | | | | | | | | | | | | <= | 0 |
| | Labor | | | 3 | 5 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | <= | 175 |
| | Top Capacity | | | 1 | 1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | <= | 50 |
| PLANT 2 | Chair FC | | | | | | | 1 | | 1 | | -1 | -1 | -1 | | | | | | | | | | | | | | | | | | | | <= | 0 |
| | Inventory FY | | | | | | | | 1 | | 1 | | | | -1 | -1 | -1 | | | | | | | | | | | | | | | | | <= | 0 |
| | Small Lathe | | | | | | | | | | | 0.8 | 1.3 | 0.2 | 1.2 | 1.7 | 0.5 | | | | | | | | | | | | | | | | | <= | 140 |
| | Large Lathe | | | | | | | | | | | 0.5 | 0.2 | 1.3 | 0.7 | 0.3 | 1.5 | | | | | | | | | | | | | | | | | <= | 90 |
| | Chair Bottom Carver | | | | | | | | | | | 0.4 | 0.4 | 0.4 | 1 | 1 | 1 | | | | | | | | | | | | | | | | | <= | 120 |
| | Labor | | | | | | | | | | | 1 | 1.05 | 1.1 | 0.8 | 0.82 | 0.84 | | | | | | | | | | | | | | | | | <= | 125 |
| PLANT 3 | Table FC | | | | | | | | | | | | | | | | | 1 | | | | 1 | | | | -1 | | | | | | | | <= | 0 |
| | Inventory FY | | | | | | | | | | | | | | | | | | 1 | | | | 1 | | | | -1 | | | | | | | <= | 0 |
| | Chair FC | | | | | | | | | | | | | | | | | | | 1 | | | | 1 | | | | -1 | -1 | -1 | | | | <= | 0 |
| | Inventory FY | | | | | | | | | | | | | | | | | | | | 1 | | | | 1 | | | | | | -1 | -1 | -1 | <= | 0 |
| | Small Lathe | | | | | | | | | | | | | | | | | | | | | | | | | | | 0.8 | 1.3 | 0.2 | 1.2 | 1.7 | 0.5 | <= | 130 |
| | Large Lathe | | | | | | | | | | | | | | | | | | | | | | | | | | | 0.5 | 0.2 | 1.3 | 0.7 | 0.3 | 1.5 | <= | 100 |
| | Chair Bottom Carver | | | | | | | | | | | | | | | | | | | | | | | | | | | 0.4 | 0.4 | 0.4 | 1 | 1 | 1 | <= | 110 |
| | Labor | | | | | | | | | | | | | | | | | | | | | | | | | 3 | 5 | 1 | 1.05 | 1.1 | 0.80 | 0.82 | 0.84 | <= | 210 |
| | Top Capacity | | | | | | | | | | | | | | | | | | | | | | | | | 1 | 1 | | | | | | | <= | 40 |



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**Table 7.25. Primal Solution to the Block Diagonal Problem**

Objective = 36206.9

| Variable | | Value | Reduced Cost | Equation | | Slack | Shadow Price |
|---|---|---|---|---|---|---|---|
| Plant1 | Sell FC set | 24.40 | 0 | Plant1 | FC Tables | 0 | 212 |
| | Sell FY set | 29.01 | 0 | | FY Tables | 0 | 320 |
| | Make FC Table | 24.40 | 0 | | FC Chairs | 0 | 97 |
| | Make FY Table | 20.36 | 0 | | FY Chairs | 0 | 130 |
| | Sell FC Table | 0 | -12 | | Labor | 0 | 44 |
| | Sell FY Table | 0 | -20 | | Top Cap | 5.240 | 0 |
| Plant2 | Trans FC Chair | 62.23 | 0 | Plant2 | FC Chair | 0 | 92 |
| | Trans FY Chair | 78.2 | 0 | | FY Chair | 0 | 125 |
| | Sell FC Chair | 0 | -10 | | Sm Lathe | 0 | 47.77 |
| | Sell FY Chair | 0 | -20 | | Lrg Lathe | 0 | 38.83 |
| | Make FC Table | 0 | -58.11 | | Chair Bot | 16.907 | 0 |
| | Make FY Table | 0 | -96.85 | | Labor | 0 | 19.37 |
| | Make FC Chair N | 62.23 | 0 | Plant3 | FC Table | 0 | 200 |
| | Make FC Chair MS | 0 | -14.2 | | FY Table | 0 | 300 |
| | Make FC Chair ML | 0 | -5.04 | | FC Chair | 0 | 90 |
| | Make FY Chair N | 73.02 | 0 | | FY Chair | 0 | 123 |
| | Make FY Chair MS | 0 | -10.24 | | Sm Lathe | 0 | 18.50 |
| | Make FY Chair ML | 5.18 | 0 | | Lrg Lathe | 0 | 12.19 |
| Plant3 | Trans FC Table | 0 | -8 | | Chair Bot | 0 | 35.27 |
| | Trans FY Table | 8.649 | 0 | | Labor | 0 | 40.00 |
| | Trans FC Chair | 35.37 | 0 | | Top Cap | 20.562 | 0 |
| | Trans FY Chair | 95.85 | 0 | | | | |
| | Sell FC Table | 0 | 0 | | | | |
| | Sell FY Table | 10.79 | 0 | | | | |
| | Sell FC Chair | 0 | -8 | | | | |
| | Sell FY Chair | 0 | -18 | | | | |
| | Make FC Table | 0 | 0 | | | | |
| | Make FY Table | 19.44 | 0 | | | | |
| | Make FC Chair N | 35.37 | 0 | | | | |
| | Make FC Chair MS | 0 | -8.59 | | | | |
| | Make FC Chair ML | 0 | -3.35 | | | | |
| | Make FY Chair N | 76.83 | 0 | | | | |
| | Make FY Chair MS | 0 | -6.68 | | | | |
| | Make FY Chair ML | 19.02 | 0 | | | | |



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