Appendix I: Using Summation Notation With GAMS ................................................................. AI-1
  AI.1 Summation Mechanics ................................................................................................. AI-1
    AI.1.1 Sum of an Item ........................................................................................................ AI-1
    AI.1.2 Multiple Sums .......................................................................................................... AI-2
    AI.1.3 Sum of Two Items ..................................................................................................... AI-2
  AI.2 Summation Notation Rules .......................................................................................... AI-3
    AI.2.1 For a Scalar Equation .............................................................................................. AI-3
    AI.2.2. For a Family of Equations ..................................................................................... AI-4
  AI.4 Defining and Using Variables ...................................................................................... AI-7
  AI.5 Equations .................................................................................................................... AI-8
  AI.6 Cautions and Extensions ............................................................................................. AI-9
Appendix I: Using Summation Notation With GAMS

Summation notation is difficult for some students to use and follow. Here we present notes on the mechanics of summation notation usage and some rules for proper use. This discussion is cast within the GAMS framework with presentation equivalents of common summation expressions and error messages caused by improper summation. All of the GAMS statements used herein are shown in Table 1 and are in file NOTATION.

AI.1 Summation Mechanics

Summation notation is a short hand way of expressing sums of algebraic terms involving subscripted items. In order to cover the mechanics of summation notation it is useful to have a set of subscripted items and associated numerical values. Thus, let us define some data

\[
x_1 = 1 \quad y_{11} = 2 \quad y_{12} = 3 \\
x_2 = 2 \quad y_{21} = 4 \quad y_{22} = 1 \\
x_3 = 3 \quad y_{31} = 1 \quad y_{32} = 4.
\]

Now let us define a variety of summation expressions.

AI.1.1 Sum of an Item.

Suppose we wished to sum all values of x. This would be written as

\[
\sum_{i=1}^{3} x_i = x_1 + x_2 + x_3 = 1 + 2 + 3 = 6
\]

or in GAMS

```
SUM1 = SUM(I, X(I));
```

For short hand purposes if i was to be summed over all possible values, we would write this as

\[
\sum_{i} x_i.
\]

We might also express a sum as follows which indicates all of the i are summed over except i=3
\[
\sum_{i=1}^{3} x_i = 3.
\]

In GAMS, this is more difficult to express where one has to write a conditional (\$) operation or define a subset as follows

```gams
SUM1 = SUM(I$(ORD(I.NE.3)), X(I));
```

or

```gams
SET SUBSETI(I) /1, 2/;
SUM1 = SUM(SUBSETI, X(SUBSETI(I)));
```

### A1.1.2 Multiple Sums

Sums over two indices consider all combinations of those items
\[
\sum_{i} \sum_{j} y_{ij} = y_{11} + y_{12} + y_{21} + y_{22} + y_{31} + y_{32} = 15.
\]

The equivalent GAMS expression is

```gams
SUM2 = SUM((I,J), Y(I,J));
```

### A1.1.3 Sum of Two Items

Suppose we wished to sum over two items completely where they shared a subscript
\[
\sum_{i} (x_i + \sum_{j} y_{ij}) = \sum_{i} x_i + \sum_{j} y_{ij} = x_1 + y_{11} + y_{12} + x_2 + y_{21} + y_{22} + x_3 + y_{31} + y_{32} = 21.
\]

The equivalent GAMS expression is as follows

```gams
SUM3 = SUM(I, X(I)+SUM(J, Y(I, J)));
```

or

```gams
SUM3 = SUM(I, X(I)) + SUM((I,J), Y(I,J));
```

On the other hand, if we wished to sum the results only for the \(i^{th}\) element and call it \(A_i\) then

\[
A_i = x_i + \sum_{j} y_{ij} = x_i + y_{i1} + y_{i2}
\]

or in GAMS

\[
A(I) = X(I) + SUM(J, Y(I,J));
\]

which would yield a vector \([6, 7, 8]\) of results.
Sums over common subscripts can be collapsed or taken apart

\[ \sum_{i} (x_{i} + z_{i}) = \sum_{i} x_{i} + \sum_{i} z_{i} \]

or

\[ \text{SUM4} = \text{SUM}(I, X(I) + Z(I)); \]

or

\[ \text{SUM4} = \text{SUM}(I, X(I)) + \text{SUM}(I, Z(I)); \]

**AI.2 Summation Notation Rules**

Certain rules apply when writing summation notation equations. The applicable rules depend on whether the final result is an unsubscripted scalar or a subscripted family of results determined by multiple equations.

**AI.2.1 For a Scalar Equation**

\[ B_1 = \sum_{i} \sum_{j} \sum_{k} p_{ijk} + \sum_{m} \sum_{n} q_{mn}. \]

All subscripts must be dealt with in each term. Thus, it is proper to define the equation

However, the following equations are wrong

\[ B_2 = p_{ijk} + q_{mn} \]

\[ B_3 = \sum_{i} \sum_{j} p_{ijk} + \sum_{m} \sum_{n} q_{mn}. \]

In the case of the first equation, the result would really have the subscripts \(i, j, k, m, n\), while the second equation result would have to have a \(k\) subscript on \(B_3\) or a sum over \(k\) to be proper.

Equivalent GAMS commands for the above equation expressions are

\[
\text{EQB1.. } B_1 = E= \text{SUM}((I,J,K), P(I,J,K)) + \text{SUM}((M,N), Q(M,N));
\]

\[
\text{EQB2.. } B_2 = E= P(I,J,K) + Q(M,N);
\]

\[
\text{EQB3.. } B_3 = E= \text{SUM}((I,J), P(I,J,K)) + \text{SUM}((M,N), Q(M,N));
\]

Here, the first equation expression is correct, while the last two equation expressions are incorrect. If you run GAMS with the above commands, you would encounter GAMS error messages \$149 which says
"UNCONTROLLED SET ENTERED AS CONSTANT" meaning that you have not somehow dealt with all the subscripts in the equation.

**AI.2.2. For a Family of Equations**

Several rules apply when one is working with a family of equations.

1. The members of the family must be specified with an indication of the subscripts which define each equation. This is done by indicating all the conditions for which the equations exist in a "for" condition. For example, suppose we define an equation which sets all C's equal to 2. This is done by saying

\[ C_i = 2 \quad \text{for all } i \quad \text{or} \quad C_i = 2 \quad \text{for } i = 1, 2, \ldots, n. \]

Similarly, if we wish to set a 2-dimensional variable equal to a constant, we would state

\[ D_{ij} = 2 \quad \text{for all } i \text{ and } j, \]

while stating that for each row of the matrix \( E_{ij} \) we have the same values \( F_i \) is defined by

\[ E_{1ij} = F_i \quad \text{for all } i \text{ and } j. \]

The equivalent GAMS commands for the above expressions are

```
EQUATIONS
  EQC(I)   EQUATION C
  EQD(I,J) EQUATION D
  EQE1(I,J) EQUATION E1;
EQC(I).. C(I) =E= 2;
EQD(I,J).. D(I,J) =E= 2;
EQE1(I,J).. E1(I,J) =E= F(I);
```

On the other hand, it is wrong to state

\[ E_{2ij} = 2 \]

without conditions on \( i \) and \( j \). The equivalent GAMS commands for the above incorrect expressions are

```
EQUATION
  EQE2   EQUATION E2;
EQE2..  E2(I,J) =E= 2;
```

Here you would get error message $149 which says "UNCONTROLLED SET
2. When writing an equation with a for statement, all subscripts which are not in the for

\[
\sum_j \sum_k p_{ijk} = G i_i \quad \text{for all } i
\]

\[
\sum_k p_{ijk} = H i_i \quad \text{for all } i \text{ and } j
\]

statement must be summed over. Consequently, it is proper to write

\[
p_{ijk} = G 2_i \quad \text{for all } i
\]

\[
\sum_k p_{ijk} = H 2_i \quad \text{for all } i.
\]

The equivalent GAMS commands for the above equations are

```gams
EQUATIONS
  EQG1(I) EQUATION G1
  EQH1(I,J) EQUATION H1
  EQG2(I) EQUATION G2
  EQH2(I) EQUATION H2;
EQG1(I)..  G1(I) =E= SUM((J,K), P(I,J,K));
EQH1(I,J).. H1(I,J) =E= SUM(K, P(I,J,K));
EQG2(I)..  G2(I) =E= P(I,J,K);
EQH2(I)..  H2(I) =E= SUM(K, P(I,J,K));
```

in which the first two equations are correct, while the last two equations are wrong and

error messages $149 "UNCONTROLLED SET ENTERED AS CONSTANT"$ would

again be realized.

3. In any term of an equation, the result after executing the mathematical operations in that
term must be of a dimension less than or equal to the family definition in the for

statement. For example, it is proper to write
\[
\sum_j \sum_k p_{ijk} = L1 \quad \text{for all } i
\]

\[
\sum_j \sum_k r_{ijkm} + \sum_j s_{ijm} = N_{im} \quad \text{for all } i \text{ and } m
\]

but wrong to write

\[
p_{ijk} = L2 \quad \text{for all } i.
\]

Thus, for the following expressions, the first two equations are appropriate but the last equation would give you error message $149 "UNCONTROLLED SET ENTERED AS CONSTANT."

4. When the dimension is less than the family definition this implies the same term appears in multiple equations. For example, in the equation

\[
2 + \sum_j \sum_k p_{ijk} + \sum_j s_{ijm} = O_{im} \quad \text{for all } i \text{ and } m,
\]

the 2 term appears in every equation and the sum involving \( p \) is common when \( m \) varies.

Equivalent GAMS commands are as follows

\[
\begin{align*}
\text{EQUATION} \\
\text{EQLI(I)}... & \quad \text{L1(I) =E= SUM((J,K), P(IJK));} \\
\text{EQN(I,M)}... & \quad \text{N(I,M) =E= \text{SUM((J,K), R(IJK,K,M))} + \text{SUM (J,S(IJK,M));}} \\
\text{EQL2(I)}... & \quad \text{L2 =E= P(IJK):}
\end{align*}
\]

5. In an equation you can never sum over the parameter that determines the family of equations. It is certainly wrong to write

\[
\sum_k \sum_j \sum_i p_{ijk} = W_i \quad \text{for all } i.
\]

Or, equivalently, the following expressions are wrong and will result in error.
message $125 which says "SET IS UNDER CONTROL ALREADY."

```gams
EQW(I)... W(I) =E= SUM(I,J,K), P(I,J,K));
```

**AI.3 Defining Subscripts**

In setting up a set of equations and variables use the following principles. Define a subscript for each physical phenomena set which has multiple members, i.e.,

Let
- \(i\) denote production processes of which there are \(I\)
- \(j\) denote locations of which there are \(J\)
- \(k\) denote products of which there are \(K\)
- \(m\) denote sales locations of which there are \(M\).

Equivalent GAMS commands are

```gams
SET I /1*20/
J /1*30/
K /1*5/
M /CHICAGO, BOSTON/;
```

Define different subscripts when you are either considering subsets of the subscript set or different physical phenomena.

**AI.4 Defining and Using Variables**

1. Define a unique symbol with a subscript for each manipulatable item.

For example:

\(p_{ijk}\) = production using process \(i\) at location \(j\) while producing good \(k\).

Or, equivalently,

```gams
PARAMETER P(I,J,K)
```

or

```gams
PARAMETER PRODUCTION(PROCESS, LOCATION, GOOD)
```

Here, for documentation purposes, the second expression is preferred.
2. Make sure that variable has the same subscript in each place it occurs.

Thus it is proper to write

\[ \text{Max} \sum_{i} \sum_{j} \sum_{k} t_{ijk} \]

\[ \sum_{i} \sum_{j} t_{ijk} = 3 \quad \text{for all } k \]

but wrong to write

\[ \text{Max} \sum_{i} \sum_{j} t_{ij} \]

\[ \sum_{i} \sum_{j} t_{ijk} = 3 \quad \text{for all } k \]

\[ t_{ijk} \in 0. \]

The second model would cause error message $148$ indicating "DIMENSION DIFFERENT."

3. The authors feel it is a bad practice to define different items with the same symbol but varying subscripts. We think you should never use the same symbol for two different items as follows

\[ u_{ij} = \text{amount of tires transported from } i \text{ to } j \text{ and} \]

\[ u_{kj} = \text{amount of chickens transported from } k \text{ to } j. \]

GAMS would not permit this, giving error $150$ "Symbolic Equations Redefined."

**AI.5 Equations**

Modelers should carefully identify the conditions under which each equation exists and use subscripts to identify those conditions. We do not think modelers should try to overly compact the families of equations. For example, it is OK to define
\[ \sum_j a_{ij} x_j \# b_i \]

for all \( i \), where \( a_{ij} \) is use of water by period and labor by period, where \( i \) denotes water periods and labor periods and \( b_i \) simultaneously contains water and labor availability by period. But we find it is better to define

\[ \sum_j d_{ij} x_j \# c_i \]

\[ \sum_j f_{ij} x_j \# h_i \]

where \( i \) denotes period,

\( d_{ij} \) denotes water use and \( c_i \) water availability,

\( f_{ij} \) denotes labor use and \( h_i \) labor availability.

### AI.6 Cautions and Extensions

1. Be careful when you sum over terms which do not contain the subscript you are summing over. This is equivalent to multiplying a term by the number of items in the sum.

\[ \sum_{j=1}^{N} x_i = N x_i \]

\[ \sum_{j=1}^{3} X_2 = 3(2) = 6 \]

Or, in GAMS

\[ \text{SUM5A} = \text{SUM(J, X("2")}; \]

2. Be careful when you have a term in a family of equations which is of a lesser dimension than the family, this term will occur in each equation. For example, the expression

\[ \sum_j x_j = z_i \quad \text{for } i = 1,2,3 \]
implies that simultaneously

\[ \sum_j x_j = z_1 \]
\[ \sum_j x_j = z_2 \]
\[ \sum_j x_j = z_3. \]

3. The same rules as outlined above apply to product cases

\[ \prod_{i=1}^{3} x_i = x_1 * x_2 * x_3. \]

Or, equivalently,

\[ \text{PRODUCTX} = \text{PROD(I, X(I))}; \]

4. The following relationships also hold for summation

a. \[ \sum_i K x_i = K \sum_i x_i \]

b. \[ \sum_{i=1}^{n} KP = K \sum_{i=1}^{n} P = K n P \]

c. \[ \sum_i \sum_j (v_{ij} + y_{ij}) = \sum_i \sum_j v_{ij} + \sum_i \sum_j y_{ij} \]

d. \[ \sum_i \sum_j (x_i + y_{ij}) = n \sum_i x_i + \sum_i \sum_j y_{ij} \quad \text{when } j = 1, 2, \ldots, n \]
### Table 1. Sample GAMS Commands for Summation Notation Expressions

```gams
* *************************************************************
** THIS FILE CONTAINS GAMS EXAMPLES IN SUPPORT **
** OF THE NOTES USING THE SUMMATION NOTATION **
* *************************************************************

SETS
I /1*3/
J /1*2/
K /1*2/
M /1*2/
N /1*3/

PARAMETERS
X(I) /1 1,2,3 2,3 3/
Z(I) /1 2,2,4,3 6/

TABLE Y(I,J)
      1    2
     1    2    3
     2    4    1
     3    1    4;

TABLE V(I,J)
      1    2
     1    2    3
     2    4    1
     3    1    4;

TABLE P(I, J, K)
      1.1 1.2 2.1 2.2
    1 1 3 5 7
    2 2 4 6 8
    3 1 2 3 4;

TABLE Q(M, N)
      1   2   3
    1   1 10   5
    2 10   5   1;

* *******************************************************
** AI.1.1 SUM OF AN ITEM **
* *******************************************************

PARAMETER
SUM1    SUM OF AN ITEM;
SUM1 = SUM(I, X(I));
DISPLAY SUM1;

* *******************************************************
** AI.1.2 MULTIPLE SUMS **
* *******************************************************

PARAMETER
SUM2    MULTIPLE SUMS;
SUM2 = SUM((I, J), Y(I, J));
DISPLAY SUM2;
```
Table 1. Sample GAMS Commands for Summation Notation Expressions (continued)

62  **********************************
63  ** AI.1.3 SUM OF TWO ITEMS **
64  **********************************
65
66  PARAMETERS
67     SUM3A    SUM OF TWO ITEMS-1
68     SUM3B    SUM OF TWO ITEMS-1
69     A(I)     SUM OF TWO ITEMS-2
70     SUM4A    SUM OF TWO ITEMS-3
71     SUM4B    SUM OF TWO ITEMS-3;
72     SUM3A    = SUM(I, X(I)+SUM(J, Y(I, J)));
73     SUM3B    = SUM(I, X(I)) + SUM ((I,J), Y(I,J));
74     A(I)     = X(I) + SUM(J, Y(I,J));
75     SUM4A    = SUM(I, X(I)+Z(I));
76     SUM4B    = SUM(I, X(I)) + SUM(I, Z(I));
77     DISPLAY SUM3A, SUM3B, A, SUM4A, SUM4B;
78
79  **********************************
80  ** AI.2.1 FOR A SCALER EQUATION **
81  **********************************
82
83  PARAMETERS
84     B1   SUM FOR A SCALER EQUATION-1;
85     B1 = SUM((I,J,K), P(I,J,K)) + SUM((M,N), Q(M,N));
86     DISPLAY B1;
87
88  * $ONTEXT
89  * THE FOLLOWING SUMMATION NOTATIONS ARE INCORRECT
90  * IF YOU TURN THESE COMMANDS ON, YOU WILL ENCOUNTER
91  * ERROR MESSAGES
92  * PARAMETERS
93  *  B2   SUM FOR A SCALER EQUATION-2
94  *  B3   SUM FOR A SCALER EQUATION-3;
95  *  B2 = P(I,J,K) + Q(M,N);
96  *  B3 = SUM((I,J), P(I,J,K)) + SUM((M,N), Q(M,N));
97  *  DISPLAY B2, B3;
98  * $OFFTEXT
99
100  **************************************
101  ** A.I.2.2 FOR A FAMILY OF EQUATIONS **
102  **************************************
103
104  VARIABLES      C(I), D(I,J), E1(I,J), F(J);
105  EQUATIONS
106     EQC(I)    EQUATION C
107     EQD(I,J)  EQUATION D
108     EQE1(I,J) EQUATION E1;
109     EQC(I)..  C(I) =E= 2;
110     EQD(I,J).. D(I,J) =E= 2;
111     EQE1(I,J).. E1(I,J) =E= F(J);
112
113  * $ONTEXT
114  * THE FOLLOWING EXPRESSION IS INCORRECT
115  * ERROR MESSAGES WILL BE ENCOUNTERED
116

copyright 1997 Bruce A. McCarl and Thomas H. Spreen.
Table 1. Sample GAMS Commands for Summation Notation Expressions (continued)

<table>
<thead>
<tr>
<th>Line</th>
<th>Command</th>
</tr>
</thead>
<tbody>
<tr>
<td>117</td>
<td>* VARIABLES E2(I,J);</td>
</tr>
<tr>
<td>118</td>
<td>* EQUATION</td>
</tr>
<tr>
<td>119</td>
<td>* EQE2 EQUATION E2;</td>
</tr>
<tr>
<td>120</td>
<td>* EQE2.. E2(I,J) =E= 2;</td>
</tr>
<tr>
<td>121</td>
<td>* $OFFTEXT</td>
</tr>
<tr>
<td>122</td>
<td>VARIABLES G1(I), H1(I,J);</td>
</tr>
<tr>
<td>123</td>
<td>EQUATIONS</td>
</tr>
<tr>
<td>124</td>
<td>EQG1(I) EQUATION G1</td>
</tr>
<tr>
<td>125</td>
<td>EQH1(I,J) EQUATION H1</td>
</tr>
<tr>
<td>126</td>
<td>EQG1(I) =E= SUM((J,K), P(I,J,K));</td>
</tr>
<tr>
<td>127</td>
<td>EQH1(I,J) =E= SUM(K, P(I,J,K));</td>
</tr>
<tr>
<td>128</td>
<td>$OFFTEXT</td>
</tr>
<tr>
<td>129</td>
<td>THE FOLLOWING EXPRESSIONS ARE INCORRECT</td>
</tr>
<tr>
<td>130</td>
<td>ERROR MESSAGES WILL BE ENCOUNTERED</td>
</tr>
<tr>
<td>131</td>
<td>VARIABLES G2(I), H2(I);</td>
</tr>
<tr>
<td>132</td>
<td>EQUATIONS</td>
</tr>
<tr>
<td>133</td>
<td>EQG2(I) EQUATION G2</td>
</tr>
<tr>
<td>134</td>
<td>EQH2(I) EQUATION H2</td>
</tr>
<tr>
<td>135</td>
<td>EQG2(I) =E= P(I,J,K);</td>
</tr>
<tr>
<td>136</td>
<td>EQH2(I) =E= SUM(K, P(I,J,K));</td>
</tr>
<tr>
<td>137</td>
<td>$OFFTEXT</td>
</tr>
<tr>
<td>138</td>
<td>VARIABLES L1(I), U(I,M), R(I,J,K,M), S(I,J,M);</td>
</tr>
<tr>
<td>139</td>
<td>EQUATIONS</td>
</tr>
<tr>
<td>140</td>
<td>EQL1(I) EQUATION L1</td>
</tr>
<tr>
<td>141</td>
<td>EQL2(I) EQUATION L2</td>
</tr>
<tr>
<td>142</td>
<td>EQL2(I) =E= P(I,J,K);</td>
</tr>
<tr>
<td>143</td>
<td>EQL2(I) =E= P(I,J,K);</td>
</tr>
<tr>
<td>144</td>
<td>$OFFTEXT</td>
</tr>
<tr>
<td>145</td>
<td>VARIABLE L2</td>
</tr>
<tr>
<td>146</td>
<td>EQUATION</td>
</tr>
<tr>
<td>147</td>
<td>EQO(I,M) EQUATION O;</td>
</tr>
<tr>
<td>148</td>
<td>EQO(I,M) =E= SUM((J,K), P(I,J,K)) + SUM(J, S(I,J,M));</td>
</tr>
<tr>
<td>149</td>
<td>$OFFTEXT</td>
</tr>
<tr>
<td>150</td>
<td>THE FOLLOWING EXPRESSION IS INCORRECT</td>
</tr>
<tr>
<td>151</td>
<td>GAMS ERROR MESSAGES WILL BE ENCOUNTERED</td>
</tr>
<tr>
<td>152</td>
<td>VARIABLE W(I);</td>
</tr>
<tr>
<td>153</td>
<td>EQUATION</td>
</tr>
<tr>
<td>154</td>
<td>EQW(I) EQUATION W;</td>
</tr>
<tr>
<td>155</td>
<td>EQW(I) =E= SUM((I,J,K), P(I,J,K));</td>
</tr>
<tr>
<td>156</td>
<td>$OFFTEXT</td>
</tr>
</tbody>
</table>

copyright 1997 Bruce A. McCarl and Thomas H. Spreen.
Table 1. Sample GAMS Commands for Summation Notation Expressions (continued)

```
172 ***************************************
173 ** AI.4 DEFINING AND USING VARIABLES **
174 ***************************************
175
176 VARIABLES
177 OBJ1        OBJECTIVE FUNCTION VALUE
178 T(I,J,K)   DECISION VARIABLE;
179
180 EQUATIONS
181 OBJFUNC1   OBJECTIVE FUNCTION
182 CONST(K)   CONSTRAINT;
183
184 OBJFUNC1.. OBJ1 =E= SUM((I,J,K), T(I,J,K));
185 CONST(K)..  SUM((I,J), T(I,J,K)) =E= 3;
186
187 MODEL EXAMPLE1 /ALL/;
188 SOLVE EXAMPLE1 USING LP MAXIMIZING OBJ1;
189 DISPLAY T.L;
190
191 * $ONTEXT
192 * THE FOLLOWING COMMANDS ARE INCORRECT
193 * THEY WILL RESULT IN ERROR MESSAGES
194 * VARIABLES
195 *   OBJ2        OBJECTIVE FUNCTION VALUE
196 *   TT(I,J,K)   DECISION VARIABLE;
197 * POSITIVE VARIABLE TT;
198 * EQUATIONS
199 *   OBJFUNC2   OBJECTIVE FUNCTION
200 *   CONSTT(K)  CONSTRAINT;
201 *   OBJFUNC2.. OBJ2 =E= SUM((I,J), TT(I,J));
202 *   CONSTT(K).. SUM((I,J), TT(I,J,K)) =E= 3;
203 * MODEL EXAMPLE2 /ALL/;
204 SOLVE EXAMPLE2 USING LP MAXIMIZING OBJ2;
205 DISPLAY TT.L;
206 * $OFFTEXT
207
208 **********************************
209 ** AI.6 CAUTIONS AND EXTENSIONS **
210 **********************************
211
212 PARAMETER
213 SUM5A  CAUTIONS AND EXTENSIONS-1;
214 SUM5A = SUM(J, X("2"));
215 DISPLAY SUM5A;
216
217 PARAMETER
218 PRODUCT6 CAUTIONS AND EXTENSIONS-2;
219 PRODUCT6 = PROD(I, X(I));
220 DISPLAY PRODUCT6;
221
222 PARAMETERS
223 SUM7A  CAUTIONS AND EXTENSIONS-3
224 SUM7B  CAUTIONS AND EXTENSIONS-3
225 SUM8A  CAUTIONS AND EXTENSIONS-4
226 SUM8B  CAUTIONS AND EXTENSIONS-4
227 SUM8C  CAUTIONS AND EXTENSIONS-4
228 SUM9A  CAUTIONS AND EXTENSIONS-5
```
### Table 1. Sample GAMS Commands for Summation Notation Expressions (continued)

<table>
<thead>
<tr>
<th>Line</th>
<th>Command</th>
</tr>
</thead>
<tbody>
<tr>
<td>226</td>
<td>SUM9A = SUM((I,J), V(I,J) + Y(I,J));</td>
</tr>
<tr>
<td>227</td>
<td>SUM9B = SUM((I,J), V(I,J)) + SUM((I,J), Y(I,J));</td>
</tr>
<tr>
<td>228</td>
<td>SUM10A = SUM((I,J), X(I) + Y(I,J));</td>
</tr>
<tr>
<td>229</td>
<td>SUM10B = 2*SUM(I, X(I)) + SUM((I,J), Y(I,J));</td>
</tr>
<tr>
<td>230</td>
<td>DISPLAY SUM7A, SUM7B, SUM8A, SUM8B, SUM8C,</td>
</tr>
<tr>
<td>231</td>
<td>SUM9A, SUM9B, SUM10A, SUM10B;</td>
</tr>
</tbody>
</table>