Title of the paper: Economics of Homeland Security: Carcass Disposal and the Design of Animal Disease Defense

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Abstract

In an effort to bolster confidence and protect the nation the U.S. government through agencies like the Department of Homeland Security is identifying vulnerabilities and evolving strategies for protection. Agricultural food supply is one identified vulnerable area, and animal disease defense is one of the major concerns there under. Should an outbreak of animal disease occur, it is likely to have a mass slaughter and disposal of animal carcasses. The current policy, mainly including slaughter policy and strict movement bans, may be not sufficient to control disease spread at a reasonable cost. We address the issue modeling vaccination as a supporting strategy with later slaughter of animals and argue that vaccination can decrease slaughter and disposal cost in the case of emergency. Our results show that (a) vaccination gains time to slow down the flow of slaughter, thereafter the disposal operation of animal carcasses. By smoothing slaughter/disposal flow, vaccination likely decreases slaughter and disposal cost; and (b) Vaccination is a more effective cost saving mechanism when the marginal cost of vaccination falls, the even size of disease outbreak is larger, the disease is more contagious and spreads faster, and/or vaccines are more effective in controlling disease spread.
1. Introduction

New Yorkers, Washingtonians, Americans in general, and the whole world were shocked and terrified by September 11th, 2001 terrorist attack on the World Trade Center and the Pentagon. Americans did not subsequently feel safe and at peace. Subsequently in an effort to bolster confidence and protect the nation the U.S. government through agencies like the Department of Homeland Security is identifying vulnerabilities and evolving strategies for protection. Agricultural food supply is one identified vulnerable area, and animal disease defense is one of the major concerns there under.

The Department of Homeland Security currently lists foot and mouth disease (FMD), Rift valley fever, avian influenza, and Brucella as priority threats to U.S. agriculture (Breeze 2004). Outbreaks of such diseases can have large economic implications as data on FMD outbreaks in the United Kingdom revealed. Namely,

(a) The 1967/68 outbreak caused the slaughter of 434,000 animals, leading to a direct cost of £35 million borne by the Ministry of Agriculture, Fisheries and Food and an indirect cost of £150 million borne by the livestock industry (Doel and Pullen 1990).

(b) The 2001 outbreak resulted in the slaughter of 6.6 million animals (Scudamore et al. 2002) and a £3 billion cost to the UK government and a £5 billion cost to the private sector (NAO report 2002).

Should a major disease outbreak occur, it is crucial to have an effective strategy to control the disease spread and manage the resultant affected animals. From an economic sense such a strategy would be designed to minimize the costs arising from (a) livestock losses; (b) government, industry and consumer economic impacts; (c) public health hazards; and (d) environmental damages. And disposal of slaughtered animals is part of this strategy.

Disease management strategies vary across the world. Vaccination has been widely used in some Asian, Africa and South American countries to control endemic FMD disease (Doel and Pullen 1990). However, in “disease-free” countries in North and Central American including the United States, the European Union, Australia and New Zealand, the basic disease control policy is slaughter of all infected and susceptible infected animals (Breeze 2004). In the case of a large outbreak such an approach mandates the slaughter of
a large number of animals, which induces a large carcass disposal issue i.e. how, at a reasonable cost, do you dispose of 6 million carcasses without damaging air, water, and land quality. Such an issue may alter the optimal disease control management system establishing a tradeoff between disease management costs and carcass disposal costs. This raises the economic issue addressed in this paper, namely, we investigate the way that the carcass disposal issue influences the design of the total disease management system.

2. Background - disease management and carcass disposal

There are various technologies that may be employed to dispose contaminated animal carcasses, including burial, incineration, composting, rendering, lactic acid fermentation, alkaline hydrolysis, and anaerobic digestion, as discussed in the recent carcass disposal review done by the National Agricultural Bio-security Center Consortium (2004). These alternatives embody some pre-outbreak activities. Namely, disposal facilities can be constructed and located before an outbreak occurs. However, such facilities can be expensive and typically have limited capacity. Pre outbreak actions may be difficult to justify given the infrequency of major outbreaks.

Carcass disposal concerns can also influence the type of disease control management strategy employed. One can use strategies that reduce the rate of slaughter so that the needed rate of carcass disposal can be reduced which in turn reduces the immediate severity of the carcass disposal problem as well as the needed facilities to handle disposal. Vaccination of potentially infected and susceptible infected animals is one of these strategies. Even though the emergency plan in some disease free countries such as the United Kingdom regards vaccination as a supporting strategy, vaccination is not considered as a main option because of the following disadvantages: (a) vaccinated animals cannot be distinguished from infected animals. However, this disadvantage may be relaxed since USDA scientists developed a test that can distinguish FMD vaccinated animals from infected animals and commercial tests are now available (Breeze 2004); (b) Vaccinated animals would need to be slaughtered anyhow given the current stamp out policy to maintain a countries "disease free" status; and (c) Some vaccinated and infected animals are still potentially contagious which reduces the disease management
effectiveness relative to immediate slaughter (for further discussion see Doel, Williams, Barnett 1993, Elbakidze 2004, and APHIS 2002).

Carcass disposal in the case of a large outbreak can generates a tremendous operational concern and source of cost. For example, in the 2001 FMD UK outbreak a large scale incineration process was undertaken and in turn extensively publicized that caused substantial tourism losses (NAO report 2002). Vaccination in conjunction with later slaughter can buy time and lighten the disposal requirement but poses tradeoffs between the costs of disease control and carcass disposal. Consider the following simplified problem statement: suppose we can dispose all carcasses within a day at an extremely high cost, or within a couple days at a much lower cost. Disease management policy should consider whether it is better to have a mechanism to delay slaughter/disposal to achieve the lower disposal cost while somewhat less effectively controlling disease spread.

This setting leads naturally to the following questions: (1) is it technically feasible and economically effective to slaughter all infected and susceptibly infected animals within a given proximity of the outbreak? If not, what are other choices could we have? In this study, we examine vaccination as a supporting strategy to buy time in conjunction with later slaughter to reduce the carcass disposal load. Initially we develop a two-period model to examine this question then later a multiple-period model. In each setting, we minimize total cost by choosing the optimal amount of animals to be slaughtered or vaccinated by period, given

- the cost and capacity of carcass disposal,
- the cost of slaughter and vaccination,
- the initial event size, i.e., the number of infected and susceptibly infected animals,
- the disease spread caused by vaccinated and non-vaccinated animals, and
- the assumption that animals must eventually be slaughtered whether vaccinated or not.

3. Model

Vaccination while being widely used in some Asian, African, and South American countries where FMD is endemic is not a recommended practice in the “disease-free”
countries including the United Kingdom and the United States. Rather written policies in those countries employ strict movement controls and slaughter of all infected and contact animals. For example, if FMD virus were found in the United States, all animals in a radius of up to 3 kilometers around the infected farm, including the affected herd, cattle, sheep, goats, swine, and susceptible wildlife, whether they are infected or not, would be killed and disposed (Breeze, 2004). However, the potential for large natural or deliberately caused terrorism induced outbreaks in multiple locations raises the possibility of mass slaughter and carcass disposal events. The following facts drawn from the UK experience under the 2001 FMD outbreak tell the possibilities of such events:

(a) The 2001 FMD outbreak caused the slaughter of 6.6 million animals (Scudamore et al. 2002) and a mass backlog of slaughter and disposal. Figure 1 shows the weekly amount of slaughter and carcass disposal over the course of the FMD outbreak. More than half of animals were awaiting slaughter (see the left panel) and more than 1/3 of animals were awaiting disposal of (see the right panel) in the 6\textsuperscript{th} week. “At the height of the outbreak the daily weight of carcass moved was over half the weight of the ammunition the armed services supplies during the entire Gulf War” (NAO report 2002, page 50). This mass backlog suggests a potential value of vaccination as a supporting strategy.

![Graph showing weekly slaughter and carcass disposal](image)

**Table 1:** Slaughter and carcass disposal over time during the 2001 UK FMD outbreak

Data Source: Scudamore et al (2002)

(b) All animals slaughtered were not necessarily infected or even in contact with infected animals. Figure 2 shows the number of animals slaughtered due to different reasons. Among all the other categories, welfare slaughter, which means that animals are
killed because they could not be sent to the market given movement restrictions, accounted for approximately 39% of the total slaughter and disposal of animals in the 2001 UK FMD outbreak. Even among animals slaughtered for disease control purposes (those on infected premises, dangerous contact contiguous premises, and dangerous contact non-contiguous premises) a large proportion of animals were healthy at the time of slaughter. Vaccinating some animals properly will slow the slaughter/disposal operation and might control disease spread while potentially easing slaughter/disposal burden and reducing the cost.

![Chart showing animals slaughtered for various purposes](chart.png)

**Table 2:** Animals slaughtered for disease control and welfare purposes

<table>
<thead>
<tr>
<th>Category</th>
<th>Thousand slaughtered</th>
</tr>
</thead>
<tbody>
<tr>
<td>Infected premises</td>
<td>1291</td>
</tr>
<tr>
<td>Dangerous contact contiguous premises</td>
<td>1237</td>
</tr>
<tr>
<td>Dangerous contact non-contiguous premises</td>
<td>1510</td>
</tr>
<tr>
<td>Slaughter on suspicion</td>
<td>125</td>
</tr>
<tr>
<td>For welfare reasons</td>
<td>2293</td>
</tr>
</tbody>
</table>


(c) The mass slaughter and disposal largely through incineration was the subject of extensive press and television coverage, which induced losses to the economy including a large reduction in tourism. The 2001 FMD outbreak resulted in an

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2 The data compiling has the following considerations: (a) The totals for dangerous contact non-contiguous premises include the 3-kilometer cull; (b) The figures exclude approximately 4,000 other animals (mainly goats and deer) slaughtered for disease control purposes and approximately 3,000 other animals for welfare purposes; (c) The figures exclude slaughtered new born lambs and calves who were not counted in the database of the Department for Environment, Food and Rural Affairs and the Rural Payment Agency because their value, for compensation purpose, was included in the valuation assigned to their mothers.
estimated cost of £4.5 to £5.4 billion for Media and Sport; and £2.7 to £3.2 billion to business directly affected by tourist and leisure (NAO report 2002). If instead vaccination was employed effectively, this might reduce the spectacular nature of the event and, thus may reduce the indirect damage to other sectors such as tourism.

3.1. Incorporating vaccination

The effectiveness of vaccination depends on various factors, including (a) disease spread rates caused by vaccinated and non-vaccinated animals, (b) the scale of the initially infected and contact animals, (c) the relationship of environmental damage and slaughter volume, (d) the costs and capacity for slaughter and disposal, and (e) the magnitude of vaccination costs, among other factors.

To model such a decision we use both a two-period setting and a multi-period setting. In setting up the model we make the following assumptions:

- The initial total number of infected and contact animals to be slaughtered is \( Q \).
- Each slaughtered/disposed animal has a monetary loss of \( p \). Although the outbreak will affect the market value of livestock, we assume \( p \) is constant to make it simple.
- The disease control authority decides the optimal number of animals to be slaughtered and vaccinated by period.
- Welfare slaughter is not required i.e. that there is sufficient feed and capacity to store the vaccinated animals.
- The literature suggests two models of FMD disease spread: exponential form (Anderson and May 1991) and Reed-Frost form (Thrushfield 1995, Carpenter et al. 2004). To capture the spatial patterns of FMD disease spread, some researchers, including Bates et al. (2001), and Schoenbaum and Disney (2003), distinguish disease contact and spread into three categories: (1) direct contact caused by movement of animals and other direct contact of animals within a herd and among herds; (2) indirect contact caused by movements of vehicles and people within a herd and among herds; and (3) airborne of contagious FMD virus. We assume that the total infected and susceptibly infected animals in the next period \( Q_{t+1} \) consists of two components: (a) the remaining infected and contact animals from the
previous period \((Q_t - s_t)\) where \(s_t\) represents the amount of slaughter/disposal at
time \(t\), and (b) the newly infected animals resulting from the disease
spread \(\alpha(Q_t - s_t)\):

\[
Q_{t+1} = (1 + \alpha)(Q_t - s_t),
\]

Here the value of \(\alpha\) varies with and without vaccination because vaccinated animals
are much less contagious (Breeze 2004),

\[
\alpha = \begin{cases} 
\alpha_H & \text{if vaccination is not employed} \\
\alpha_L & \text{if vaccination is employed} 
\end{cases}.
\]

To gain insight into the role of vaccination, we elaborate both the two-period and
multiple-period settings to solve the slaughter and carcass disposal problem dynamically.
That is, policymakers have to make the following two decisions: (a) whether to employ
vaccination as a supporting disease control and carcass management strategy; and (b) how
many animals to be slaughtered/disposed and vaccinated in each period over the course of
an FMD outbreak.

### 3.2. Two-period setting

In the two-period setting, we assume that policymakers have two options: (a) a
slaughter of all infected and contact animals within a proximity of infected animals in the
first period; and (b) vaccination of some animals in the first period to buy time to lessen
the operational pressure and reduce the total disposal cost. However, all infected and
contact animals must be slaughtered and disposed in the second period. The cost
minimization problem with vaccination is

\[
\min \left[ SC(s_1) + VC(v_1) + EC(s_1) + p_s s_1 \right] + \frac{1}{1 + r} \left[ SC(s_2) + EC(s_2) + p_s s_2 \right],
\]

where \(SC(s)\) denotes the slaughter and carcass disposal cost, \(VC(v)\) is the total cost of
vaccinating \(v\) units of animals, and \(EC(s)\) captures the environmental damages resulting
from the slaughter and disposal of animals. All these three cost functions are increasing.
and convex. That is, $\frac{dSC(s)}{ds} > 0$ and $\frac{d^2 SC(s)}{ds^2} \geq 0$ \(^3\), $\frac{dVC(v)}{dv} > 0$ and $\frac{d^2 VC(v)}{dv^2} \geq 0$, and

\[
\frac{dEC(s)}{ds} > 0 \text{ and } \frac{d^2 EC(s)}{ds^2} \geq 0.
\]

$r$ denotes the time value of money, which is a proxy for the value of delaying the slaughter and disposal of a head animal to the next period.

The model solution optimally allocates all initially infected and contact animals within a given proximity of the infected animals for slaughter or vaccination, which is shown in equations (4-a). All vaccinated animals in the first period along with newly infected due to disease spread by the vaccinated animals and associated contact animals will be slaughtered and disposed of in the second period, as given by equation (4-b).

\[
s_1 + v_1 = Q \quad \text{in the first period,} \tag{4-a}
\]

\[
s_2 = (1 + \alpha)v_1 \quad \text{in the second period.} \tag{4-b}
\]

Minimizing equation (3) subject to equations (4-a) and (4-b) yields the optimality condition below:

\[
\frac{dSC(s_1)}{d s_1} + \frac{dEC(s_1)}{d s_1} + p = \frac{dVC(v_1)}{dv_1} + \frac{1 + \alpha}{1 + r} \left( \frac{dSC(s_2)}{d s_2} + \frac{dEC(s_2)}{d s_2} + p \right). \tag{5}
\]

Three terms on the left side of equation (5) represent the gain of postponing the slaughter of one additional animal to the next period, including the marginal slaughter/disposal cost, the marginal environmental damage, and the livestock value in the first period. The four terms on the right side is the present value of loss resulting from vaccinating one additional animal and slaughter/dispose this animal in the next period, consisting of: (a) the marginal vaccination cost in the first period; and (b) the discounted marginal slaughter/disposal cost, (c) the discounted marginal environmental damage, and (d) the discounted livestock value loss. The optimal number of slaughter and disposal of animals in the first period ($s_1$) is achieved when the present value of the potential gain equals the potential loss of vaccinating one additional animal to the next period to kill and

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\(^3\) The amount slaughter and disposal of animals should be at most equal to carcass disposal capacity. Otherwise, the total cost function should include the additional cost of building up new operation capacity.
dispose. Assuming, the optimal number of animals vaccinated in the first period is $v_i^* = Q - s_i^*$, it is optimal to employ vaccination when $s_i^* < Q$ or $v_i^* > 0$.

In the absence of vaccination all the subject animals $Q$ have to be killed and disposed in the first period. Thus, the total slaughter and disposal is $Q$ at the cost of $SC(Q) + EC(Q)$. Should vaccination be used, some animals are slaughtered and the remaining animals are vaccinated in the first period, and then disease is stamped out in the second period. Therefore, the total slaughter and disposal is $s_i^* + s_2^*$ and the corresponding total cost is the net present value of slaughter and disposal, vaccination, and environmental costs in two periods. Let $\Delta_v$ denote the difference in total number of animals slaughtered and disposed of and $\Delta_c$ denote the cost difference between these two options. $\Delta_c$ measures the cost reduction should vaccination be used and, hence, represents the value of vaccination. Algebraically, $\Delta_v$ and $\Delta_c$ are written below:

$$\Delta_v = (s_i^* + s_2^*) - Q = [s_i^* + (1 + \alpha)(Q - s_i^*)] - Q = \alpha(Q - s_i^*), \quad (6-a)$$

$$\Delta_c = \left[\frac{SC(Q) + EC(Q)}{Total \ cost \ without \ vaccination} - \left(\frac{SC(s_i^*) + VC(s_i^*) + EC(s_i^*) + s_i^* p}{Total \ cost \ in \ the \ first \ period}\right)\right] + \frac{1}{1 + r}\left[\frac{SC(s_2^*) + EC(s_2^*) + s_2^* p}{Total \ cost \ in \ the \ second \ period}\right]. \quad (6-b)$$

The magnitudes of these $\Delta$ measures depend upon the rate of disease spread, the time value of money, the initial size of FMD outbreak, the livestock value, etc.

**Proposition 1:** In the two-period setting,

- the amount of vaccinated animals becomes smaller (conversely the larger the first period slaughter becomes) when we have (a) increases in the disease spread rate caused by vaccinated animals, and/or (b) decreases in the time value of money.

$$\left\{\frac{d v_i^*}{d \alpha_L} < 0, \frac{d v_i^*}{d r} > 0, \text{ and } \frac{d s_i^*}{d \alpha_L} > 0, \frac{d s_i^*}{d r} < 0\right\}. \quad (a)$$

An increase in the initial size of disease outbreak will result in a larger amount of slaughter and vaccinations in the first period.

$$\left\{\frac{d v_i^*}{d Q} > 0 \text{ and } \frac{d s_i^*}{d Q} > 0\right\}. \quad (b)$$
• the total number of animals slaughtered and subsequently disposed of increases with an increase in (a) the event size and/or (b) the time value of money
\[
\left( \frac{d \Delta_n}{dQ} > 0 \text{ and } \frac{d \Delta_n}{dr} > 0 \right).
\]

• the value of vaccination is greater when we have (a) an decrease in the disease spread rate caused by vaccinated animals; (b) an increase in the event size; and/or (c) an increase in the time value of money \( \left\{ \frac{d \Delta_n}{d\alpha_L} < 0, \frac{d \Delta_n}{dQ} > 0 \text{ and } \frac{d \Delta_n}{dr} > 0 \right\} \).

• when the disease spread rate caused by vaccinated animals is higher than the time value \( (\alpha_L > r) \), an increase in the value of livestock induces a higher amount of slaughter and lower vaccination in the first period, a smaller amount of total slaughter/disposal of animals, and a lower cost saving resulting from vaccination.
\[
\left( \frac{d s^*_i}{dp} > 0, \frac{d v^*_i}{dp} < 0, \frac{d \Delta_i}{dp} < 0 \text{ and } \frac{d \Delta_i}{dp} < 0 \right) \text{. Otherwise, when } \alpha_L < r, \frac{d s^*_i}{dp} < 0, \frac{d v^*_i}{dp} > 0 \text{, and } \frac{d \Delta_i}{dp} > 0 \text{.}
\]

Proof: See Appendix A.

Proposition 1 suggests the following results:

(a) An increase in the spread rate caused by vaccinated animals may cause a more wide spread and, hence, a possibly high volume of animals slaughtered. On the other hand, when vaccinated animals spread disease faster, the disease control authority takes action by slaughtering and disposing a great amount of animals in the first period, which may reduce the total amount of slaughter and disposal. Thus, the effect of an increase in the spread rate among vaccinated animals is undetermined depending on the magnitude of these two effects. However, vaccination is more valuable when vaccinated animals spread disease slower.

(b) When the event size is greater in terms of the number of initially infected and contact animals, both the number of slaughter/disposal and the corresponding total cost will increase under both options. However, employing vaccination decreases cost more.
(c) The higher discount rate, the more valuable vaccination becomes even though the total slaughter and disposal may increases. Vaccination gains time to consider multiple alternate courses of action. It is more likely to employ more cost effectively disposal of carcasses. Because of the environmental regulations and public health concerns, on-farm burial was generally not used in the 2001 outbreak. Instead, seven mass burial pits were built in England (5), Scotland (1) and Wales (1) at a construction cost of £79 million; and the cost of restoration and management in the future are estimated at £35 million during the course of the 2001 FMD outbreak (NAO report 2002). We speculate that UK could have constructed the disposal capacity at a lower cost if the great time pressure did not exist.

(d) Considering the value of livestock loss, postponing the slaughter and disposal of one additional animal to the next period results in a gain of $p$ in the current period and a loss of $\frac{1+\alpha_L}{1+r} p$ in its present value. Therefore, the net livestock loss because of the later slaughter using vaccination is $\frac{1+\alpha_L}{1+r} p - p = \frac{\alpha_L - r}{1+r} p$. Hence, if the disease spread rate caused by vaccinated animals is higher than the time value, a higher livestock value increases the amount of slaughter/disposal and decreases the amount of vaccination in the current period. Moreover, the difference in the total slaughter and disposal of animals becomes smaller and vaccination is less valuable. If $\alpha_L = r$, the livestock value has no effect on the optimal solution at all.

3.3. Multiple-period setting

Should an FMD outbreak occur in the United States resulting from intentional terrorism attacks, we expect a mass slaughter and disposal of animals, and it is unlikely to stamp out the disease within a day or a week. A multiple-period setting is warranted in which the authority chooses the number of slaughter and vaccination if it is employed to minimize the total cost given the time requirement to stamp out the disease.

Let $i$ denote two options of in a multiple-period setting: (a) $i = n$ when vaccination is excluded from disease control and carcass disposal management; and (b) $i = v$ when vaccination is used as a supporting strategy. We assume that an infected animal is either
killed/disposed or vaccinated under the second option, i.e. no animal is carried on to the next period if it is not vaccinated. The cost comparison between these two options quantifies the value of vaccination. Let $Q^i_t$ denote the total infected and contact animals, and $s^i_t$ be the number of animals slaughtered and disposed at time $t$ given an option $i$.

$Q^i_t - s^i_t$ represents the total amount of infected and contact animals carried on to the next period, and $Q^i_t - s^i_t$ is the total vaccinated animals at time $t$. The change in the number of infected and contact animals is

$$\dot{Q}^i_t = -s^i_t + \alpha(Q^i_t - s^i_t),$$

where $\alpha$ is the rate of disease spread as defined in equation (2). Equation (7) decomposes the change of the total infected and contact animals into two components: (a) a decrease because of the current slaughter $s^i_t$ and (b) an increase resulting from the disease spread $\alpha(Q^i_t - s^i_t)$. The authority aims to control disease and manage carcass disposal in a timely manner to avoid the spill-over effects on the other sectors including tourism industry. We assume that the FMD virus has to be stamped-out within a time period $T$.

**Option 1 -- vaccination is not allowed**

The first option assumes that vaccination is excluded from the strategy set. Given that the disease has to be stamped out by the time period $T$, the authority decides the optimal slaughter and disposal of animals in each period. The cost minimization problem is

$$\min_{\{s, z\}} \int_{t_0}^T e^{-rt} \left[ SC(s^w_t) + EC(s^w_t) + p s^w_t \right] dt$$

s.t. $Q^w_t - s^w_t = -s^w_t + \alpha(Q^w_t - s^w_t)$.  

(8-a)

Based on equations (8-a) and (8-b), the Hamiltonian equation is

$$H = \left[ SC(s^w_t) + EC(s^w_t) + p s^w_t \right] + \lambda \left[ -s^w_t + \alpha(Q^w_t - s^w_t) \right],$$

(9)

The first order necessary conditions for an internal solution are

$$\frac{\partial H}{\partial s^w_t} = (SC + EC + p) - (1 + \alpha) \lambda = 0,$$

(10-a)
\[
\frac{\partial H}{\partial Q_{it}^w} = \lambda \alpha_H = r\lambda - \dot{\lambda},
\]
\[\tag{10-b}\]

\[
\frac{\partial H}{\partial \lambda} = \alpha_H \dot{Q}_{it}^w - (1 + \alpha_H) s_{it}^w = \dot{Q}_{i}^w.
\]
\[\tag{10-c}\]

Equation (10-a) suggests that the optimal slaughter is achieved when the marginal cost \((SC^*+EC^*+p)\) equals the gain \((1 + \alpha_H)\dot{\lambda}\) resulting from a slower spread caused by vaccinated animals. Based on equations (10-a), (10-b), and (10-c), we can derive the optimal dynamic solution for the number of slaughtered and the number of total infected and contact animals at time \(t\):

\[
S_{it}^w = \frac{(r - \alpha_H)(SC^*+EC^*+p)}{SC^*+EC^*},
\]
\[\tag{11-a}\]

\[
\dot{Q}_{i}^w = \alpha_H \dot{Q}_{i}^w - (1 + \alpha_H) s_{it}^w
\]
\[\tag{11-b}\]

Equation (11-a) implies that the change in the number of slaughtered animals increases (decreases) over time if the time value is greater (smaller) than the speed of disease spread. This suggests that more animals will be slaughtered in the later periods if the time value of money increases.

**Option II – vaccination is employed as a supporting strategy**

When vaccination is used in conjunction with later slaughter of animals, the net present value of the total event cost flow is minimized by choosing the optimal number of animals to be slaughtered \(s_{it}^v\) and to be vaccinated \(v_{it}^v\) at each time period \(t\). The cost minimization problem is given below:

\[
\min_{\{s_{it}^v, v_{it}^v\}} \int_0^T e^{-rt} \left[ SC(s_{it}^v) + VC(v_{it}^v) + EC(s_{it}^v) + p s_{it}^v \right] dt,
\]
\[\tag{12-a}\]

s.t. \( \dot{Q}_{i}^v = -s_{it}^v + \alpha_L (Q_{i}^v - s_{it}^v) \).
\[\tag{12-b}\]

Based on equations (12-a) and (12-b) we write the Hamiltonian equation:

\[
H = \left[ SC(s_{it}^v) + VC(Q_{i}^v - s_{it}^v) + EC(s_{it}^v) + p s_{it}^v \right] + \dot{\lambda} \left(-s_{it}^v + \alpha_L (Q_{i}^v - s_{it}^v)\right).
\]
\[\tag{13}\]

The first order necessary conditions for an internal solution are
Based on equations (14-a), (14-b), and (14-c), we derive the following dynamics:

\[
\dot{s}_t^i = \frac{V'' \dot{Q}_t^i - (1 + r)VC' + (r - \alpha)(SC' + EC' + p)}{SC'' + VC'' + EC''},
\]

\[
\dot{v}_t^i = \frac{(SC'' + EC'') \dot{Q}_t^i + (1 + r)VC' - (r - \alpha)(SC' + EC' + p)}{SC'' + VC'' + EC''},
\]

\[
\dot{Q}_t^i = \alpha Q_t^i - (1 + \alpha) s_t^i.
\]

As show in equation (15-a), when the dynamics of the amount of slaughter and disposal achieves its stability if there is any, the discounted marginal gain of postpone the slaughter and disposal of one head of animal \( \left( \frac{VC'' \dot{Q}_t^i + (r - \alpha)(SC' + EC' + p)}{SC'' + VC'' + EC''} \right) \) equals to the discounted marginal cost of vaccination \( \left( \frac{(1 + r)VC'}{SC' + VC'' + EC''} \right) \).

**Comparison between two options in the multiple-period setting**

To compare these two options and quantify the value of vaccination, we make following additional assumptions of cost terms: (a) a constant variable cost and a zero fixed cost of vaccination, i.e. \( VC' = vc \) and \( VC'' = 0 \); (b) a zero environmental cost that leads to underestimate the vaccinations value; and (c) a quadratic cost function of slaughter \( S(q^i) = a + bs_t^i + c s_t^i^2 \) such that both \( b \) and \( c \) are positive.

**Proposition 2:** In the multiple-period setting, when the time value of money is less than twice of the disease spread rate \( 0 < r < 2\alpha \), to stamp out the disease, it is necessary to slaughter and dispose at least \( \frac{2\alpha - r}{1 + \alpha} \) of currently infected and susceptibly infected.
animals in each period. Otherwise, when \( r \geq 2\alpha \), the disease will be stamped out regardless the ratio between \( Q'_i \) and \( s'_i \).

**Proof:** See Appendix B.

Proposition 2 implies that in order to bring disease under control when \( 0 < r < 2\alpha \), the disease control authority should at least slaughter and disposal \( \frac{100(2\alpha_H - r)}{1 + \alpha_H} \) percent of currently infected and contact animals when vaccination is not allowed, and \( \frac{100(2\alpha_L - r)}{1 + \alpha_L} \) percent when vaccination is employed. Therefore, a higher percentage of currently infected and contact animals are likely to be killed and disposed when vaccination is not allowed since \( \frac{2\alpha_L - r}{1 + \alpha_L} < \frac{2\alpha_H - r}{1 + \alpha_H} \). If in case, \( \frac{100(2\alpha_L - r)}{1 + \alpha_L} \) percentage of currently infected and contact animals already exceeds the operation capacity, it is less likely to bring disease under control unless new slaughter/disposal capacity is established.

As a supporting strategy in conjunction with later slaughter of animals in a total disease control management, our analytical results suggest the following advantages of vaccination:

(a) **It permits slowing down the flow of carcasses for disposal while controlling disease spread.** Many animals killed and disposed are likely not infected at all. In the 2001 FMD outbreak in the United Kingdom, less than 1% disposed animals were known to be infected (NAO report, Scudamore et al. 2002). Vaccination of these animals is feasible even given the concerns of disease spread. On the other hand, slowing down the slaughter and disposal operation lessens the pressure on the current existing capacity and likely reduces the spill-over effect on other industry.

(b) **It gains time allowing cost reductions for carcass disposal.** Vaccination could lessen pressure on the expensive facility construction. This extra time may permit use of cheaper and more environmental friendly options.
4. Simulation Results

The analytical results show that the desirability and the value of vaccination depends on the time value of money, cost terms of slaughter, disposal and vaccination, rates of disease spread caused by vaccinated and non-vaccinated animals, and the number of initially infected and contact animals. We run various simulations to conduct sensitive analysis of each factor. We choose the following value of parameters for the base case:

(a) **Time requirement and the time value of money**: We assume that FMD disease has to be stamped out within two weeks. The time value of money is $r=12\%$.

(b) **Value of slaughter and disposal cost**: The average slaughter cost per head is estimated at $130$ per head (Lambert 2002). The carcass disposal review provides the range of disposal cost for each technology (page 22 in Chapter 9). We convert the cost range per ton of carcasses into the cost per head in Table 2. The average cost is used for each technology, and the median ($63.5$) of the average costs is used for the base case.

(c) **Vaccination cost**: Vaccination costs consist of the cost of vaccines and the cost of administrating vaccinations in the field. In the United States, the North American Vaccine Bank (NAVB) stores viral antigens for FMD and other diseases and the Foreign Animal Disease Diagnostic Laboratory at Plum Island, New York identifies the viral subtype. If vaccinations for the viral subtype are not available at NAVB or a similar international vaccine bank, then the viral antigen has to be manufactured, which will lead to a higher vaccination cost. The vaccination administration cost depends on various factors such as transportation costs, institutional efficiency of delivering appropriate vaccines and operate vaccinations, etc. Vaccination cost per head ranges from $1.2$ when using the current $15$ FMD virus types (Breeze 2004) to $6$ estimated by Schoenbaum and Disney (2003). We suspect that vaccination cost per head in Shoenbaum and Disney (2003) is mainly the cost of veterinary service and ignore vaccine costs and transportation costs of shipping vaccines to the designated locations. Thus, in the base case, we assume the average vaccination cost is $15$ per head.

Table 3: Disposal cost under different disposal technology
<table>
<thead>
<tr>
<th>Carcass Disposal technology</th>
<th>range of cost (per ton)</th>
<th>range of cost (per head)</th>
<th>average cost (per head)</th>
</tr>
</thead>
<tbody>
<tr>
<td>burial</td>
<td>$15-200</td>
<td>$4-50</td>
<td>$27</td>
</tr>
<tr>
<td>landfill</td>
<td>$10-500</td>
<td>$2.5-125</td>
<td>$64</td>
</tr>
<tr>
<td>open burning</td>
<td>$200-725</td>
<td>$50-181</td>
<td>$116</td>
</tr>
<tr>
<td>fixed-facility incineration</td>
<td>$35-2000</td>
<td>$9-500</td>
<td>$255</td>
</tr>
<tr>
<td>air-curtain incineration</td>
<td>$140-510</td>
<td>$35-128</td>
<td>$82</td>
</tr>
<tr>
<td>bin- and in-vessel composting</td>
<td>$6-230</td>
<td>$1.5-58</td>
<td>$30</td>
</tr>
<tr>
<td>window composting</td>
<td>$10-105</td>
<td>$2.5-26</td>
<td>$14</td>
</tr>
<tr>
<td>rendering</td>
<td>$40-460</td>
<td>$10-115</td>
<td>$63</td>
</tr>
<tr>
<td>fermentation</td>
<td>$65-650</td>
<td>$16-163</td>
<td>$90</td>
</tr>
<tr>
<td>anaerobic digestion</td>
<td>$25-125</td>
<td>$6-31</td>
<td>$19</td>
</tr>
<tr>
<td>alkaline hydrolysis</td>
<td>$40-320</td>
<td>$10-80</td>
<td>$45</td>
</tr>
</tbody>
</table>


(d) **Environmental cost**: The environmental cost resulting from mass slaughter and disposal was set to zero due to the limited information and knowledge of the environmental damage. Hence, the value of vaccination as a supporting strategy could be underestimated.

(e) **Value of livestock loss per head**: Based on the USDA-NASS 2004 Statistics of Cattle, Hogs, and Sheep (USDA), we use $p = $819 to quantify the average livestock value per head.

(f) **The size of initial event**: We assume that the number of initially infected and contact animals is $Q = 100$.

(g) **Disease spread rates**: We assume that disease spread rate caused by vaccinated and non-vaccinated animals are $\alpha_L = 9\%$ and $\alpha_U = 11\%$, respectively. These values reflect that vaccinated animals shed less and, thus less contagious than non-vaccinated animals.

Figure 3 trace the number of slaughter/disposal of animals ($s_i$) and the total infected and contact animals ($Q_i$) at each period for option $i$. The horizontal and vertical axes represent time and the number of animals, respectively. Under the second option when vaccination is used, lines with circles and diamonds traces the dynamics of $Q_i$ and
The difference between these two lines represents the number of animals vaccinated and, thus, the area between these two lines shows the total vaccination. In turn lines with squares and pentagrams represent the dynamic outcomes ($Q^n_v$ and $\xi^n_v$) when vaccination is not allowed. The area under the line with diamonds (pentagrams) indicates the total slaughter and disposal of the outbreak when vaccination is (isn’t) used. This basic case shows that (a) the authority starts to slaughter and dispose animals in the 7th day under the non-vaccination option, and the 11th day when vaccination is employed; (b) vaccination decreases the total slaughter and disposal of animals, which is mainly because vaccinated animals are less contagious.

Let define two ratios,

$$\mu_l = \frac{TQ_v - TQ_{nv}}{TQ_{nv}} \times 100\%,$$  \hspace{1cm} (16-a)

$$\mu_k = \frac{TC_{nv} - TC_v}{TC_{nv}} \times 100\%,$$  \hspace{1cm} (16-b)

Table 4: Dynamics of the number of slaughter and the total infected and contact animals during the course of an FMD outbreak (base case)

(Value of parameters: $\bar{Q} = 100$, $r = 0.12$, $\alpha_{H} = 9\%$, $\alpha_L = 9\%$)

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where $TQ_i$ and $TC_i$ represent the total number of slaughter and disposal of animals and the net present value of the total cost under the option $i$. $\mu_i > 0$ implies that vaccination causes more animals slaughtered and disposed during the course of FMD disease; and $\mu > 0$ suggests that vaccination reduces the total cost.

We provide two cases in which the cost function of slaughter and disposal of animals is either linear or quadratic. It is obvious to assume the linear function form since we only find the average slaughter and disposal cost. However, simulations assuming the quadratic function form allow us to capture diseconomy of scale.

**Case 1 when the slaughter and disposal cost function is linear**

**Proposition 3:** When the slaughter and disposal cost per head is a constant, if $\alpha_H \geq r$ is satisfied, all initially infected and susceptibly infected animals are slaughtered and disposed in the first period and vaccination always decreases the total cost.

*Proof: See Appendix C.*

Assuming a constant slaughter and disposal cost per head, similarly as we discussed for Proposition 1, postponing the slaughter and disposal of one head of animal from the current period to next period causes (a) a present value of gain $b + p$ where $b$ is the current marginal slaughter/disposal cost, and (b) a present value of loss $\frac{1+\alpha_H}{1+r}(b + p)$.

Thus, if $\alpha_H \geq r$, the potential loss is greater than the potential gain and, hence, the authority will kill and dispose this animal in the current period rather than postponing to the next period. Proposition 3 suggests that the vaccination option is more economical strategy when the disease spread rate caused by non-vaccinated animals is greater than the time value of money. When $\alpha_H < r$, not all initial infected and susceptibly infected animals are killed and disposed in the first period. Hence, under certain conditions, a no-vaccination option may cost less. We conduct six sets of total 61 simulations varying one parameter at a time while controlling for other factors to quantify its impacts (see Table 2 for the specifications), and simulation results are illustrated in Figure 4.

**Table 5: Specifications of 61 simulations**

21
<table>
<thead>
<tr>
<th>Targeted Parameter</th>
<th>Value of the targeted parameter</th>
<th>Case #</th>
<th>Location in Figure 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>time value of money</td>
<td>( r = [0, 2%, 4%, 6%, 8%, 12%, 16%, 20%, 25%, 30%] )</td>
<td>1-10</td>
<td>1(^{st}) block</td>
</tr>
<tr>
<td>average vaccination cost</td>
<td>( vc = [0, 1.5, 6, 10, 15, 20, 30, 40, 50, 60] )</td>
<td>11-20</td>
<td>2(^{nd}) block</td>
</tr>
<tr>
<td>disease spread rate caused by vaccinated animals</td>
<td>( \alpha_L = [1%, 2%, 3%, 4%, 5%, 6%, 7%, 8%, 9%, 10%] )</td>
<td>21-30</td>
<td>3(^{rd}) block</td>
</tr>
<tr>
<td>disease spread rates caused by non- and vaccinated animals</td>
<td>( (\alpha_L, \alpha_H) = [(1%, 3%), (2%, 4%), (3%, 5%), (4%, 6%), (5%, 7%), )  ( (6%, 9%), (7%, 10%), (8%, 11%), )  9%, 11%), (10%, 12%)] )</td>
<td>31-40</td>
<td>4(^{th}) block</td>
</tr>
<tr>
<td>initial event size</td>
<td>( \bar{Q} = [100, 200, 300, 400, 500, 600, 700, 800, 900, 1000] )</td>
<td>41-50</td>
<td>5(^{th}) block</td>
</tr>
<tr>
<td>disposal technology</td>
<td>( dc = [14, 19, 27, 30, 45, 63, 64, 82, 90, 116, 255] )</td>
<td>51-61</td>
<td>6(^{th}) block</td>
</tr>
</tbody>
</table>

Table 6: The ratio of quantity and cost between with and without vaccination for each scenario (\( \mu_q \) and \( \mu_c \)) when the slaughter and disposal cost function is linear.

Figures 4(a) and 4(b) trace \( \mu_q \) and \( \mu_c \) where the horizontal axis indicates the case number of 61 simulations. These two panels suggest the following results:

1. **Effects of the time value of money**: As suggested by Proposition 3, in the first five simulations where \( \alpha_H \geq r \), a no-vaccination option results in the slaughter and
disposal of all initially infected and contact animals in the first period (100 heads), and vaccination is a cost-saving mechanism (slightly above zero). When \( \alpha_i < r \) is satisfied, whether vaccination can reduce the total cost is not straightforward. Delaying slaughter facilitates disease spread because of the volume effect, but vaccination can curtail the spread because vaccinated animals shed less. Simulations 5-10 in Figure 4 show that vaccination increases the total slaughter and disposal of animals by approximately 20%, and the value of vaccination in terms of cost-saving decreases as the time value of money goes up.

(2) **Effects of vaccination costs:** Simulations 11-20 show that a high vaccination cost increases the total amount of slaughter and disposal of animals and decreases the value of vaccination, which meet the expectation formulized in Proposition 1.

(3) **Effects of disease spread:** We examine the following effects of disease spread rates:

- As specified in simulations 21-30, the disease spread caused by vaccinated animals changes while the rate under the no-vaccination option is fixed.
- Simulations 31-40 change the disease spread rates by both vaccinated and non-vaccinated animals while their differences keep constant.

The third block in both panels of Figure 4 shows that vaccination causes a smaller increase in the total slaughter and disposal of animals and a smaller cost reduction as vaccinated animals becomes more contagious. Simulations in the fourth block suggest that vaccination is more valuable as the overall disease spread goes faster.

(4) **Effects of the number of initial infected and susceptibly infected animals:** FMD virus is 20 times more infectious than human smallpox (Breeze, 2004). Infected animals shed enormous amounts of virus, and they can easily infect other animals in the same herd and among herds by direct or indirect contact. FMD virus could also infect animals within a large premise by contamination of water, soil, etc. We vary the initial event size in simulations 40-41. It is obvious that vaccination becomes more valuable as the even size increases.

(5) **Effects of carcass disposal technology and costs:** Table 1 suggests that there are various carcass disposal technologies associated with different disposal costs.
Vaccination likely result in a larger amount slaughter and disposal of animals, and the possible cost saving resulting from vaccinating animals will decreases if the carcass disposal is more expensive. However, the values of model parameters suggest that the carcass disposal cost even under more expensive technology is dominated by the sum of the slaughter cost and the livestock loss per head. Simulations 50-61 suggests that vaccination cuts down the total cost more under a more expensive technology.

**Case 2 when the slaughter and disposal cost function is quadratic**

It is reasonable to expect a quadratic term in the slaughter and disposal cost function. For example, assuming a zero fixed cost, adding a positive (negative) quadratic term in the cost function will allow us to capture diseconomy (economy) of scale. However, the only cost figures we found in the literature is the constant average cost per head. In this case, we assume that the slaughter and disposal cost function is

$$SC_t = (b + p)s_t + c s_t^2 = ($130 + $63)s_t + s_t^2 / 2.$$  \hspace{1cm} (17)

Equation (17) implies that the slaughter and disposal operation exhibits diseconomy of scale. Figure 5 illustrate the comparison of the total amount of animal slaughter and disposal (the left panel) and the total cost (the right panel) with and without vaccination. The general patterns of the quantity ratio and the cost ratio ($\mu_q$ and $\mu_c$) are similar as in Figure 4.

![Graphs](image)

(a) quantity ratio ($\mu_q$)  
(b) cost ratio ($\mu_c$)
Table 7: The ratio of quantity and cost between with and without vaccination for each scenario (μ_t and μ_s) when the slaughter and disposal function is quadratic

Discussion on livestock value loss

The livestock loss is an important cost factor. Under the two-period setting where vaccination is employed to buy one additional period for slaughter and carcass disposal, when the disease spread rate caused by vaccinated animals is lower (higher) than the time value, an increase in the livestock value per head increases (decreases) the value of vaccination in reducing the total cost. The 2001 UK FMD outbreak revealed that at least 39% of animals were not infected at all and they had to be killed because of movement bans (NAO report). Naturally, the question is that how much is the potential gain if we could do something with these animals or their carcasses. Assuming that the welfare slaughter is 39%, we compare the total cost between the following scenarios:

(a) The authority cannot distinguish uninfected animals from others. They kill and dispose all infected and susceptibly infected animals.

(b) The disease control authority somehow can distinguish infected and uninfected animals. These 39% carcasses are saved for other uses such as doggie food and their value per head is 20% or 50% of the livestock value loss per head (p).

Results in Table 3 show that the authority can save 5% to 16% costs if they can utilize the carcasses of possibly uninfected animals. However, we should be cautious about the numbers since the net cost saving depends on the cost of distinguishing uninfected animals from infected ones.

Table 3: The cost saving (%) between scenarios in which the authority can and cannot distinguish uninfected animals from others

(Values of parameters: Ω = 100, α_H = 11%, α_L = 9%, r = 12%, slaughter and disposal cost=130+63=193, p=891, c=0 for the linear case and c=0.5 for the quadratic case)

<table>
<thead>
<tr>
<th>Assumption</th>
<th>Linear case</th>
<th>Quadratic case</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>w/ vaccination</td>
<td>w/o vaccination</td>
</tr>
<tr>
<td>Value=0.2p</td>
<td>5.21%</td>
<td>6.32%</td>
</tr>
<tr>
<td>Value=0.5p</td>
<td>13.52%</td>
<td>15.82%</td>
</tr>
</tbody>
</table>
5. Concluding Remarks and Policy Implications

In this study, we argue that vaccination could be a supporting strategy in FMD outbreak emergency. While controlling disease spread since vaccinated animals shed less, vaccination could gain time to slow down the flow of slaughter, thereafter the disposal operation of animal carcasses. Thus, employing vaccination may allow policy makers to seek a set of lower cost and environmental friendly strategies to control disease and manage carcass disposal. The potential supporting role of vaccinations in a large scale disease outbreak has been recognized since the 1967/68 UK FMD outbreak.\(^4\) This paper investigates the question economically showing cases where vaccinations could play an important role in reducing the total costs. The main cases where value is attained are summarized below:

(a) If we only use vaccination to buy one period, the total slaughter and disposal of animals is greater (see Proposition 1). Vaccination is likely a more effective cost saving mechanism if vaccinated animals spread disease much slower, the initial event size is greater, the time value of money is bigger, the livestock value per head increases when the disease spread is lower than the time value of money.

(b) In the multiple-period setting, vaccination generally increases the total amount of slaughter and disposal of animals but not always. Vaccination becomes more valuable in reducing the total cost when the costs of vaccination fall, the disease outbreak becomes larger, the vaccines are more effective in controlling disease spread, and/or the disease in general spreads faster.

Vaccination would be even more valuable if we overcame the following limitation in our model: we included nonzero environmental damages as we feel environmental costs fall if time pressures were removed.

\(^4\) The Northumberland Committee was established to review the outbreak and its control and eradication responses of the 1968/69 FMD outbreak in England. The committee recommended vaccination as a supporting mechanism for FMD outbreak control. Ever since then, European Union law permits the use of emergency vaccination as part of a stamping out policy where appropriate (NAO report 2002).
Welfare slaughter accounts for a substantial percentage of the total slaughter and disposal of animals. Animals killed due to the welfare purpose are not infected at all, and they are in the wrong place at the wrong time because of movement bans. If the authority can distinguish uninfected animals from other and have a differentiate meat market coming from uninfected animals or use these carcasses for other purpose such as doggie food, vaccination may be more valuable, and the total cost of event could be lower. However, policy makers shall consider screening costs of animals and trade disadvantage.

Should the authority agree on the value of vaccination as a supporting strategy conjunction with the later slaughter of animals, there are several remaining feasibility questions:

- **Could rules be relaxed to lessen or even diminish the trade disadvantage?** The International Office of Epizootics (OIE), is the WTO named agency that sets standards to prevent international spread of livestock diseases. International rules strongly favor “disease free” countries and are restrictive toward animal exports of countries where FMD is endemic. Therefore, countries like UK and US mainly rely on movement bans and slaughter policy to maintain their disease free status to take trade advantages. The trade disadvantage of vaccination was the main reason that the UK Farmers’ Unions opposed to vaccination during the 2001 FMD outbreak (NAO report). However, the relevant rules about FMD penalizing vaccination and encouraging mass slaughter do not reflect technology advances and economic rationale. Scientists associated with the USDA developed a test that can distinguish vaccinated animals from infected ones in 1994 (Breeze 2004), which will lessen the worry of disease spread through trade. To reflect the model technology and economic considerations, it may be of OIE’s interest to relax international trade regulations related to FMD diseases and allow disease-free but vaccinated animal into the world trade.

- **Can the authority supply vaccines in a timely manner?** When an FMD outbreak occurs, the feasibility of vaccination as a supporting strategy requires the availability of vaccines. There are two types of vaccine reserves (Doel and Fullen 1990): (1) conventional commercial FMD vaccine that has a 12-month shelf life;
and (2) concentrated inactivated vaccines with a 15-year predicted shelf life. The latter one are held and managed by a consortium of three countries including Canada, Mexico and the United States at the North American Vaccine Bank, NAVB (Breeze 2004), and a consortium of seven countries including Australia, Eire, New Zealand, Norway, Sweden and the United Kingdom at the Pairbright Laboratory of the AFRC Institute for Animal Health (Doel and Fullen 1990). The threat of bioterrorism imposes some pressure on vaccine reserves and, thus these FMD vaccine banks may need to take another look on their reserves.

- **Could the Authority deliver vaccines into infected and contact regions in a timely manner?** Even if there is enough vaccine matching the virus strings identified in infected animals, it will take time to move the vaccines to the needed points for use. Breeze (2004) argues it will take 1-2 days for transportation for a specimen and preliminary diagnosis at the Plum Island Foreign Animal Disease Diagnostic Laboratory; 2 days to determine the virus subtype; 4 days to produce the vaccine and deliver it to the outbreak location; and at least 1 day to administer the vaccine within the initial area designated for vaccination. Therefore, we need a minimum of 8 or 9 days to employ vaccination even if virus subtypes are available in NAVB. NAVB may need to design and establish a faster response procedure.

Our model implicitly assumes that the current slaughter and disposal capacity can handle operation flows. However, because of infrequent outbreaks, the pre-event capacities are very limited, let along environmental and legal concerns to dump contaminated animals carcasses. Therefore, it is likely to face a shortage of slaughter and disposal facilities. Policy makers have two options: either to build up slaughter and disposal facilities ex ante that will not be used if there is no outbreak; or to build up slaughter and carcass facility ex post that could be substantially costly. It is important to determine the optimal investment in disposal facilities ex ante, which can be the future research direction.

**Reference:**

APHIS (2002). Foot-and-Mouth Disease Vaccine at

Veterinary Services, United States department of Agriculture. Last access on May 3rd, 2005.


Appendix A: Proof of Proposition 1

The comparative static analysis of equation (4) yields the following inequalities:

\[
\frac{d s_t^*}{d \alpha} = \frac{1}{(1+r)SOC} \left( \frac{dSC(s_2)}{d_s} + \frac{dEC(s_2)}{d_s} + p + (1+\alpha) \left( \frac{d^2SC(s_2)}{d_s^2} + \frac{d^2EC(s_2)}{d_s^2} \right) \right) > 0 \quad (A-1)
\]

\[
\frac{d s_t^*}{d Q} = \frac{1}{SOC} \left[ \frac{dVC(v_1)}{d_v} + \frac{(1+\alpha)^2}{1+r} \left( \frac{d^2SC(s_2)}{d_s^2} + \frac{d^2EC(s_2)}{d_s^2} \right) \right] > 0; \quad (A-2)
\]

\[
\frac{d s_t^*}{dr} = -\frac{1+\alpha}{(1+r)SOC} \left[ \frac{dSC(s_2)}{d_s} + \frac{dEC(s_2)}{d_s} + p \right] < 0, \quad (A-3)
\]

\[
\frac{d s_t^*}{dp} = \frac{\alpha - r}{(1+r)SOC} \begin{cases} > 0 & \text{if } \alpha > r \\ = 0 & \text{if } \alpha = r \\ < 0 & \text{if } \alpha < r \end{cases}, \quad (A-4)
\]

where \(SOC>0\) is the second order condition of the cost minimization problem in equation (3). Taking the derivative of \(\Delta_e\) in equation (6-a) yields the following inequalities:

\[
\frac{d \Delta_e}{d \alpha} = -\frac{(\bar{Q} - s_1^*)}{1+r} \left( \frac{\partial SC}{\partial s_2} (s_2^*) + \frac{\partial EC}{\partial s_2} (s_2^*) \right) < 0, \quad (A-5)
\]

\[
\frac{d \Delta_e}{d Q} = \left( \frac{dSC}{d s_1} (\bar{Q}) + \frac{dEC}{d s_1} (\bar{Q}) + p \right) - \left( \frac{dVC}{d_v} (v_1) + \frac{1+\alpha}{1+r} \left( \frac{dSC}{d_s} (s_2^*) + \frac{dEC}{d_s} (s_2^*) + p s_2^* \right) \right), \quad (A-6)
\]

\[
= \left( \frac{\partial SC}{\partial s_1} (\bar{Q}) - \frac{\partial SC}{\partial s_1} (s_1^*) \right) + \left( \frac{\partial EC}{\partial s_1} (\bar{Q}) - \frac{\partial EC}{\partial s_1} (s_1^*) \right) > 0 \text{ by substituting the FOC} \]

\[
\frac{d \Delta_e}{dr} = \frac{1}{(1+r)} \left( \frac{\partial SC}{\partial s_2} (s_2^*) + \frac{\partial EC}{\partial s_2} (s_2^*) + p s_2^* \right) > 0, \quad (A-7)
\]

\[
\frac{d \Delta_e}{dp} = -\frac{\alpha - r}{1+r} (\bar{Q} - s_1^*) \begin{cases} > 0 & \text{if } \alpha > r \\ = 0 & \text{if } \alpha = r \\ < 0 & \text{if } \alpha < r \end{cases}, \quad (A-8)
\]

Differentiate \(\Delta q\) in equation (6-a) with respect to \(r\) and \(p\) yields

\[
\frac{d \Delta q}{dr} = -\frac{\alpha}{d s_1^*} \frac{d s_1^*}{dr} > 0 \left( \text{since } \frac{d s_1^*}{dr} < 0 \right), \quad \text{and} \quad (A-9)
\]

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\[
\frac{d\Delta q}{dp} = -\alpha_L \frac{ds_1}{dp} \begin{cases} 
> 0 & \text{if } \alpha_L < r \\
0 & \text{if } \alpha_L = r \\
< 0 & \text{if } \alpha_L > r.
\end{cases}
\]  
(A-10)

Differentiate equation (6-a) with respect to \( \bar{Q} \) and then substituting equation (A-2) and SOC yields the following inequality:

\[
\frac{d\Delta q}{d\bar{Q}} = -\alpha_L \left( 1 - \frac{d s_1}{d\bar{Q}} \right) = \frac{\alpha_L}{SOC} \left( \frac{d^2 SC(s_1)}{d s_1^2} + \frac{d^2 EC(s_1)}{d s_1^2} \right) > 0. \quad (A-11)
\]

Differentiating equation (6-a) with respect to \( \alpha_L \) results in the following equation:

\[
\frac{d\Delta q}{d\alpha_L} = -\alpha_L \frac{d s_1}{d\bar{Q}} + (\bar{Q} - s_1). \quad (A-12)
\]

The sign of equation (A-12) is undetermined given all our assumptions.

We summarize inequalities in (A-1)-(A-10), (A-7), (A-11) and (A-12) in Table 4.

**Table 4: Comparative Static Analysis for the two-period setting**

<table>
<thead>
<tr>
<th></th>
<th>Disease spread ((\alpha_L))</th>
<th>Initially infected and contact animals ((\bar{Q}))</th>
<th>Time value ((r))</th>
<th>Livestock price per unit ((p))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\alpha_L &gt; r)</td>
<td>(\alpha_L &lt; r)</td>
<td>(\alpha_L = r)</td>
<td>(\alpha_L = r)</td>
</tr>
<tr>
<td>Amount of slaughter and disposal in the first period ((s_1'))</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Amount of vaccinated animals in the first period ((v_1'))</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Difference in total slaughter and disposal ((\Delta_q))</td>
<td>+/-</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Value of vaccination ((\Delta_c))</td>
<td>-</td>
<td>+</td>
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**Appendix B: Proof of Proposition 2**

We make the following assumptions: (a) the cost of slaughter and disposal is given by \( SC = a + b s_i' + c \left( s_i' \right)^2 \) where \( s_i' \) is the amount of slaughter and disposal of animals at time \( t \) given the option \( i \); (b) the adverse environmental impact is excluded because of the difficulty of quantifying its value; (c) the value of livestock per head is \( p \); and (d) the
average vaccination cost per head is $vc$. Based on equation (11-a) and (11-b), and (15-a) and (15-c), we write out the following dynamics of the corresponding control and state variables under two options below:

\[
\begin{align*}
    i = \nu v \\
    \dot{s}_i^w &= (r - \alpha_{ul}) s_i^w + \frac{(r - \alpha_{ul})(b + p)}{2c} \\
    \dot{Q}_i^w &= \alpha_{ul} Q_i^w - (1 + \alpha_{ul}) s_i^w \\
    i = \nu v \\
    \dot{s}_i^c &= (r - \alpha_{ul}) s_i^c + \frac{(r - \alpha_{ul})(b + p) - VC(1 + r)}{2c} \\
    \dot{Q}_i^c &= \alpha_{ul} Q_i^c - (1 + \alpha_{ul}) s_i^c
\end{align*}
\]

(S-1)

Solving the dynamics yields the equilibrium $[s_i^w, Q_i^w]$ under option $i$:

\[
\begin{bmatrix}
    s_i^w \\
    Q_i^w
\end{bmatrix} = \begin{bmatrix}
    -\frac{b + p}{2c} \\
    -\left(1 + \frac{1}{\alpha_{ul}}\right)\frac{b + p}{2c}
\end{bmatrix}
\quad \text{and} \quad
\begin{bmatrix}
    s_i^c \\
    Q_i^c
\end{bmatrix} = \begin{bmatrix}
    -\frac{b + p + (1 + r)VC}{2c} \\
    -\left(1 + \frac{1}{\alpha_{ul}}\right)\frac{b + p - (1 + r)VC}{2c}
\end{bmatrix}.
\]

Defining $M_i^j = s_i^j - s_i^c$ and $L_i^j = Q_i^j - Q_i^c$, we can re-arrange equations (B-1) below:

\[
\begin{align*}
    \dot{M}_i^j &= (r - \alpha_{ul}) M_i^j, \\
    \dot{L}_i^j &= \alpha_{ul} Q_i^j - (1 + \alpha_{ul}) M_i^j,
\end{align*}
\]

(B-2a)

(B-2b)

where $i = H$ if $i = \nu v$ and $i = L$ if $i = \nu v$. Furthermore, equations (B-2a) and (B-2b) can be re-arranged below:

\[
\begin{bmatrix}
    \dot{M}_i^j \\
    \dot{L}_i^j
\end{bmatrix} = \begin{bmatrix}
    r - \alpha_{ul} & 0 \\
    -(1 + \alpha_{ul}) & \alpha_{ul}
\end{bmatrix} \begin{bmatrix}
    M_i^j \\
    L_i^j
\end{bmatrix} = U \begin{bmatrix}
    M_i^j \\
    L_i^j
\end{bmatrix}.
\]

(B-3)

Hence, the dynamics under two different options are given by the homogeneous equation (B-3). Their solutions are determined by eigenvalue $\lambda_i^1, \lambda_i^2$ and eigenvector $[v_1^i, v_2^i]$ of matrix $U$, where

\[
\lambda_i^1 = r - \alpha_{ul} \quad \text{and} \quad \lambda_i^2 = \alpha_{ul},
\]

(B-4a)
\[ V'_1 = \begin{bmatrix} \frac{1}{(1 + \alpha_j)/(2 \alpha_j - r)} \end{bmatrix} \quad \text{and} \quad V'_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \]  

(B-4b)

Figure 6 illustrates the solution under the option \( i \) that depends on the value of \( r \) and \( \alpha_j \). The vertical axis represents the total amount of infected and susceptibly infected animal \( Q'_i \), and the horizontal axis shows the total amount of slaughter and disposal of animals \( s'_i \) over time. The intercept of two dynamics \( \dot{Q}'_i = 0 \) and \( s'_i = 0 \) characterizes the equilibrium point \( E_i \), and \( V'_1(E) \) and \( V'_2(E) \) show the eigenvectors through the equilibrium. All L-shaped directional arrows suggest the trajectory of \( \dot{Q}'_i \) and \( s'_i \).
Figure 6: Phase Diagrams of the total infected and susceptible infected animals \( (Q_i) \) and the total amount of slaughter and disposal of animals \( (s_i) \) at time \( t \) under option \( i \). As shown in Figure 6, except the first case under which \( 0 < r < \alpha_j \), where the equilibrium \( E \) is a saddle point, all other cases have an unstable equilibrium. Furthermore, these five phase diagrams suggest the following findings:

(a) When \( 0 < r < 2\alpha_j \) (including plots 12(a) and 12(b)), the total infected and susceptible infected animals \( (Q_i) \) will reach zero in the first quadrant if

\[
\frac{s_i}{Q_i} \geq \frac{2\alpha_j - r}{1 + \alpha_j}
\]

is satisfied.

(b) When \( r \geq 2\alpha_j \), regardless the ratio between \( Q_i \) and \( s_i \), the total infected and susceptible infected animals \( (Q_i) \) can always reach zero in the first quadrant.

These two findings conclude Proposition 2.

Appendix C: Proof of Proposition 3

Assuming a constant average slaughter and disposal cost per head \( (b) \), the total cost is

\[
SC = (b + p) \sum_{i=1}^{r} \frac{s_i}{(1+r)^{i-1}} = (b + p) \sum_{i=1}^{r} \left( \frac{1 + \alpha_H}{1 + r} \right)^{i-1} q_i = (b + p) \sum_{i=1}^{r} \left( \frac{1 + \alpha_H}{1 + r} \right)^{i-1} q_i, \quad (C-1)
\]
Such that \( q_i = \frac{s_i}{(1+\alpha_H)^{t_i}} \) and \( \sum_{i=1}^{t} q_i = \bar{Q} \). The total cost if all initially infected and susceptibly infected animals are killed and disposed in the first period is

\[
SC_1 = (b + p)\bar{Q} = (b + p)\sum_{i=1}^{t} q_i.
\]  

(C-2)

Comparing equations (C-1) and (C-2), we know that \( SC_1 \leq SC \) if \( \alpha_H \geq r \). That is, it is a least cost option to slaughter all initially infected and susceptibly infected animals in the first period when \( \alpha_H \geq r \) given that vaccination is not allowed.

If vaccination is used as a supporting strategy, it is one of alternative choices that policy makers can kill all initially infected and contact animals in the first period. Thus, the total cost when vaccination is employed is at most as high as that under the no-vaccination option if \( \alpha_H \geq r \). Therefore, when \( \alpha_H \geq r \) is satisfied, vaccination can always decrease the total cost.