Economics of Homeland Security: Carcass Disposal and the Design of Animal Disease Defense

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Abstract

In an effort to bolster confidence and protect the nation the U.S. government through agencies like the Department of Homeland Security is identifying vulnerabilities and evolving strategies for protection. Agricultural food supply is one identified vulnerable area, and animal disease defense is one of the major concerns there under. Should a major outbreak of animal disease occur, it is likely to involve mass slaughter and disposal of animal carcasses. Current policy indicates that all subject animals are to be immediately slaughtered but this may overwhelm the capacity to cleanly and safely dispose of carcasses. We address the way that carcass disposal concerns may modify the choice of disease control strategy specifically evaluating vaccination as a time buying strategy with later slaughter of animals. Our results show that (a) vaccination does gain valuable time by slowing down the flow of slaughtered animals decreasing the total event cost; and (b) vaccination becomes more desirable the larger the outbreak, the faster spreading the disease, and/or the more effective or cheaper the vaccine.

1 Introduction

Carcass disposal can be a major concern in the face of an animal disease outbreak. Namely,

(a) The 1967/68 outbreak caused the slaughter of 434,000 animals, leading to a direct cost of £35 million borne by the Ministry of Agriculture, Fisheries and Food and an indirect cost of £150 million borne by the livestock industry (Doel and Pullen 1990).

(b) The 2001 outbreak resulted in the slaughter of 6.6 million animals (Scudamore et al. 2002) and a £3 billion cost to the UK government and a £5 billion cost to the private sector (NAO report 2002).

Should a major disease outbreak occur whether inadvertent or intentional, it is crucial to have an effective disease control and infected carcass disposal strategies. From an economic sense such strategies would be designed to minimize the costs arising from

(a) livestock losses; (b) economic impacts; (c) government costs; (d) public health hazards; and (e) environmental damages. Disposal of slaughtered animals is part of this strategy.

Disease management approaches vary across the world. Vaccination has been widely used in some Asian, Africa and South American countries to control endemic FMD disease (Doel and Pullen 1990). However, in "disease-free" countries in North and Central American including the US, the European Union, Australia and New Zealand, the basic disease control policy is slaughter of all infected and contact animals (Breeze 2004). In the case of a large outbreak this stamp-out policy mandates the slaughter of numerous animals, which induces a large carcass disposal issue i.e. how using the UK case as an example do you dispose of 6 million carcasses at a reasonable cost without damaging air, water, and land quality. The carcass disposal strategy is interactive with the disease management strategy and tradeoffs may occur between disease management costs and carcass disposal costs. In turn such considerations may alter the optimal disease control management system. This constitutes the economic issue addressed in this paper, namely, we investigate the way that the carcass disposal issue influences the design of the disease management system.

2 Background - disease management and carcass disposal

There are various technologies that may be employed to dispose of contaminated animal carcasses, including burial, incineration, composting, rendering, lactic acid fermentation, alkaline hydrolysis, and anaerobic digestion (NABSCC 2004). These alternatives embody some pre-outbreak activities. Namely, disposal facilities can be constructed and located before an outbreak occurs. However, such facilities can be expensive and typically have limited capacity. Extensive pre outbreak actions may be difficult to justify given the infrequency of major outbreaks.

Carcass disposal demands can be manipulated by altering the disease control management strategy employed. Strategies that reduce the rate of slaughter reduce the needed rate of carcass disposal, the immediate severity of the carcass disposal problem and the needed facilities to handle disposal. Vaccination of potentially infected and contact animals is such a strategy. Even though the emergency plan in some disease free countries such as the United Kingdom regards vaccination as a supporting strategy, vaccination is not considered as a main option because of the following disadvantages: (a) vaccinated animals traditionally could not be distinguished from infected animals and thus needed be slaughtered for disease control and to maintain disease free status. However, Breeze (2004) argues this is no longer the case as a recent developed and commercialized test can distinguish FMD vaccinated animals from infected animals; and (b) Some vaccinated animals may be already infected or could still catch the disease, which reduces disease management effectiveness relative to immediate slaughter (for further discussion see Doel, Williams, Barnett 1993, Elbakidze 2004, and APHIS 2002).

Carcass disposal during a large outbreak can generate a tremendous operational concern and cost source. For example, in the 2001 UK outbreak large scale incineration was undertaken and media coverage led to substantial tourism losses (NAO report 2002). Vaccination in conjunction with later slaughter can buy time and lighten the disposal requirement but poses tradeoffs between disease control and carcass disposal. Consider the following simplified problem statement: suppose we can dispose all carcasses within a day at an extremely high cost, or within a couple days at a much lower cost. Disease management policy should consider whether it is better to have a mechanism to delay slaughter/disposal to achieve the lower disposal cost while somewhat less effectively controlling disease spread.

This setting leads naturally to the following questions: Is it technically feasible and economically effective to slaughter all infected and contact animals within proximity of the outbreak? If not, what other choices are there? Here, we examine vaccination as a supporting strategy to buy time and reduce the immediate carcass disposal load. Initially we develop a two-period model to examine this question then later a multiple-period model. In this model we minimize total cost by choosing the optimal amount of animals to be slaughtered or vaccinated by period, given

- whether or not vaccination is employed.
- the cost and capacity of carcass disposal,
- the cost of slaughter and vaccination,
- the initial event size, i.e., the number of initially infected and contact animals,
- the disease spread caused by vaccinated and non-vaccinated animals, and

- the assumption that animals must eventually be killed whether vaccinated or not.

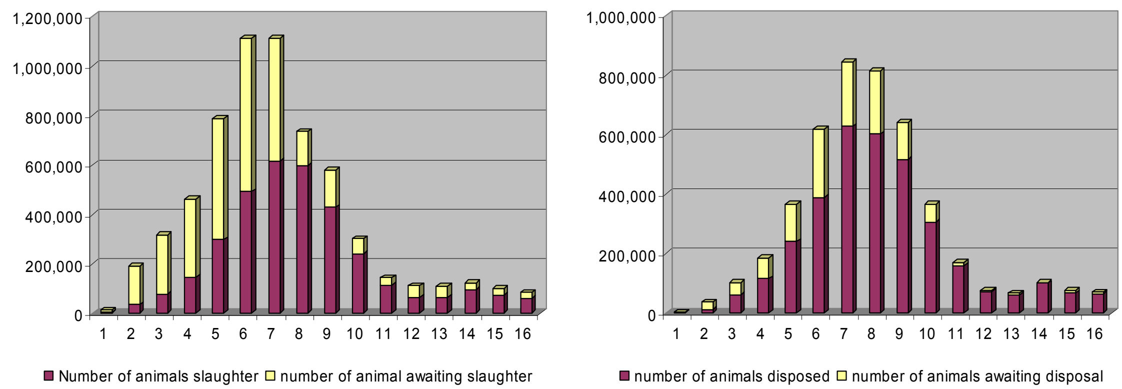
We also conduct sensitivity analysis under the following scenarios: (a) vaccinated animals are eventually killed. However, some of these uncontaminated animal carcasses are saved and used for alternative purposes and, hence, they have some salvage value; and (b) some vaccinated animals are healthy, and they are not killed.

3 Model

Vaccination is not a recommended practice in the "disease-free" countries including the UK and US. Written disease management policies therein employ strict movement controls along with slaughter of all infected and contact animals. For example, if FMD were found in the US, all animals in a radius of up to 3 kilometers around the infected farm, including the affected herd, cattle, sheep, goats, swine, and susceptible wildlife, whether they are infected or not, would be killed and their carcasses disposed (Breeze, 2004). However, outbreak events can create quite a carcass disposal burden. Consider the following drawn from the UK experience under the 2001 FMD outbreak:

(a) The 2001 FMD outbreak caused the slaughter of 6.6 million animals (Scudamore et al. 2002) and a mass backlog of slaughter and disposal. Figure 1 shows the weekly amount of slaughter and carcass disposal over the course of the FMD outbreak. More than 400,000 animals were awaiting slaughter (see the left panel) and more than 200,000 animals were awaiting disposal (see the right panel) between the 6th and 9th week. Collectively, "At the height of the outbreak the daily weight of carcass moved was over half the weight of the ammunition the armed services supplies during the entire Gulf War" (NAO report 2002, page 50). This mass backlog suggests a potential value of vaccination as a supporting strategy.

(b) The mass slaughter and disposal largely through involving incineration was the subject of extensive media coverage, and in turn a large reduction in tourism. The 2001 FMD outbreak resulted in an estimated lost tourism cost of £4.5 to £5.4; and £2.7 to £3.2 billion to business directly affected by tourist and leisure (NAO report 2002). Vaccination might have reduced the spectacular nature of the event and, thus may reduce the tourism damage.



(a) Number of animals slaughtered and awaiting for slaughter of

(b) Number of animals disposed and awaiting for disposal of

Figure 1: Weekly slaughter and carcass disposal during the 2001 UK FMD outbreak
Data Source: Scudamore et al (2002)

On the other hand, adopting vaccination with a later slaughter of animals will not spread the disease significantly due to the following reasons: (a) vaccinated animals are much less contagious than unvaccinated animals; and (b) The 2001 UK FMD outbreak revealed that a significant percentage of animals slaughtered and disposed are not infected at all, and only approximately 1% of animals are infected. Thus, it is likely to be feasible to vaccinate animals while bring the disease under control.

4 Modeling including vaccination

Depiction of the decisions involved with disease control, carcass disposal and vaccination requires that one develop a model that is inclusive of (a) disease spread rates caused by vaccinated and non-vaccinated animals, (b) the scale of the disease outbreak, (c) the relationship between environmental damage and slaughter and disposal volume, (d) the costs and capacity for slaughter and disposal, and (e) the magnitude of vaccination costs. In particular, choosing the number of animals to be slaughtered and vaccinated if vaccination is adopted in each period, policymakers minimize the total social cost including disease management costs, the market value of slaughtered animals, vaccination costs, carcass disposal cost, and environmental damages

To model such a decision we use both a two-period and a multi-period version. In setting up the model we make the following assumptions:

- The initial total number of infected and contact animals to be slaughtered is $\overline{Q}$.

- The value in terms of lost market revenue from each slaughtered/disposed animal is *p*. For simplicity we assume *p* is constant though outbreak presence even though the outbreak scale will affect market values of existing and remaining livestock.
- Welfare slaughter is not required, i.e., that there is sufficient feed and capacity to store the vaccinated animals.
- The literature suggests two models of FMD disease spread: exponential form (Anderson and May 1991) and Reed-Frost form (Thrushfield 1995, Carpenter et al. 2004). To capture the spatial patterns of FMD disease spread, some researchers, including Bates et al. (2001) and Schoenbaum and Disney (2003), distinguish disease contact and spread into three categories: (1) direct contact caused by movement of animals and other direct contact of animals within a herd and among herds; (2) indirect contact caused by movements of vehicles and people within a herd and among herds; and (3) airborne of contagious FMD virus. We assume that the total infected and contact animals in the next period (Q_{t+1}) consists of two components: (a) the remaining infected and contact animals from the previous period $(Q_t - s_t)$ where s_t represents the amount of slaughter/disposal at time *t*, and (b) the newly infected and contact animals resulting from the disease spread $\alpha(Q_t - s_t)$:

  $$Q_{t+1} = (1+\alpha)(Q_t - s_t), \tag{1}$$

  Here the value of α varies with and without vaccination because vaccinated animals are much less contagious (Breeze 2004),

  $$\alpha = \begin{cases} \alpha_H & \text{if vaccination is not employed} \\ \alpha_L & \text{if vaccination is employed} \end{cases}, \tag{2}$$

  where $\alpha_H > \alpha_L$.

To gain insight into the role of vaccination, we elaborate both the two-period and multiple-period settings to solve the slaughter and carcass disposal problem dynamically. That is, policymakers have to make the following two decisions: (a) whether to employ vaccination as a supporting disease control and carcass management strategy; and (b) how many animals to be slaughtered/disposed and vaccinated in each period over the course of an FMD outbreak.

4.1 Two-period model

In the two-period model, we assume that policymakers have two options: (a) a slaughter of all the infected and contact animals within proximity of infected animals in the first period; and (b) vaccination of some number of the contact animals in the first period to lessen the operational pressure and reduce carcass disposal cost, and slaughter of all remaining infected, vaccinated, and contact animals along with disposal in the second period. The cost minimization problem with vaccination is

$$\min_{s_1, v_1, s_2} \underbrace{\left[SC(s_1) + VC(v_1) + EC(s_1) + p\, s_1\right]}_{\text{Total cost in the first period}} + \frac{1}{1+r} \underbrace{\left[SC(s_2) + EC(s_2) + p\, s_2\right]}_{\text{Total cost in the second priod}}, \tag{3}$$

where $SC(s)$ denotes the slaughter and carcass disposal cost, $VC(v)$ is the total cost of vaccinating v units of animals, and $EC(s)$ captures the environmental damages resulting from the slaughter and disposal of animals. r denotes the time value of money, which is a proxy for the value of delaying the slaughter and disposal of a head animal to the next period. We assume all the three cost functions are increasing and convex. That is, we assume that $\frac{dSC(s)}{ds} > 0$ and $\frac{d^2 SC(s)}{d s^2} \geq 0$[1], $\frac{dVC(v)}{dv} > 0$ and $\frac{d^2 VC(v)}{d v^2} \geq 0$, and $\frac{dEC(s)}{ds} > 0$ and $\frac{d^2 EC(s)}{d s^2} \geq 0$.

Restrictions on the model allocates all initially infected and contact animals within proximity of the infected animals for slaughter or vaccination in period 1 (equation 4-a):

$$s_1 + v_1 = \overline{Q} \tag{4-a}$$

[1] The number of animals disposed of in each period is assumed to be at most equal to carcass disposal capacity. Hence, the disposal capacity is not a binding resource.

Furthermore, in period 2 all vaccinated animals carried over from the first period along with any newly infected/contact animals due to disease spread will be slaughtered and disposed of (equation 4-b):

$$s_2 = (1+\alpha_L)v_1 \quad . \tag{4-b}$$

Minimizing cost as given in equation (3) subject to equations (4-a) and (4-b) yields the optimality condition below:

$$\frac{dSC(s_1)}{d\,s_1} + \frac{dEC(s_1)}{d\,s_1} + p = \frac{dVC(v_1)}{dv_1} + \frac{1+\alpha_L}{1+r}\left(\frac{dSC(s_2)}{d\,s_2} + \frac{dEC(s_2)}{d\,s_2} + p\right). \tag{5}$$

The three terms on the left-hand side of equation (5) represent the gain from postponing the slaughter of an additional animal to the next period, including the marginal slaughter/disposal cost, the marginal environmental damage, and the livestock value per head in the first period. The four terms on the right-hand side give the present value of the loss resulting from vaccinating one additional animal, including the marginal vaccination cost in the first period, and the marginal slaughter/disposal cost, the marginal environmental damage, and the livestock value loss in the second period. The optimal number of slaughtered animals and disposed carcasses in the first period (s_1^*) is developed when the present value of the potential gain of vaccinating one additional animal and then slaughtering and disposing of it in period 2 equals the potential loss of doing so. The optimal number of animals vaccinated in the first period is $v_1^* = \overline{Q} - s_1^*$.

When vaccination is not an option all subject animals $\overline{Q}$ have to be killed and disposed of in the first period. Thus, total slaughter and disposal is $\overline{Q}$ incurring a cost of $SC(\overline{Q}) + EC(\overline{Q}) + p\overline{Q}$. When vaccination is an option, in the first period some animals are slaughtered and the remaining ones vaccinated, and then all vaccinated animals plus any secondarily infected are slaughtered and disposed of in period 2. Therefore, the total slaughter and disposal is $s_1^* + s_2^*$ and the corresponding total cost is the net present value of slaughter and disposal, vaccination, and environmental costs in the two periods. Let Δ_q denote the difference in the total number of animals slaughtered and disposed of. Similarly let Δ_c be the difference in total cost between these two options or the cost reduction should vaccination be used. Algebraically, Δ_q and Δ_c are:

$$\Delta_q = (s_1^* + s_2^*) - \overline{Q} = [s_1^* + (1+\alpha_L)(\overline{Q} - s_1^*)] - \overline{Q} = \alpha(\overline{Q} - s_1^*), \tag{6-a}$$

$$\Delta_c = \underbrace{[SC(\overline{Q}) + EC(\overline{Q}) + \overline{Q}p]}_{\text{Total cost without vaccination}} - \left(\underbrace{[SC(s_1^*) + VC(v_1^*) + EC(s_1^*) + s_1^* p]}_{\text{Total cost in the first period}} + \frac{1}{1+r} \underbrace{[SC(s_2^*) + EC(s_2^*) + s_2^* p]}_{\text{Total cost in the second priod}} \right). \tag{6-b}$$

The magnitudes of these Δ measures depend upon the rate of disease spread, the time value of money, the initial size of FMD outbreak, the livestock value, etc.

Proposition 1: In the two-period setting,

- the amount of vaccinated animals becomes smaller (conversely the first period slaughter becomes larger) when we have either: (a) an increase in the disease spread rate among vaccinated animals, and/or (b) decreases in the time value of money $\left(\frac{d v_1^*}{d\alpha_L} < 0, \frac{d v_1^*}{dr} > 0, \text{and} \frac{d s_1^*}{d\alpha_L} > 0, \frac{d s_1^*}{dr} < 0 \right)$. An increase in the initial size of disease outbreak will result in a larger amount of slaughter and vaccinations in the first period $\left(\frac{d v_1^*}{d\overline{Q}} > 0 \text{ and } \frac{d s_1^*}{d\overline{Q}} > 0 \right)$.
- the total number of animals slaughtered and subsequently disposed of increases with an increase in (a) event size and/or (b) time value of money $\left(\frac{d \Delta_q}{d\overline{Q}} > 0 \text{ and } \frac{d \Delta_q}{dr} > 0 \right)$.
- the value of vaccination is greater when we have (a) a decrease in the disease spread rate caused by vaccinated animals; (b) an increase in the event size; and/or (c) an increase in the time value of money $\left(\frac{d \Delta_c}{d\alpha_L} < 0, \frac{d \Delta_c}{d\overline{Q}} > 0 \text{ and } \frac{d \Delta_c}{dr} > 0 \right)$.
- when the disease spread rate from vaccinated animals exceeds the time value of money ($\alpha_L > r$), an increase in the value of livestock induces more slaughter and less vaccination in the first period, a smaller amount of total slaughter/disposal of animals, and a lower cost saving from vaccination.

$\left(\frac{d\, s_1^*}{dp} > 0,\ \frac{d\, v_1^*}{dp} < 0,\ \frac{d\,\Delta_q}{dp} < 0 \text{ and } \frac{d\,\Delta_c}{dp} < 0\right)$. Otherwise, when $\alpha_L < r$, $\frac{d\, s_1^*}{dp} < 0$, $\frac{d\, v_1^*}{dp} > 0$, $\frac{d\,\Delta_q}{dp} > 0$ and $\frac{d\,\Delta_c}{dp} > 0$.

Proof: See Appendix A.

Proposition 1 suggests the following results:

- Vaccination is always more valuable when the spread rate from vaccinated animals becomes slower. However, whether an increase in the disease spread rate caused by vaccinated animals leads to a larger volume of animals slaughtered depending on the following tradeoff: an increase in the disease spread rate from vaccinated animals causes a greater number of slaughter. However, the disease control authority may slaughter and dispose of a greater number of animals in the first period, which reduces total slaughter and disposal.

- When the event size is greater in terms of the number of initially infected and contact animals, both the number of slaughter/disposal and the corresponding total cost will increase under both options. However, employing vaccination decreases cost more the larger the event size.

- The higher the discount rate, the more valuable vaccination becomes even though the total slaughter and disposal increases. Vaccination buys time and permits lower cost of carcass disposal. Because of the environmental regulations and public health concerns, on-farm burial was generally not used in the 2001 UK FMD outbreak. Instead, seven mass burial pits were built at a construction cost of £79 million; and the cost of restoration and management in the future were estimated at £35 million (NAO report 2002). It is likely that disposal capacity costs would have fallen if time pressure could have been reduced.

- Suppose the initial equilibrium is achieved when equation (5) is satisfied. A one-unit increase of the livestock value increases the gain by p (see the left-hand side of equation (5)) and the loss by $\frac{1+\alpha_L}{1+r}p$ of postponing the slaughter of one additional animal to the next period. Therefore, the net livestock loss increases by

$\frac{1+\alpha_L}{1+r}p - p = \frac{\alpha_L - r}{1+r}p$. Since the daily compound interest rate is generally smaller than disease spread rate, an increase of livestock value will cause more animals slaughtered and disposed in the first period.

4.2 Multiple-period setting

Should a major disease outbreak occur it is unlikely to stamp out the disease within a day or a week (recent simulations show events lasting 80-100 days). The multiple-period model version deals with such longer time periods.

In the multiple-period setting the policy employed (denoted as *i*) can potentially use vaccination (*i=v)* or not (*i=nv*). Let Q_t^i denote the total infected and contact animals in period *t*, and s_t^i be the number of animals slaughtered and disposed of at time *t* given policy *i*. $Q_t^{nv} - s_t^{nv}$ represents the total amount of infected and contact animals carried on to the next period, and $Q_t^v - s_t^v$ is the total vaccinated animals at time *t* assuming we have capacity to vaccinate all animals. The change in the number of infected and contact animals is

$$\dot{Q}_t^i = -s_t^i + \alpha(Q_t^i - s_t^i), \tag{7}$$

where *α* is the rate of disease spread as above in equation (2). Equation (7) decomposes the change in the total number of infected and contact animals into two components: (a) a deduction accounting for current slaughter s_t^i and (b) an increase resulting from the disease spread $\alpha(Q_t^i - s_t^i)$. The authority aims to minimize total economic cost. We assume that the outbreak will be over by the terminal time period *T*.

4.2.1 Policy Option 1 -- vaccination is not allowed

The first policy option assumes that vaccination is not used. Given that the disease has to be stamped out by the time period *T*, the authority decides the optimal slaughter and disposal of animals in each period. The cost minimization problem then becomes

$$\min_{\{s_t\}_{t=1}^{T}} \int_{t=0}^{T} e^{-rt}\left[SC\left(s_t^{nv}\right) + EC\left(s_t^{nv}\right) + p\, s_t^{nv}\right] dt \tag{8-a}$$

$$\text{s.t. } \dot{Q}_t^{nv} = -s_t^{nv} + \alpha_H (Q_t^{nv} - s_t^{nv}). \tag{8-b}$$

The resultant Hamiltonian is

$$H = \left[SC\left(s_t^{nv}\right) + EC\left(s_t^{nv}\right) + p\, s_t^{nv}\right] + \lambda\left(-s_t^{nv} + \alpha_H (Q_t^{nv} - s_t^{nv})\right), \tag{9}$$

where λ is the co-state variable associated with $\dot{Q}_t^{nv}$. The marginal impact of animal slaughter on the Hamiltonian is

$$\partial H / \partial s_t^{nv} = \left(SC' + EC' + p\right) - (1 + \alpha_H)\lambda . \tag{10-a}$$

If the expression is positive, the authority should slaughter all the animals and vaccination is not an optimal choice. If it is negative, it is optimal to maintain the disease endemic. The internal solution is pursued when equation (10) equals zero, i.e., the marginal cost $\left(SC' + EC' + p\right)$ equals the gain $(1 + \alpha_H)\lambda$ from a decreases in Q_t^{nv} because of the slaughter. Additional conditions for the internal solution are given below:,

$$\partial H / \partial Q_t^{nv} = \lambda \alpha_H = r\lambda - \dot{\lambda}, \tag{10-b}$$

$$\partial H / \partial \lambda = \alpha_H Q_t^{nv} - (1 + \alpha_H) s_t^{nv} = \dot{Q}_t^{nv}. \tag{10-c}$$

Based on equations (10-a), (10-b), and (10-c), we can derive the optimal dynamic solution for the number of slaughtered and the costate variable λ at time t:

$$\dot{s}_t^{nv} = \frac{(r - \alpha_H)(SC' + EC' + p)}{SC'' + EC''}, \tag{11-a}$$

$$\dot{\lambda}_t^{nv} = \frac{r - \alpha_H}{1 + \alpha_H}(SC' + EC' + p). \tag{11-b}$$

General speaking, the disease spread rate exceed the daily interest rate. Therefore, Equation (11-a) implies that the number of slaughtered animals decreases over time and, thus, more animals will be slaughtered in the early periods. Equation (11-b) reflects the inter-temporal change of the optimal marginal impact of Q_t^{nv}, i.e., the marginal impact of Q_t^{nv} on the total event cost generally decreases as *t* increases.

4.2.2 Policy Option II -- vaccination is employed as a supporting strategy

When vaccination is used in conjunction with a later slaughter of all vaccinated animals, the net present value of the total event cost flow is minimized by choosing the optimal number of animals to be slaughtered s_t^v and to be vaccinated v_t^v at each time period *t*, where $s_t^v + v_t^v = Q_t^v$. The cost minimization problem is given below:

$$\min_{\{s_t\}_{t=1}^T} \int_{t=0}^{T} e^{-rt} \left[SC(s_t^v) + VC(Q_t^v - v_t^v) + EC(s_t^v) + p\, s_t^v \right] dt\,, \tag{12-a}$$

$$\text{s.t. } \dot{Q}_t^v = -s_t^v + \alpha_L (Q_t^v - s_t^v)\,. \tag{12-b}$$

The Hamiltonian equation in this case is:

$$H = \left[SC(s_t^v) + VC(Q_t^v - s_t^v) + EC(s_t^v) + p\, s_t^v \right] + \lambda\left(- s_t^v + \alpha_L (Q_t^v - s_t^v)\right). \tag{13}$$

The first order necessary conditions for an internal solution are

$$\partial H / s_t^v = \left(SC' - VC' + EC' + p \right) - (1 + \alpha_L)\lambda = 0\,, \tag{14-a}$$

$$\partial H / Q_t^v = VC' + \lambda \alpha_L = -\dot{\lambda} + r\lambda\,, \tag{14-b}$$

$$\partial H / \partial \lambda = \alpha\, Q_t^v - (1 + \alpha_L)\, s_t^v = \dot{Q}_t^v\,. \tag{14-c}$$

Similar as equation (10-a), Equation (14-a) shows that the marginal impact of slaughter on the Hamiltonian, and it equals to zero when the internal solution is achieved. Based on equations (14-a), (14-b), and (14-c) we can derive the dynamics of two control variables (s_t^v and v_t^v) and one costate variable associated with Q_t^v:

$$\dot{s}_t^v = \frac{VC''\dot{Q}_t^v - (1 + r)VC' + (r - \alpha_L)(SC' + EC' + p)}{SC'' + VC'' + EC''}\,, \tag{15-a}$$

$$\dot{v}_t^v = \frac{(SC'' + EC'')\,\dot{Q}_t^v + (1 + r)VC' - (r - \alpha_L)(SC' + EC' + p)}{SC'' + VC'' + EC''}\,, \tag{15-b}$$

$$\lambda_t^v = -\frac{\alpha_L - r}{1 + \alpha_L}(SC' - VC' + EC' + p) - VC'\,, \tag{15-c}$$

Equation (15-a) indicates, when the amount of slaughter and disposal is dynamically stable that the discounted marginal gain of postponing slaughter and disposal of one animal is $\left(\dfrac{VC''\dot{Q}_t^v + (r-\alpha)(SC'+EC'+p)}{SC''+VC''+EC''} \right)$ and equals the discounted marginal cost of vaccination $\left(\dfrac{(1+r)VC'}{SC''+VC''+EC''} \right)$.

4.2.3 Comparison between two options in the multiple-period setting

To compare these two options and quantify the value of vaccination, we make following additional assumptions on the cost terms: (a) vaccination is done at a constant variable cost and zero fixed cost, i.e. $VC^{'} = vc$ and $VC^{''} = 0$; (b) there is a zero environmental cost from carcass disposal (this will lead to an underestimate of the value of vaccination); and (c) slaughter cost is quadratic -- $S(q_{st}) = a + b\,s_t + c\,s_t^2$ where both *b* and *c* are positive. Given these assumptions, we analyze the dynamics under two policies:

- When vaccination is not used, the dynamics of the control variable in equation (10-a) can be re-written below: $\dot{s}_t^{nv} = (r-\alpha_H)\,s_t^{nv} + \dfrac{(r-\alpha_H)(b+p)}{2c}$. The intercepts of the two dynamics $\dot{s}_t^{nv} = 0$ and $\dot{Q}_t^{nv} = 0$ characterizes the equilibrium and eigenvectors, $v_1^{nv}(E)$ and $v_2^{v}(E)$, where $V_1^{nv} = \begin{bmatrix} 1 \\ (1+\alpha_H)/(2\alpha_H - r) \end{bmatrix}$ and $V_2^{nv} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.
- Similarly, when vaccination is used, the dynamics of the control variable in equation (15-a) can be rewritten as $\dot{s}_t^{v} = (r-\alpha_L)\,s_t^{v} + \dfrac{(r-\alpha_L)(b+p) - VC(1+r)}{2c}$. The intercepts of the two dynamics $\dot{s}_t^{v} = 0$ and $\dot{Q}_t^{v} = 0$ characterizes the equilibrium and eigenvectors, $v_1^{v}(E)$ and $v_2^{v}(E)$, where $V_1^{v} = \begin{bmatrix} 1 \\ (1+\alpha_L)/(2\alpha_L - r) \end{bmatrix}$ and $V_2^{v} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

The detailed analysis of dynamics is given in Appendix B. Since the daily interest rate is generally lower than the disease spread rate, we are able to identify the appropriate phase diagram in Figure 2. The vertical and horizontal axis show the total amount of infected and contact animals (Q_t^i) and the total amount of slaughter/disposal of animals (s_t^i), respectively. All L-shaped directional arrows suggest the trajectory of $\dot{s}_t^i$ and $\dot{Q}_t^i$.

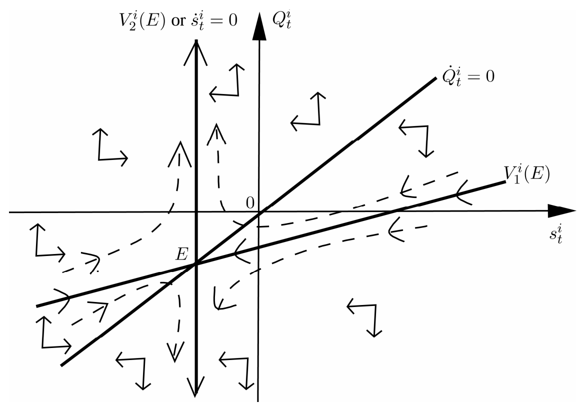


Figure 2: Phase diagram of the total infected and contact animals (Q_t^i) and the total amount of slaughter/disposal of animals (s_t^i) under policy option *i* when $0 < r < \alpha_L < \alpha_H$

Proposition 2: In the multiple-period setting, when the time value of money is less than the rate of disease spread rate ($0 < r < \alpha$), the authority can stamp out the disease if at least $\frac{2\alpha - r}{1+\alpha}$ of the currently infected and contact animals are slaughtered and disposed in each period.

Proof: See Appendix B.

Proposition 2 implies the sufficient condition to stamp out the diseases: the disease control authority should at least slaughter and dispose of $\frac{100(2\alpha_H - r)}{1+\alpha_H}$ percent of the currently infected and contact animals when vaccination is not allowed, and $\frac{100(2\alpha_L - r)}{1+\alpha_L}$ percent when vaccination is allowed. Thus, a higher percentage of the currently infected and contact animals are killed and disposed of at each period when vaccination is not allowed since $\frac{2\alpha_L - r}{1+\alpha_L} < \frac{2\alpha_H - r}{1+\alpha_H}$. This finding agrees with out intuition: There are two ways to control the disease spread, either kill infected and contact animals or vaccinate them. When vaccination is not allowed, the authority can only rely on slaughter/disposal of animals to bring the disease outbreak under control and, thus, a higher proportion of animals will be killed. Furthermore, we are aware of the following: (a) Given the percentage, if the required number of animals need to be slaughtered/disposed exceeds the operational capacity, the disease will not be brought under control unless new slaughter/disposal capacity is established; and (b) Even thought the no-vaccination option causes a higher proportion of animals to be killed in each period, it does not implies this option results in a larger volume of slaughter and disposal of animals.

As a supporting strategy in conjunction with a later slaughter of animals in a total disease control management, our analytical results that vaccination *slows the flow of carcasses for disposal and consequent cost while still controlling disease spread.* Many animals killed and disposed are likely not infected at all. In the 2001 FMD outbreak in the United Kingdom, less than 1% disposed animals were known to be infected (NAO report, Scudamore et al. 2002). Vaccination of these animals is feasible even given the concerns of disease spread. On the other hand, slowing down the slaughter and disposal operation lessens the pressure on the current existing capacity and likely reduces both facility construction and carcass disposal cost as well as environmental and other spill-over effects.

5 Simulation Results

The analytical results show the value and extent of vaccination depends on the time value of money; costs of slaughter, disposal and vaccination; rate of disease spread from vaccinated and non-vaccinated animals, and the number of initially infected/contact animals. To study empirical magnitudes we performed numerical simulations. We choose the following value of parameters for the benchmark case:

(a) *The size of initial event, time horizon, and time value of money*: We assume that the disease outbreak with the initial number of initially infected and contact animals $\overline{Q} = 100$ has to be stamped out within ten weeks. The time value of money that is measured by the weekly compounded interest rate is r=4%.

(b) Slaughter and disposal cost: The average slaughter cost per head is estimated at \$130 per head (Lambert 2002). The NABCC report provides a range of disposal cost per ton cross a number of disposal technologies (page 22 in Chapter 9). We converted the cost per ton into cost per head with the results given in Table 2, and its median value ($63) is used for the benchmark case. We also consider alternative technologies and use the midrange cost for each technology in simulations. It is reasonable that slaughter/disposal cost rises at an increasing rate with event size. Thus, we examine cases with a quadratic cost function that exhibited such characteristics. However, the only cost figures we found in the literature are constant per head. We assume that the slaughter/disposal cost function is

$$SC_t = (b + p)\, s_t + c\, s_t^2 = (\$130 + \$63)\, s_t + s_t^2 / 2\,. \tag{17}$$

Table 1: Disposal cost under different disposal technology

Carcass Disposal technology	Cost range per ton [1]	Cost range per head	Median cost per head
Burial	$15-200	$4-50	$27
Landfill	$10-500	$2.5-125	$64
open burning	$200-725	$50-181	$116
fixed-facility incineration	$35-2000	$9-500	$255
air-curtain incineration	$140-510	$35-128	$82
bin- and in-vessel composting	$6-230	$1.5-58	$30
window composting	$10-105	$2.5-26	$14
rendering	$40-460	$10-115	$63
fermentation	$65-650	$16-163	$90
anaerobic digestion	$25-125	$6-31	$19
alkaline hydrolysis	$40-320	$10-80	$45

Source: Carcass Disposal Review, page 22, Chapter 9.
Note: The current cost range may differ substantially because there is a dramatic increase in fuel price after this report was done, especially for thermal destruction including open burning, fixed-facility incineration, and air-curtain incineration.

(c) *Vaccination cost*: Vaccination costs consist of vaccines and injection costs. Breezes argued that the cost of vaccines is $1.20 per head when using the current 15 FMD virus types; and Schoenbaum and Disney (2003) estimated that the veterinary service per head costs $6. We use the average vaccination cost $6 per head in the benchmark case. However, we do aware that this cost number ignores certain cost component, let along transportation costs of shipping vaccines to the designated locations. Therefore, we employ various cost estimates in the sensitive analysis.

(d) *Environmental cost*: Due to limited information on and knowledge of environmental damage the environmental cost resulting from mass slaughter and carcass disposal was set to zero. Hence, the value of vaccination as a supporting strategy could be underestimated.

(e) *Value of livestock loss per head*: Based on the USDA-NASS 2004 Statistics of Cattle, Hogs, and Sheep (USDA), we use *p*=$819 to quantify the average swine livestock value per head.

(f) *Disease spread rates*: We did not find concrete number of disease spread rates in the literature. It could be spread rates vary case by case depending on the situation then. The NAO report documents the weekly newly confirmed infected premises. Given these 32 weekly data, we are able to calculate the average daily spread rate among premises that is roughly 5%. Since various disease management and control mechanisms were undertaken along this 32-week period, we assume that disease spread rate among vaccinated animals is 5% instead ($\alpha_L = 5\%$). We assume that disease spread rate caused by vaccinated animals are $\alpha_H = 10\%$. The assumptions of disease spread rate reflect that vaccinated animals shed less and, thus less contagious than non-vaccinated animals.

Under the benchmark case (see Simulations 1d, 2b, 3b, 4c, and 5a in Table 2), the adoption of vaccination during the course of disease outbreak caused 1.91% more of slaughter and disposal of animals (the total amount of slaughter and disposal of animals is 209 when vaccination is not used versus 213 when vaccination is used), a longer disease prevalence (four weeks without vaccination versus six weeks with vaccination), and a reduced total event cost by 2.80% ($217,900 versus $211,800). Therefore, it is better off for the authority to use vaccination in the benchmark case.

Table 2: Simulation results on various cases

(Parameters for the benchmark case: weekly interest rate: $r = 0.04$; slaughter cost per head=$130; disposal cost per head=$63.5; vaccination cost per head=$6; livestock value per head=$819; initial event size=100; disease spread rate caused by vaccinated animals=0.05; and disease spread rate caused by non-vaccinated animals=0.10. Only one parameter varies in the each set of simulations.)

Simu. No.	Changing parameter	Policy I: w/o vaccination			Policy II: w/ vacciation			Comparison of quantity & cost	
		Number of animal killed (heads)	Total Cost (1000$)	Disease prevelance (weeks)	Number of animal killed (heads)	Total Cost (1000$)	Disease prevelance (Weeks)	RQ (%)	RC (%)
Interest rate									
1a	0.01	206	219.9	2	205	216.6	3	-0.49	1.53
1b	0.02	206	219.3	2	206	215.3	4	0.00	1.82
1c	0.03	207	218.6	3	207	213.9	4	0.00	2.18
1d	0.04	209	217.9	4	209	211.8	6	0.00	2.80
1e	0.05	211	217.0	5	214	208.6	8	1.42	3.86
Disease spread rate caused by vaccinated animals									
2a	0.01	209	217.9	4	213	182.0	8	1.91	16.49
2b	0.03	209	217.9	4	226	202.3	8	8.13	7.14
2c	0.05	209	217.9	4	213	211.8	6	1.91	2.80
2d	0.07	209	217.9	4	209	215.3	4	0.00	1.19
2e	0.09	209	217.9	4	208	217.5	3	-0.48	0.19
Disease spread rate caused by non-vaccinated animals									
3a	0.05	217	210.1	8	213	211.8	6	-1.84	-0.81
3b	0.10	209	217.9	4	213	211.8	6	1.91	2.80
3c	0.15	207	220.7	2	213	211.8	6	2.90	4.05
3d	0.20	205	222.3	2	213	211.8	6	3.90	4.72
3e	0.25	200	222.4	1	213	211.8	6	6.50	4.77
Initial event size									
4a	200	209	217.9	4	213	211.8	6	1.91	2.80
4b	400	430	453.9	4	439	432.9	8	2.09	4.63
4c	600	659	703.0	5	673	660.8	8	2.12	6.00
4d	800	896	963.4	6	910	894.7	8	1.56	7.13
4e	1000	1141	1234.0	8	1144	1134.7	8	0.26	8.05
Constant average vaccination cost									
5a	1.5	209	217.9	4	216	210.6	6	3.35	3.36
5b	6.0	209	217.9	4	213	211.8	6	1.91	2.80
5c	10.0	209	217.9	4	211	212.7	5	0.96	2.40
5d	15.0	209	217.9	4	208	214.4	5	-0.48	1.62
5e	20.0	209	217.9	4	206	215.6	3	-1.44	1.03
Constant average disposal cost relating to different disposal technology									
6a	burial ($14)	209	207.8	4	213	201.9	6	1.91	2.88
6b	landfill ($19)	209	208.9	4	213	202.9	6	1.91	2.87
6c	open burning ($27)	209	210.5	4	213	204.5	6	1.91	2.86
6d	fixed-facility incineration ($30)	209	211.1	4	213	205.1	6	1.91	2.85
6e	air-curtain incineration ($45)	209	214.2	4	213	208.1	6	1.91	2.83
6f	bin-/invessel composting ($63)	209	217.9	4	213	211.8	6	1.91	2.80
6g	window compositing ($64)	208	218.0	3	213	212.0	6	2.40	2.77
6h	rendering ($82)	208	221.7	3	213	215.6	6	2.40	2.75
6i	fermentation ($90)	208	223.3	3	213	217.2	6	2.40	2.73
6j	anaerobic digestion ($116)	208	228.7	3	213	222.5	6	2.40	2.69
6k	alkaline hydrolysis ($255)	207	257.1	2	212	250.6	5	2.42	2.52

We also conduct six sets of simulations to examine the sensitivities. In each set of simulations we only change one parameter keeping all the other factors the same as in the benchmark case. Table 2 also summarizes results of various simulations.

(1) *Effects of the time value of money*: Simulations 1a-1e vary the weekly interest rate that ranges from 0.01 in Simulation 1a and 0.05 in Simulation 1e. The results show that the possibly additional increase in the total amount of slaughter/disposal of animals as well the cost reduction goes up as the weekly interest rate increases: total amount of slaughter/disposal of animals decreased by 0.49% and the total event cost decreased by 1.53% in Simulation 1a when r=0.01 versus 1.42% and 1.53% in Simulation 1e when r=0.05.

(2) *Effects of disease spread*: We examine the following effects of disease spread rates:

- Simulations 2a-2e vary the disease spread rate caused by vaccinated animals while holding the rate from non-vaccinated animals constant at 10%. Results show that as vaccinated animals becomes more contagious that vaccination causes a progressively smaller increase in total slaughter/ disposal and a progressively smaller cost reduction.
- Simulations 3a-3e vary the disease spread rate caused by non-vaccinated animals while holding the rate from vaccinated animals constant at 5%. Results shows that vaccination becomes more valuable when non-vaccinated animals are more contagious. Furthermore, when non-vaccinated and vaccinated animals spread the disease at the same rate, vaccination increases the total event cost indeed (Simulation 3a).

Therefore, these two sets of simulations show that vaccination is more valuable if vaccines can relative reduce the disease spread more in comparison with no vaccination.

(3) *Effects of the number of initial infected and contact animals*: We vary the initial event size in simulations 3a-3e. Results shows that vaccination tends to increases the total amount of slaughter/disposal of animals less and decrease the total event cost more as the initial event size goes up. That is, vaccination is shown to be more and more valuable as the event size increases.

(4) *Effects of variable cost of vaccination and disposal of animals*: We vary the constant average vaccination cost in Simulation 5q-5e. Results show that a high vaccination cost curbs the possible increases of the total amount of slaughter/disposal of animals (the total amount increased by 3.35% using vaccination in Simulation 5a when the average vaccination cost is $1.5 versus decreases by 1.44% in Simulation 5e when the average vaccination cost goes up to $20). A high vaccination cost also decreases the value of vaccination, which meet the expectation formulized in Proposition 1 (saving cost by 3.36% in Simulation 5a versus 1.03% in Simulation 5e). On the other hand, there are a number of possible carcass disposal technologies (Table 1) each exhibiting different disposal costs. Thus, we take the median value of the average disposal cost for each disposal technology in Simulations 6a-6k. Results suggest that the possible cost saving resulting from vaccinating animals will decrease if the carcass disposal is more expensive.

5.1.1 Discussion on livestock value loss

The market value of lost livestock is an important cost factor. In the two-period setting where vaccination is employed, when the disease spread rate caused by vaccinated animals is higher than the time value, an increase in the livestock value per head decreases the value of vaccination. The 2001 UK FMD outbreak revealed that at least 39% of animals were not infected at all and they had to be killed because of movement bans (NAO report). Naturally, the question is that how much is the potential gain if we could do something with these animals and/or their carcasses. Keeping all the other parameters the same value as in the benchmark case, we assume that the welfare slaughter is 39% and compare the total cost between the following three scenarios:

(a) The authority cannot distinguish uninfected animals from others. They kill and dispose all infected and contact animals.

(b) The disease control authority somehow can distinguish infected and uninfected animals. These 39% carcasses are saved for later use and the salvage value of such carcasses is 20% or 50% of the livestock value loss per head (p).

Results in Table 3 show that total cost is reduced by 5% (Simulation 7a) to 27% (Simulation 7d) when the 39% of the carcasses can be used and need not be disposed. The results do suggest that there is substantial cost saving room if the authority can distinguish carcasses and find a way to salvage uninfected animal carcasses. However, we ignore the possible cost to storage those uninfected carcasses when the movement ban is imposed.

Table 3: The cost saving (%) when 39% of the animals go into a discounted use

(Values of parameters: $\overline{Q} = 100$, $\alpha_H = 11\%$, $\alpha_L = 9\%$, $r = 12\%$, slaughter and disposal cost=130+63=193, p=891, c=0 for the linear case and c=0.5 for the quadratic case)

Simu. No.	salvage value of undisposed carcasses (% of livestock value per head)	Policy I: w/o vaccination			Policy II: w/ vacciation			Comparison of quantity & cost	
		Number of animal killed (heads)	Total Cost (1000$)	Disease prevelance (weeks)	Number of animal killed (heads)	Total Cost (1000$)	Disease prevelance (Weeks)	RQ (%)	RC (%)
7a	0	209	217.9	4	213	206.8	6	1.91	5.09
7b	0.25	209	217.9	4	214	190.7	6	2.39	12.48
7c	0.5	209	217.9	4	214	174.5	6	2.39	19.92
7d	0.75	209	217.9	4	214	158.3	6	2.39	27.35

6 An Aside Feasibility of Vaccination

Should the authority agree on the value of vaccination as a supporting strategy conjunction with the later slaughter of animals, there are several remaining feasibility questions:

Could rules be relaxed to lessen or even diminish the trade disadvantage? The International Office of Epizootics (OIE), is the WTO named agency that sets standards to prevent international spread of livestock diseases. International rules strongly favor “disease free” countries and are restrictive toward animal exports of countries where FMD is endemic. Therefore, countries like UK and US mainly rely on movement bans and slaughter policy to maintain their disease free status to take trade advantages. The trade disadvantage of vaccination was the main reason that the UK Farmers’ Unions opposed to vaccination during the 2001 FMD outbreak (NAO report). However, the relevant rules about FMD penalizing vaccination and encouraging mass slaughter do not reflect technology advances and economic rationale. Scientists associated with the USDA developed a test that can distinguish vaccinated animals from infected ones in 1994 (Breeze 2004), which will lessen the worry of disease spread through trade. To reflect the model technology and economic considerations, it may be of OIE’s interest to relax international trade regulations related to FMD diseases and allow disease-free but vaccinated animal into the world trade.

Can the authority supply vaccines in a timely manner? When an FMD outbreak occurs, the feasibility of vaccination as a supporting strategy requires the availability of vaccines. There are two types of vaccine reserves (Doel and Fullen 1990): (1) conventional commercial FMD vaccine that has a 12-month shelf life; and (2) concentrated inactivated vaccines with a 15-year predicted shelf life. The latter one are held and managed by a consortium of three countries including Canada, Mexico and the US at the North American Vaccine Bank, NAVB (Breeze 2004), and a consortium of seven countries including Australia, Eire, New Zealand, Norway, Sweden and the United Kingdom at the Pairbright Laboratory of the AFRC Institute for Animal Health (Doel and Fullen 1990). The threat of bioterrorism imposes some pressure on vaccine reserves and, thus these FMD vaccine banks may need to take another look on their reserves.

Could the Authority deliver vaccines into infected and contact regions in a timely manner? Even if there is enough vaccine matching the virus strings identified in infected animals, it will take time to move the vaccines to the needed points for use. Breeze (2004) argues it will take 1-2 days for transportation for a specimen and preliminary diagnosis at the Plum Island Foreign Animal Disease Diagnostic Laboratory; 2 days to determine the virus subtype; 4 days to produce the vaccine and deliver it to the outbreak location; and at least 1 day to administer the vaccine within the initial area designated for vaccination. Therefore, we need a minimum of 8 or 9 days to employ vaccination even if virus subtypes are available in NAVB. NAVB may need to design and establish a faster response procedure.

7 Concluding Remarks and Policy Implications

Current US disease control policy calls for mass and immediate slaughter in the case of a disease outbreak. In this study, we examine the value that vaccination can play as a supporting strategy during a major outbreak. Namely vaccination can be used to slow down the flow of slaughter, thereafter the load on the carcass disposal operation and in turn total disposal cost and the incidence of environmental effects as has been recognized since the 1967/68 UK FMD outbreak.[2] This paper investigates the question economically examining when vaccination reduces total event cost. The main cases where total cost is reduced are summarized below:

(a) If we only use vaccination to buy one period, the total slaughter and disposal of animals is greater (see Proposition 1). Vaccination is likely a more effective cost saving mechanism if vaccinated animals spread disease much slower, the initial event size is greater, the time value of money is bigger, the livestock value per head decreases when the disease spread is higher than the time value of money.

(b) In the multiple-period setting, vaccination generally increases the total amount of slaughter and disposal of animals but not always. Vaccination becomes more valuable in reducing the total cost when the costs of vaccination fall, the disease outbreak becomes larger, the vaccines are more effective in controlling disease spread, and/or non-vaccinated animals spread the disease relatively much faster than vaccinated animals. Our results also show that the disease control authority can stamp out the disease if they kill/dispose a critical mass of animals, i.e., at least $\frac{2\alpha - r}{1+\alpha}$ proportion of currently infected and contact animals, where α and r are disease spread rate and weekly interest rate, respectively.

Vaccination would be even more valuable if we overcame the following limitation in our model: we included nonzero environmental damages as we feel environmental costs fall if time pressures were removed.

Welfare slaughter accounts for a substantial percentage of the total slaughter and disposal of animals. Animals killed due to the welfare purpose are not infected at all, and they are in the wrong place at the wrong time because of movement bans. If the authority can distinguish uninfected animals from other and have a differentiate meat market coming

[2] The Northumberland Committee was established to review the outbreak and its control and eradication responses of the 1968/69 FMD outbreak in England. The committee recommended vaccination as a supporting mechanism for FMD outbreak control. Ever since then, European Union law permits the use of emergency vaccination as part of a stamping out policy where appropriate (NAO report 2002).

from uninfected animals or use these carcasses for other purpose such as doggie food, vaccination may be more valuable, and the total cost of event could be substantially lower. However, policy makers shall consider screening costs of animals and trade disadvantage.

Our model implicitly assumes that the current slaughter and disposal capacity can handle operation flows. However, because of infrequent outbreaks, the pre-event capacities are very limited, let along environmental and legal concerns to dump contaminated animals carcasses. Therefore, it is likely to face a shortage of slaughter and disposal facilities. Policy makers have two options: either to build up slaughter and disposal facilities ex ante that will not be used if there is no outbreak; or to build up slaughter and carcass facility ex post that could be substantially costly. It is important to determine the optimal investment in disposal facilities ex ante, which can be the future research direction.

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Appendix A: Proof of Proposition 1

The comparative static analysis of equation (4) yields the following inequalities:

$$\frac{d s_1^*}{d\alpha_L} = \frac{1}{(1+r)SOC}\left(\frac{dSC(s_2)}{d s_2} + \frac{dEC(s_2)}{d s_2} + p + (1+\alpha_L)(\overline{Q} - s_1)\left(\frac{d^2 SC(s_2)}{d s_2^2} + \frac{d^2 EC(s_2)}{d s_2^2}\right)\right) > 0 \text{;} \quad \text{(A-1)}$$

$$\frac{d s_1^*}{d\overline{Q}} = \frac{1}{SOC}\left[\frac{d^2 VC(v_1)}{d v_1^2} + \frac{(1+\alpha_L)^2}{1+r}\left(\frac{d^2 SC(s_2)}{d s_2^2} + \frac{d^2 EC(s_2)}{d s_2^2}\right)\right] > 0\,; \quad \text{(A-2)}$$

$$\frac{d s_1^*}{dr} = -\frac{1+\alpha_L}{(1+r)^2 SOC}\left[\frac{dSC(s_2)}{d s_2} + \frac{dEC(s_2)}{d s_2} + p\right] < 0\,, \quad \text{(A-3)}$$

$$\frac{d s_1^*}{dp} = \frac{\alpha_L - r}{(1+r)SOC}\begin{cases} > 0 & \text{if } \alpha_L > r \\ = 0 & \text{if } \alpha_L = r \\ < 0 & \text{if } \alpha_L < r \end{cases}, \quad \text{(A-4)}$$

where $SOC = \frac{d^2 SC(s_1)}{d s_1^2} + \frac{d^2 EC(s_1)}{d s_1^2} + \frac{d^2 VC(v_1)}{d v_1^2} + \frac{(1+\alpha_L)^2}{1+r}\left(\frac{d^2 SC(s_2)}{d s_2^2} + \frac{d^2 EC(s_2)}{d s_2^2}\right) > 0$ is the second order condition of the cost minimization problem in equation (3). Taking the derivative of Δ_c in equation (6-a) yields the following inequalities:

$$\frac{d\Delta_c}{d\alpha_L} = -\frac{\overline{Q} - s_1^*}{1+r}\left(\frac{\partial SC}{\partial s_2}(s_2^*) + \frac{\partial EC}{\partial s_2}(s_2^*)\right) < 0\,, \quad \text{(A-5)}$$

$$\begin{aligned}\frac{d\Delta_c}{d\overline{Q}} &= \left(\frac{dSC}{d s_1}(\overline{Q}) + \frac{dEC}{d s_1}(\overline{Q}) + p\right) - \left(\frac{dVC}{d v_1}(v_1^*) + \frac{1+\alpha_L}{1+r}\left(\frac{dSC}{d s_2}(s_2^*) + \frac{dEC}{d s_1}(s_2^*) + p\right)\right), \\ &= \left(\frac{dSC}{d s_1}(\overline{Q}) - \frac{dSC}{d s_1}(s_1^*)\right) + \left(\frac{dEC}{d s_1}(\overline{Q}) - \frac{dEC}{d s_1}(s_1^*)\right) > 0 \text{ by substituting the FOC}\end{aligned} \quad \text{(A-6)}$$

$$\frac{d\Delta_c}{dr} = \frac{1}{(1+r)^2}\left(SC(s_2^*) + EC(s_2^*) + p\, s_2^*\right) > 0\,, \quad \text{(A-7)}$$

$$\frac{d\,\Delta_c}{dp} = \overline{Q} - s_1^* - \frac{1}{1+r} s_2^* = \frac{r-\alpha_L}{1+r}\left(\overline{Q} - s_1^*\right)\begin{cases} >0 & \text{if } \alpha_L < r \\ =0 & \text{if } \alpha_L = r \\ <0 & \text{if } \alpha_L > r \end{cases}. \tag{A-8}$$

Differentiating Δq in equation (6-a) with respect to r, p, $\overline{Q}$, and α_L yield

$$\frac{d\Delta q}{dr} = -\alpha_L \frac{ds_1}{dr} > 0 \text{ (since } \frac{d\,s_1^*}{dr} < 0\text{) , and} \tag{A-9}$$

$$\frac{d\Delta q}{dp} = -\alpha_L \frac{ds_1}{dp}\begin{cases} >0 & \text{if } \alpha_L < r \\ =0 & \text{if } \alpha_L = r \\ <0 & \text{if } \alpha_L > r \end{cases}. \tag{A-10}$$

$$\frac{d\Delta q}{d\overline{Q}} = -\alpha_L\left(1 - \frac{d\,s_1}{d\overline{Q}}\right) = \frac{\alpha_L}{SOC}\left(\frac{d^2\,SC(s_1)}{d\,s_1^2} + \frac{d^2\,EC(s_1)}{d\,s_1^2}\right) > 0, \tag{A-11}$$

$$\frac{d\Delta q}{d\,\alpha_L} = -\alpha_L \frac{d\,s_1}{d\overline{Q}} + \left(\overline{Q} - s_1\right). \tag{A-12}$$

The sign of equation (A-12) is undetermined under our assumptions.

We summarize inequalities in (A-1)-(A-10), (A-7), (A-11) and (A-12) in Table 4.

Table 4: Comparative Static Analysis for the two-period setting

	Disease spread (α_L)	Initially infected and contact animals $(\overline{Q})$	Time value (r)	Livestock price per unit (p)		
				$\alpha_L > r$	$\alpha_L < r$	$\alpha_L = r$
Amount of slaughter and disposal in the first period (s_1^*)	+	+	-	+	-	0
Amount of vaccinated animals in the first period (v_1^*)	-	+	+	-	+	0
Difference in total slaughter and disposal (Δ_q)	+/-	+	+	-	+	0
Value of vaccination (Δ_c)	-	+	+	-	+	0

■

Appendix B: Proof of Proposition 2

We make the following assumptions: (a) the cost of slaughter and disposal is given by $SC = a + b\,s_t^i + c\left(s_t^i\right)^2$ where s_t^i is the amount of slaughter and disposal of animals at time t given policy option i; (b) the adverse environmental impact is excluded because of the difficulty of quantifying its value; (c) the value of livestock per head is p; and (d) the average vaccination cost per head is vc. Based on equation (11-a) and (11-b), and (15-a) and (15-c), we can write out the dynamics for the corresponding control and state variables under the two policy options as:

$$\begin{cases} i = nv & \begin{cases} \dot{s}_t^{nv} = (r-\alpha_H)\, s_t^{nv} + \dfrac{(r-\alpha_H)(b+p)}{2c} \\ \dot{Q}_t^{nv} = \alpha_H Q_t^{nv} - (1+\alpha_H)\, s_t^{nv} \end{cases} \\ i = v & \begin{cases} \dot{s}_t^{v} = (r-\alpha_L)\, s_t^{v} + \dfrac{(r-\alpha_L)(b+p) - VC(1+r)}{2c} \\ \dot{Q}_t^{v} = \alpha_L Q_t^{v} - (1+\alpha_L)\, s_t^{v} \end{cases} \end{cases} . \qquad \text{(B-1)}$$

Solving the dynamics yields equilibrium values for $\left[s_e^i, Q_e^i\right]$ as follows:

$$\begin{bmatrix} s_e^{nv} \\ Q_e^{nv} \end{bmatrix} = \begin{bmatrix} -\dfrac{b+p}{2c} \\ -\left(1+\dfrac{1}{\alpha_H}\right)\dfrac{b+p}{2c} \end{bmatrix} \text{ and } \begin{bmatrix} s_e^{v} \\ Q_e^{v} \end{bmatrix} = \begin{bmatrix} -\dfrac{b+p}{2c} + \dfrac{(1+r)VC}{(r-\alpha_L)} \\ -\left(1+\dfrac{1}{\alpha_L}\right)\left(\dfrac{b+p}{2c} - \dfrac{(1+r)VC}{(r-\alpha_L)}\right) \end{bmatrix}.$$

Examining the differences due to policy in slaughter/disposal $M_t^i = s_t^i - s_e^i$ and infected/contact animals $L_t^i = Q_t^i - Q_e^i$, we can re-arrange equations (B-1) below:

$$\dot{M}_t^i = (r-\alpha_j)\, M_t^i, \qquad \text{(B-2a)}$$

$$\dot{L}_t^i = \alpha_j Q_t^i - (1+\alpha_j)\, M_t^i\ , \qquad \text{(B-2b)}$$

where j=H if i=nv and j=L if i=v. Furthermore, equations (B-2a) and (B-2b) can be re-arranged below:

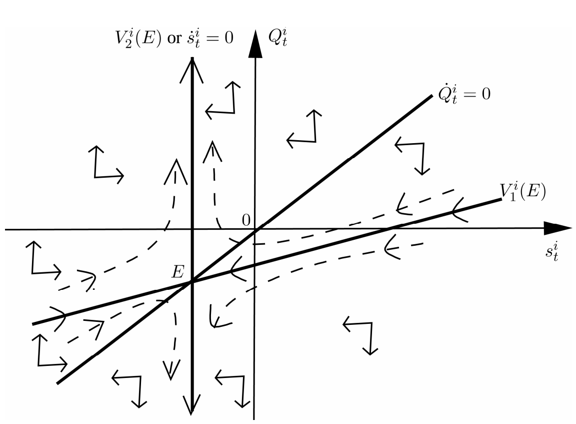
$$\begin{bmatrix} \dot{M}_t^i \\ \dot{L}_t^i \end{bmatrix} = \begin{bmatrix} r-\alpha_j & 0 \\ -(1+\alpha_j) & \alpha_j \end{bmatrix} \begin{bmatrix} M_t^i \\ L_t^i \end{bmatrix} = U \begin{bmatrix} M_t^i \\ L_t^i \end{bmatrix}. \qquad \text{(B-3)}$$

Hence, the dynamics under two different options are given by the homogeneous equation (B-3). Their solutions are determined by the eigenvalues $\left(\lambda_1^i, \lambda_2^i\right)$ and eigenvectors $\left(V_1^i, V_2^i\right)$ of matrix U, which are

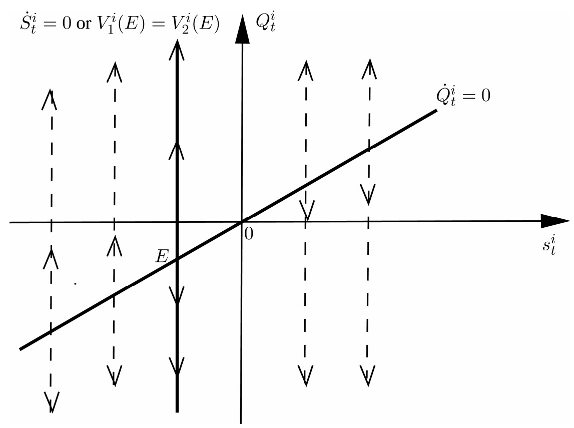
$$\lambda_1^i = r - \alpha_j \text{ and } \lambda_1^i = \alpha_j , \tag{B-4a}$$

$$V_1^i = \begin{bmatrix} 1 \\ (1+\alpha_j)/(2\alpha_j - r) \end{bmatrix} \text{ and } V_2^i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{B-4b}$$

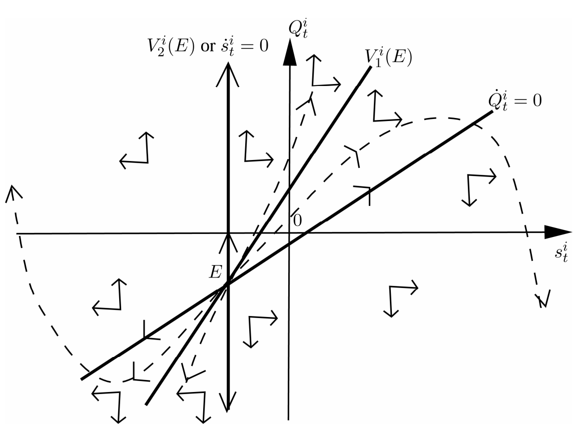
Figure 6 gives a phase diagram for the solution under policy option *i* as it depends on the value of *r* and α_j. The vertical axis represents the total amount of infected and contact animal (Q_t^i), and the horizontal axis shows the total amount of slaughter and disposal of animals (s_t^i) over time. The intercept of the two dynamics $\dot{Q}_t^i = 0$ and $\dot{s}_t^i = 0$ characterizes the equilibrium point *E*, and $V_1^i(E)$ and $V_2^i(E)$ show the eigenvectors. All L-shaped directional arrows suggest the trajectory of $\dot{Q}_t^i$ and $\dot{s}_t^i$.



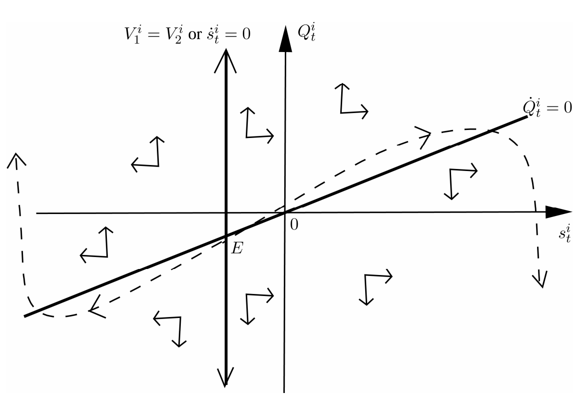
(a) $0 < r < \alpha_j$



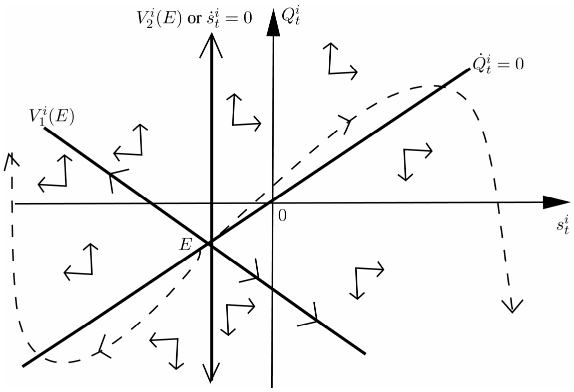
(b) $r = \alpha_j$



(c) $\alpha_j < r < 2\alpha_j$



(d) $r = 2\alpha_j$



(e) $r > 2\alpha_j$

Figure 6: Phase Diagrams of the total infected and contact animals (Q_t^i) and the total amount of slaughter/disposed of animals (s_t^i) at time *t* under policy option *i*

As shown in Figure 6, excepting for the case where $0 < r < \alpha_j$ **(**where the equilibriumis a saddle point), all cases exhibit an unstable equilibrium. Furthermore, these five phase diagrams suggest the following findings:

(a) When $0 < r < 2\alpha_j$ (including plots 12(a) and 12(b)), the total infected and contact animals (Q_t^i) will reach zero in the first quadrant if $\frac{s_t^i}{Q_t^i} \geq \frac{2\alpha_j - r}{1 + \alpha_j}$ is satisfied.

(b) When $r \geq 2\alpha_j$, regardless the ratio between Q_t^i and s_t^i, the total infected and contact animals (Q_t^i) can always reach zero in the first quadrant

This concludes discussion and proof of Proposition 2. ■