Measurement and Interpretation of Elasticities

What Are Elasticities?

- Measure of the relationship between two variables

\[ \text{Elasticity} = \frac{\text{Percentage change in } y}{\text{Percentage change in } x} \]

- Elastic vs. inelastic
- Arc vs. point

Alfred Marshall

- Popularized concepts
  - Changed the name and face of economics

- Quirks

- Elasticities
**Elasticities of Demand**

- **Own-price elasticity of demand**
  - responsiveness of changes in quantity associated with a change in the goods own price
- **Income elasticity of demand**
  - responsiveness of changes in quantity associated with a change in income
- **Cross-price elasticity of demand**
  - responsiveness of changes in quantity associated with a change in price of another good

**Own-Price Elasticity of Demand**

\[
\text{Own-price Elasticity} = \frac{\text{Percentage change in quantity}}{\text{Percentage change in own price}}
\]

\[
\text{Own-price elasticity} = \frac{(Q_a - Q_b)/(Q_a + Q_b)/2}{(P_a - P_b)/(P_a + P_b)/2}
= \frac{\Delta Q}{\Delta P} = \frac{P_a - P_b}{P_a + P_b}
\]

- Interpretation -- 1% increase in price leads to a x% change in quantity purchased over this arc

**Own-Price Elasticity**

- Consumer bundle B to A
- Change in quantity 2 to 1
- Change in price 9 to 10
- What is the own-price elasticity of demand at this arc?
Recall change in quantity = 2 to 1 and price 9 to 10

\[ \text{% change in quantity} = \frac{(1-2)/(1+2)/2}{(10-9)/(10+9)/2} = -0.667 \]

\[ \text{% change in own price} = -0.105 \]

or

\[ \frac{\Delta Q}{\Delta P} \frac{F}{Q} = \frac{(1-2) \cdot (10+9)/2}{(10-9) \cdot (1+2)/2} = -1 \cdot \frac{9.5}{1.5} = -6.33 \]

Interpretation -- 1% increase in price leads to a 6.33% decrease in quantity purchased over this arc

Own-Price Elasticity

- Bundles C to D

\[ \text{Unitary Elasticity} \]

% change in quantity = \(\frac{(5-6)/(5+6)/2}{(6-5)/(6+5)/2}\) = 0.18

\[ \text{% change in own price} = -1.00 \]

Interpretation -- 1% increase in price leads to a 1% decrease in quantity purchased over this arc

Own-Price Elasticity

- Bundles E to F

\[ \text{Interpretation -- 1% increase in price leads to a 0.29% decrease in quantity purchased over this arc} \]
Own-Price Elasticity Cont.

- Generally elasticities vary over the curve
- Negative – law of demand
- Linear demand curve - specific

\[ \Delta Q \quad \frac{P}{\Delta P} \cdot \frac{\Delta Q}{Q} \]

Own-Price Elasticity

If value of the elasticity coefficient is | Demand is said to be | %Δ in quantity is
---|---|---
Less than -1.0 | Elastic | Greater than %Δ in price
Equal to -1.0 | Unitary elastic | Same as %Δ in price
Greater than -1.0 | Inelastic | Less than %Δ in price

Use - example

- What is arc elasticity for corn between the prices of $15 (6 corn) and $20 (5 corn) / dozen?
• Calculation of arc elasticity
  – % change in Price = \((20-15)/[(20+15)/2]\) = 0.28
  – % change in Q = \((5-6)/[(5+6)/2]\) = -0.18
  – Own-price elasticity = \(-0.18/(0.28)\) = -0.63

• Elastic or inelastic
  – Why?
• Goal is to increase revenues. The current price is $17.50 / dozen, should you increase or decrease price?

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### Revenue Implications - Know

<table>
<thead>
<tr>
<th>Own-price elasticity</th>
<th>Cutting the price will</th>
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• Necessary information from earlier calculations
  – Price increase from 15 to 20
  – Quantity decreases from 6 to 5
  – Own-price elasticity = \(-0.18/(0.28)\) = -0.63
• Current price $17.50 with Q = 5.5
• Goal is to increase revenues
  – Current TR = 17.5 \times 5.5 = 96.25
  – Increase price TR = 20 \times 5 = 100
  – Decrease price TR = 15 \times 6 = 90
**Revenue Implications – Why?**

**Unit Elasticity Demand Curve**

Brings about the same % increase in the quantity demanded – definition of unit elasticity.

**Revenue Implications – Why?**

**Unit Elasticity Demand Curve**

Loss in revenue due to price change = Gain in revenue due to quantity change.

**Revenue Implications – Why?**

**Inelastic Demand Curve**

Brings about a smaller increase in the % quantity demanded – definition of inelastic.
Inelastic Demand Curve

Revenue Implications – Why?

Producer revenue falls since \( \% \Delta P \) is greater than \( \% \Delta Q \).

Revenue before the change was \( 0P_bQ_b \).

Revenue after the change was \( 0P_aQ_a \).

Elastic Demand Curve

Revenue Implications

Cut in price brings about a larger % increase in the quantity demanded.
Producer revenue increases since the gain is greater than the loss.

Revenue before the change was $0P_bCQ_b$.
Revenue after the change was $0P_aDQ_a$.

**Revenue Implications - Know**

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Relative Elasticities

- Perfectly inelastic
- Perfectly elastic

Relative Elasticities

- Relatively more inelastic
- Relatively more elastic

Long vs. Short-Run

- Demand curves tend to be more elastic (flatter) over time as consumers adjust to changing prices – Why?
Consumer Surplus

Gain in consumer surplus after the price cut is area \( PP_CD \)

Inelastic Demand Curve

Consumer surplus increased by area \( PP_CD \)

Income Elasticity of Demand

\[
\text{Income Elasticity of Demand} = \frac{\text{Percentage change in quantity}}{\text{Percentage change in income}}
\]

\[
\text{Income elasticity} = \frac{\Delta Q}{\Delta I} = \frac{I_A - I_B}{I_A + I_B}/2
\]

- Interpretation -- 1% increase in income leads to a \( x \)% change in quantity purchased over this arc

Income Elasticity Example

- Income and Corn
  - Income change 200 to 400
  - Corn quantity change 5 to 9
- What is arc income elasticity of demand?

\[
\frac{\text{% change in quantity}}{\text{% change in income}} = \frac{(Q_A - Q_B)/[(Q_A + Q_B)/2]}{(I_A - I_B)/[(I_A + I_B)/2]}
\]

\[
= \frac{0.57}{0.66} = 0.85
\]

Interpretation?
Interpreting the Income Elasticity of Demand - Know

<table>
<thead>
<tr>
<th>If the income elasticity is</th>
<th>The good is classified as</th>
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<tr>
<td>Greater than 1.0</td>
<td>A luxury and a normal good</td>
</tr>
<tr>
<td>Less than 1.0 but greater</td>
<td>A necessity and a normal good</td>
</tr>
<tr>
<td>than 0.0</td>
<td></td>
</tr>
<tr>
<td>Less than 0.0</td>
<td>An inferior good!</td>
</tr>
</tbody>
</table>

Cross-Price Elasticity of Demand

Cross-price elasticity

\[
\text{Cross-price elasticity} = \frac{\frac{\Delta Q_c}{Q_c}}{\frac{\Delta P_D}{P_D}} = \frac{\frac{\Delta Q_c}{\Delta P_D}}{\frac{P_D}{Q_c}}
\]

- Interpretation -- 1% increase in price of good D leads to a x% change in quantity purchased of good C over this arc

Cross-Price Elasticity Example

- Steak quantity and corn price
  - Corn price change from $20 to $15 / dozen
  - Steak quantity changes from 2.5 to 2.75 pounds
- What is arc cross-price elasticity of demand for steak?

\[
\text{% change in quantity of steak} \div \text{% change in corn price} = \frac{(2.75 - 2.5)/(2.75 + 2.5)/2}{(15 - 20)/(15 + 20)/2} = \frac{0.1}{-0.28} = -0.33
\]
Interpreting the Cross Price Elasticity of Demand - Know

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<td>Positive</td>
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<td>Negative</td>
<td>Complements</td>
</tr>
<tr>
<td>Zero</td>
<td>Independent</td>
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Stimulus Bill Example

- 2009 Stimulus Bill
  - Included a up to a $1500 tax credit for insulation and energy efficient windows, doors, HVAC units

- What is a tax credit?

- Why pass the bill and potential economic effects? - nonpolitical

Stimulus Bill Insulation

- Assume you have calculated the following elasticities for insulation
  - Income elasticity of demand = 1.2
  - Own-price elasticity = -0.4
  - Cross price elasticity with lumber = -0.02
  - Cross price elasticity with energy = 0.09
  - Assume tax credit decreases insulation price by 30%

- What is the effect of the stimulus bill given these elasticities? Recession has decreased incomes by 10%
Stimulus Bill Insulation

- Decrease in insulation sales – recession
  - $-10\% \times 1.2 = -12\%$ - decrease in insulation sales
- Increase in insulation sales – stimulus bill
  - $-30\% \times -0.4 = 12\%$ - increase in insulation sales
- Change in lumber sales – stimulus bill
  - $-30\% \times -0.02 = 0.6\%$ - increase in lumber sales
- Change in energy use – stimulus bill
  - $-30\% \times 0.09 = -2.7\%$ - decrease in energy use

Costs of the Bill

- Decrease in tax revenues – insulation tax credit
- Increase in tax revenues – increase in insulation sales
- Increase in tax revenues – increase in lumber sales
- Decrease in tax revenues – decrease in energy use
- Environmental / other
- Overall ?

Price Flexibility of Demand

- Price flexibility is the reciprocal of own price elasticity
  - Price flexibility = $1/(\text{own price elasticity})$

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<th>Price Flexibility of Demand</th>
<th>$\frac{\text{Percentage change in price}}{\text{Percentage change in quantity}}$</th>
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- Rearrange
  - $\% \Delta \text{ price} = \text{price flexibility} \times \% \Delta \text{ quantity}$
If the calculated elasticity is -0.25, then the price flexibility = 1/-0.25 = -4.0
Useful concept to producers to help form price expectations
Example USDA projects an additional 2% of supply will come on the market, what happens to price.

\[ \% \Delta \text{Price} = \text{price flexibility} \times \% \Delta \text{Quantity} \]
\[ = -4.0 \times (+2\%) \]
\[ = -8\% \]

If supply increases by 2%, price would fall by 8%.

Revenue Implications – Demand Elasticity and Changes in Supply

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Characteristic of agriculture

Summary - Know

- Know how to interpret all three elasticities
- Know how to interpret a price flexibility
- Understand revenue implications for producers if prices are cut (raised)
- Understand the welfare implications for consumers if prices are cut (raised)