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Option price without expected utility

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ABSTRACT

We consider the meaning of the option price, commonly acknowledged as the preferred ex ante welfare measure, in the rank-dependent expected utility (RDEU) framework. The importance of this pertains to performing benefit-cost analysis when RDEU maximizers are prevalent in society.

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1. Introduction

In this manuscript, we consider the meaning of welfare measures under risk, allowing for the possibility that preferences do not correspond to all of the axioms of the expected utility model. As a specific alternative to the expected utility (EU) model we consider the rank-dependent EU (RDEU) model introduced by Quiggin (1982). By incorporating similar features into prospect theory (Kahneman and Tversky, 1979), Tversky and Kahneman (1992) develop cumulative prospect theory (CPT), which has become another widely used non-expected utility model in decision analysis under risk.

The option price (OP), not to be confused with the price of financial options, is viewed as the desired measure of welfare when changes occur under conditions of known, and collective, risk[1] (Graham, 1981). Decisions that fall under guidelines calling for an ex ante benefit-cost analysis ideally would use estimates of the OP for changes involving risk. Relevant situations might include policies affecting all federal lands or resources, and rule changes relating to some environmental regulations. An early empirical example includes Desvousges et al. (1987), who estimated the OP for improved water quality used by recreational anglers. As a more recent example, Riddel and Shaw (2006) consider the potential loss in welfare (benefits estimated ex ante) from mortality risks tied to shipping nuclear wastes to the national high level nuclear waste repository at Yucca Mountain. Many other empirical studies examine ex ante welfare measures for human health changes that are linked to deteriorations or improvements in air or water pollution (see the review in Shaw et al., 2005). The EU is assumed as the framework in almost all of the studies that include a formal expression for the welfare measure.

In laboratory experiments and some other settings, however, many have come to view the EU axioms as restrictive and models based on them often fail to predict observed behaviors. In particular, the independence axiom is often violated by individuals who are make choices under conditions of risk or uncertainty. This led to decades of research devoted to exploration of non-EU models, either by relaxing the independence axiom or another modification (see Starmer, 2000, for a survey of non-EU models; and Shaw and Woodward, 2008, for a more recent, but focused, review). Modifications that relax some EU axioms are typically accomplished by introducing non-linear probability weighting functions. While these alternatives have been greatly investigated, their relationship to the OP has only been mentioned in a few, rare instances (see Smith, 1992).

2. Option price under expected utility

We define a surplus in state i, s_i, as a compensating variation for the availability of a public good. Let u_i^0 denote a utility function in state i when the good is not available, and u_i denote a utility function in state i when the good is available. If the income in state i is w_i, the surplus in state i can be derived from $u_i\ (w_i - s_i) = u_i^0(w_i)$. If expected utility theory holds and there are two states of the world that occur

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[1] For example, in a 2-state world with a collective risk, everyone will experience state 1 or everyone will experience state 2.

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with probabilities p and $1-p$, the expected utility of an individual after paying the state-dependent surplus is

$$pu_1(w_1 - s_1) + (1-p)u_2(w_2 - s_2) \equiv \overline{u}. \tag{1}$$

Graham (1981) defines the OP using an ex-ante willingness to pay (WTP) for the good that keeps the expected utility unchanged, i.e., the value of T such that

$$pu_1(w_1 - T) + (1-p)u_2(w_2 - T) = \overline{u}. \tag{2}$$

The OP can be greater or smaller than the expected surplus, $E(s)=ps_1+(1-p)s_2$, for a risk-averse individual (see Graham, 1981). In practice, the OP may be difficult to elicit from individuals and the expected surplus is sometimes used instead to estimate the ex-ante WTP, despite some discrepancy between the two measures. Graham (1981) uses a graphical representation of the WTP locus to illustrate the OP concept. The locus depicts possible pairs of willingness to pay, or contingent payment points. Let (x_1, x_2) be the vector of payments in states 1 and 2. The WTP locus (x_1, x_2) is constructed and developed such that

$$pu_1(w_1 - x_1) + (1-p)u_2(w_2 - x_2) = \overline{u}. \tag{3}$$

The WTP locus is smooth and concave because the agent is assumed to be a risk-averse EU maximizer. The OP can be found from the intersection of the WTP locus and the 45-degree line drawn through the origin (see Fig. 1), as it indicates an equal pair of contingent payments. The arrow-headed line is the iso-expected-value line. Therefore the value on the x_1 axis where the arrow-headed line crosses the 45-degree line is $E(s)$. For both graphs in Fig. 1, we assume that $w_1=w_2=w$, and $u_1'(w)<u_2'(w)$ for all w. In panel (a), $s_1<s_2$, while in panel (b), $s_1>s_2$.

3. The option price under rank-dependent expected utility

Next assume that there is a rank-dependent expected utility maximizer (see Quiggin, 1993, for key assumptions) whose utility function is the same as the EU agent's. The essential features of the RDEU are that there is a non-linear weighting function h: $[0,1]\rightarrow[0,1]$ which is non-decreasing with $h(0)=0$ and $h(1)=1$, and that the RDEU maximizer ranks outcomes so that one outcome relative to another matters. This type of ranking is a feature in virtually all important alternatives to the EU, including CPT. To apply the RDEU features in a state-dependent utility framework, we follow Chiu's (1996) basic methodology. Chiu ranks prospective outcomes according to their state-dependent utility levels instead of their state-dependent income levels. Here assume that there are two states and state 2 is a preferred state, i.e., $u_1(w)<u_2(w)$ for all w, and $w_1 \le w_2$. We assume further that after paying the surplus, state 2 is still preferred to state 1, i.e., $u_1(w_1-s_1)<u_2(w_2-s_2)$. If the utility function is state-dependent, the RDEU can be written as

$$h(p)u_1(w_1 - s_1) + (1-h(p))u_2(w_2 - s_2) = \overline{v}. \tag{4}$$

If $h(p)>p$, the individual overweighs low utility outcomes, and therefore underweighs the high utility outcome. As a result, the value

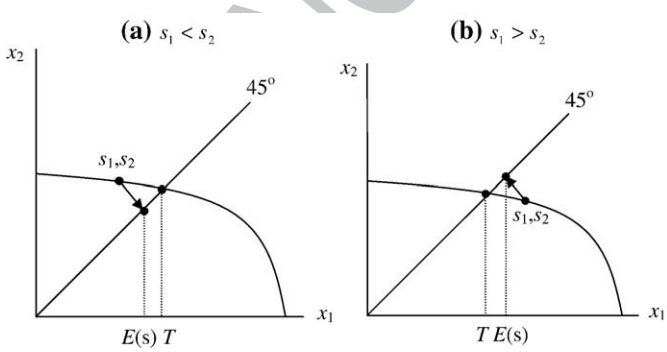

Fig. 1. WTP loci under EU.

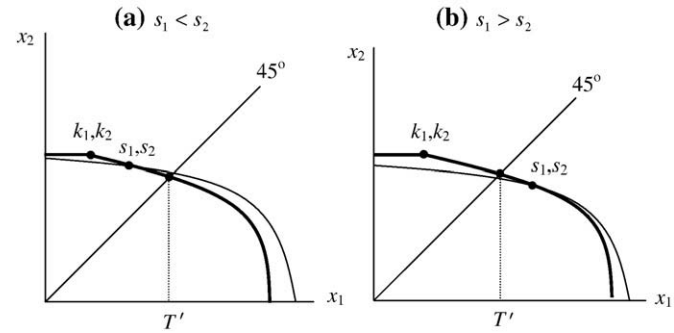

Fig. 2. WTP loci under RDEU.

of $\bar{v}$ in Eq. (4) is lower than the value of $\bar{u}$ in Eq. (1). Thus the individual is said to be *pessimistic*. Pessimistic individuals dwell on the worst case scenario, attributing more importance to it than the true probability warrants. If $h(p)<p$, the individual overweighs the high or best utility outcomes, and the individual is said to be optimistic.[2] Applying Chiu's framework to Graham's OP concept, the RDEU-OP, denoted by T' must satisfy the following equation:

$$h(p)u(w_1 - T') + (1-h(p))u(w_2 - T') = \overline{v}. \tag{5}$$

We compare the OP's of two agents who have the same surplus in each state; one is a state-dependent EU agent and another one is a state-dependent RDEU agent. Because society may well be mixed with people who are each type of agent, it is of interest to ask, which of the two types of agents has a higher OP?

Proposition 1. *Assume state-dependent utility and* $s_1<s_2$. *If an EU agent and a pessimistic (an optimistic) agent have the same surplus in each state, then the pessimistic (optimistic) agent's option price is smaller (larger) than the EU agent's. If* $s_1>s_2$, *then the pessimistic (optimistic) agent's option price is larger (smaller) than the EU agent's.*

Proof. Define $u(s_1,s_2)=pu_1(w_1-s_1)+(1-p)\,u_2(w_2-s_2)$ and $v(s_1,s_2)=h(p)u_1(w_1-s_1)+(1-h(p))\,u_2(w_2-s_2)$ with $h(p)>p$ so that u is utility for an EU agent and v is the same for a pessimistic agent. Since $u_1(w_1-s_1<u_2(w_2-s_2)$ and $h(p)>p$, then $u(s_1, s_2)>v(s_1, s_2)$. Let T and T' be the OP's of agent u and v, respectively. Then $u(s_1, s_2)=u(T, T)$ and $v(s_1, s_2)=v(T',T')=\bar{v}$. We have the following equations:

$$u(s_1, s_2) - \overline{v} = [h(p) - p][u_2(w_2 - s_2) - u_1(w_1 - s_1)] \tag{6}$$

$$u(T', T') - \overline{v} = [h(p) - p][u_2(w_2 - T') - u_1(w_1 - T')]. \tag{7}$$

Let c be a function mapping from the payment in state 1 (x_1) to the payment in state 2 (x_2), such that $v(x_1,c(x_1))=\bar{v}$. It follows that $c'<0$. Since $v(s_1, s_2)=v(T',T')$ and $s_2>s_1$, then $s_2>T'>s_1$. It also follows that $u_2(w_2-T')>u_2(w_2-s_2)$ and $u_1(w_1-s_1)>u_1(w_1-T')$. Therefore, the right-hand side of Eq. (7) is greater than that of Eq. (6). Then $u(T',T')>u(s_1,s_2)=u(T,T)$, and $T>T'$. If the agent is optimistic, then $h(p)<p$ and consequently $T'>T$. If $s_1>s_2$ the results are reversed. □

The results in Proposition 1 can be illustrated in Fig. 2. The bold curves are the WTP loci for pessimistic agents. In panel (a), $s_1<s_2$ and the pessimistic agent's OP is lower than the EU agents'. In panel (b), $s_1>s_2$ and the pessimistic agent's OP is higher than the EU agents'. Note that the WTP locus now has a kink at (k_1, k_2), defined as $u_1(w_1-k_1)=u_2(w_2-k_2)$. The WTP locus is not differentiable at (k_1,k_2) when $h(p)\neq p$.

[2] When $h(p)$ is strictly concave (convex) for all p in [0,1], the agent is pessimistic (optimistic). If the weighting function has an inverted S-shape as suggested by Tversky and Kahneman (1992), i.e., $h(p)>p$ when $p<p'$ and , $h(p)<p$ when $p>p'$, and $h(p')=p'$ for some p', then the agent is pessimistic (optimistic) when p is sufficiently small (large).

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This is similar to the kink on an indifference curve of an individual who is risk averse of order 1 defined by Segal and Spivak (1990)[3]. Segal and Spivak (1997) prove that the first-order risk aversion at (k_1,k_2) is equivalent to the local utility function that is not differentiable at (k_1,k_2).

Because expected surplus is often used in lieu of the OP, it is of interest to see whether the discrepancy between the OP and the expected surplus increases or decreases under RDEU framework. We assume that the weighting function h is known and define weighted expected surplus as $WE(s)=h(p)s_1+(1-(p))s_2$. We denote the difference between the EU-OP and the expected surplus by t and the difference between the RDEU-OP and the weighted expect surplus by t', i.e., $t=T-E(s)$ and $t'=T'-WE(s)$. We are interested in finding whether the difference is larger or smaller under RDEU. We know that $t'>t$ when $T'-T>WE(s)-E(s)$. We define the following: $\Delta p=h(p)-p$, $\Delta s=s_2-s_1$, $\Delta u_1=u_1(w_1-T)-u_1(w_1-s_1)$, and $\Delta u_2=u_2(w_2-T)\ u_2(w_2-s_2)$. We have

$$WE(s)-E(s)=-\Delta p\Delta s, \tag{8}$$

and, for a sufficiently small Δp, we have

$$T'-T=\frac{-\Delta p[\Delta u_2-\Delta u_1]}{pu_1'(w_1-T)+(1-p)u_2'(w_2-T)}. \tag{9}$$

Eqs. (8) and (9) provide us with conditions that can be used to compare the difference between the weighted expected surplus and the expected surplus and the difference between the RDEU-EU and the EU-OP. These conditions help explain the size of t and t' under EU and RDEU, respectively. For example, when an agent is pessimistic and $s_1<s_2$, we know that Δp, Δs, $\Delta u_2>0$, and $\Delta u_1<0$. Eqs. (8) and (9) tell us that, if $\Delta s>\frac{\Delta u_2-\Delta u_1}{pu_1'(w_1-T)+(1-p)u_2'(w_2-T)}$, then the reduction in the expected surplus is larger than the reduction in the OP. Therefore, the discrepancy between the OP and the expected surplus is larger under RDEU than under EU.

4. Summary

Decision makers often make policy based on benefit-cost analysis, requiring examination of benefit, or welfare measures. Such welfare measures, in the presence of well-defined risks and when the expected utility (EU) framework appropriately models behavior, are possible to identify. For many reasons, some individuals behave in a manner that is inconsistent with the axioms of the EU. In such cases, a benefit-cost analysis should be based on welfare estimates that relate to their behavior and the more appropriate, non-EU framework. In this paper, we consider the meaning of an ex ante welfare measure in the rank-dependent expected utility (RDEU) framework, finding key differences. The importance of this pertains to performing benefit-cost analysis when RDEU maximizers are prevalent in society.

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[3] For a random variable ε with $E(\varepsilon)=0$. Let $\mu(t\varepsilon)$ be the risk premium that the agent is willing to pay to avoid the risk $t\varepsilon$. The agent is risk averse of order 1 if $\partial\mu(t\varepsilon)/\partial t|_{t=0}\neq 0$.

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