Class #23 & 24

The efficient allocation of resources over time

Video

Present Value and Discounting

AGEC 350
Richard T. Woodward

A mini lecture: Present value

• A benefit or cost experienced in the future is given less weight when evaluated today because of:
  – Time preference: impatience & procrastination
  – Opportunity cost of capital: you can put money in interest-bearing accounts meaning that $100 today is worth more to you that $100 in the future.
Present value

- Suppose that you put $100 in the bank at 10% interest.
- In one year you have $100 \times 1.1 = $110
- In two years you have $110 \times 1.1 = $121
- The present value of $121 to be received in 2 years is $100.
  \[ \frac{121}{1.1^2} = 100 \]

Present Value

- In general, if \( r \) is your interest rate, the present value of a benefit \( B \) (or cost) to be received in \( n \) years is

\[ PV = \frac{B}{(1 + r)^n} \]

Team Task

1. What is the present value of $2,420 to be paid in 2 years assuming a discount rate of 10% per year?

A. $2,420
B. $2,200
C. $2,000
D. $2,662
E. $2,928
#1 Answer

- $2,420/(1+10\%)^2 =
- $2,420/(1.1)^2 =
- $2000

Team Task

2. Without using a calculator, if the discount rate is greater than 10%, will the present value be greater or less than the answer to question 1?
   A. Greater
   B. Less
   C. Same

#2 Answer

- $2,420/(1+r\%)^2$
  As r gets larger, the denominator gets bigger, so the PV gets smaller.

- Central insight – as the discount rate gets bigger, you place less weight on the future, so the present value goes down.
Team Task
3. A policy maker is considering whether to spend $5,000 today on a project that will generate $1000 next year and every year after for 5 years total? Will the project pass a benefit-cost test?
A. Yes  B. No.
C. Note enough information to answer.

#3 Answer
$5,000 today on a project that will generate $1000 for five years?

$1000/(1+r) + $1000/(1+r)^2 + $1000/(1+r)^3 + $1000/(1+r)^4 + $1000/(1+r)^5

If r=0 then PV of benefits = $5000. As long as r>0, the project will not pass a benefit cost test.

Team Task
4. A policy maker is considering whether to spend $5,000 today on a project that will generate $1000 next year and every year for 6 years total? Will the project pass a benefit-cost test?
assume a discount rate of 5%
A. Yes  B. No.
C. Not enough information to answer.
#4 Answer

Cost: $5,000 today  
Benefit: $1,000 next year and every year for 6 years total

<table>
<thead>
<tr>
<th>Year</th>
<th>Benefit</th>
<th>Cost</th>
<th>Net Benefit</th>
<th>PV (of Net Benefit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0</td>
<td>5,000.0</td>
<td>-5,000.0</td>
<td>-5,000.0</td>
</tr>
<tr>
<td>1</td>
<td>1,000.0</td>
<td>0.0</td>
<td>1,000.0</td>
<td>952.4</td>
</tr>
<tr>
<td>2</td>
<td>1,000.0</td>
<td>0.0</td>
<td>1,000.0</td>
<td>907.0</td>
</tr>
<tr>
<td>3</td>
<td>1,000.0</td>
<td>0.0</td>
<td>1,000.0</td>
<td>863.8</td>
</tr>
<tr>
<td>4</td>
<td>1,000.0</td>
<td>0.0</td>
<td>1,000.0</td>
<td>822.7</td>
</tr>
<tr>
<td>5</td>
<td>1,000.0</td>
<td>0.0</td>
<td>1,000.0</td>
<td>783.5</td>
</tr>
<tr>
<td>6</td>
<td>1,000.0</td>
<td>0.0</td>
<td>1,000.0</td>
<td>746.2</td>
</tr>
</tbody>
</table>

PV of NB 75.7

Team Task:  
Turn in to be graded

4. Suppose a project costs $10,000 immediately and then $1,000 each year for 3 years. The benefits of the project are worth $3,500 per year and occur for 4 years starting one year from now.  
What is the net present value of this project using a 10% discount rate?  
Does the project pass the benefit cost test?

Follow-up

- By how much would the immediate costs have to change to flip the answer in question 4 (round to the nearest hundred dollars)?
Team Task

6. The owner of 100 barrels of an oil reserve can sell her oil now or next year. Her cost of extraction is $7 per unit. The price of the resource today is $20 today and she believes it will increase by 10% next year. The owner’s personal discount rate is 10% per year.

Calculate: NB if extracted now of 1 barrel now and next year. PV of NB in both periods. Would she extract a barrel today or next year?

Discussion

6. The owner of 100 barrels of an oil reserve can sell her oil now or next year. Her cost of extraction is $7 per unit. The price of the resource today is $20 today and she believes it will increase by 10% next year. The owner’s personal discount rate is 10% per year.

How much oil would she extract a now and how much would she extract next year?

Hotelling’s Rule

- If decision makers seek to maximize the discounted present value of net revenue, then they will tend to extract non-renewable resources today only if the NB obtained from doing so is greater than or equal to the PV of NB of extraction in the future.
- Hence, in theory at the equilibrium the MNB today = PV MNB in the future.
<table>
<thead>
<tr>
<th></th>
<th>PV MNB</th>
<th>MNB</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>PV MNB</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>PV MNB</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>PV MNB</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>PV MNB</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>PV MNB</td>
<td></td>
</tr>
<tr>
<td></td>
<td>PV MNB</td>
<td></td>
</tr>
</tbody>
</table>
What does theory say about the price of non-renewable resources?

- Do you think there is a natural tendency of the price of oil to go up or down? Why?

What does this mean for the price?

- Tons
- Price (in future $'s)
- MNB (undiscounted)
- PV (MNB)
- Time

But...

Source: Office of Energy Statistics (on my computer at I:\Module Material\4 - Resource Allocation, static and dynamic\Dynamic Resource Allocation and Discounting\OilPrices.xls)
Another new equimarginal condition

- 1\textsuperscript{st} – \( MB = MC \) => efficiency
- 2\textsuperscript{nd} – \( MC_1 = MC_2 \) => cost effectiveness
- 3\textsuperscript{rd} - The MNB to every user should be equal at the efficient allocation.
- 4\textsuperscript{th} = The present value of MNB is equal in all periods for dynamically efficient allocation.
A Dynamically Efficient Allocation

From the video
Again, there is marginal scarcity rent at the optimum. The PV of the marginal scarcity rent will be constant.

Summary Dynamic Efficiency

- The basic idea is the same as in static efficiency. Total net benefits are maximized only if the marginal net benefits are the same to all users.
- In this case the allocation is between the present (undiscounted) and the future (discounted).
- The present value of marginal net benefits are equal across time.

Exercises using handout
Solving for the socially optimal allocation (20% discount rate) (using the graph)

7. There are 350 units of the resource available that must be used in two periods.
   a. What is the dynamically efficient allocation of the resource over two periods?
      \( Q(\text{today}) = \) _____________, \( Q(\text{next yr}) = \) ______________.
   b. At the dynamically efficient allocation, what is the marginal net benefit to the producers of the resource today and in one year?
      \( \text{MNB(today)} = \) _____________, \( \text{MNB(next yr)} = \) ______________.
      \( \text{PV(MNB(\text{today})}) = \) _____________, \( \text{PV(MNB(\text{next yr})}) = \) ______________.
   c. What price in each period would lead to the dynamically efficient allocation across the two periods?
      \( P(\text{today}) = \) _____________, \( P(\text{next yr}) = \) ______________.

\[ \frac{18}{18} = 1.2 \text{ or } \frac{18}{1.2} = 15 \]
\[ 250 + 100 = 350 \]
Today more is used, so you keep consuming until MNB is lower.

Next period less is used, so MNB is higher than it is today.

But don’t forget, the PV of MNB is equal.
Future price = $22  
Current price = $19

Another way to get the efficient intertemporal allocation

Future price = $22  
Current price = $19

Another way to get the efficient intertemporal allocation

Future price = $22  
Current price = $19

Another way to get the efficient intertemporal allocation

Future price = $22  
Current price = $19
Team Task (Graded)
The socially optimal allocation (20% discount rate)

There are 900 units of the resource available that must be used in two periods.

a. What is the dynamically efficient allocation of the resource over two periods?
   Q(today) = _____________, Q(next yr) = ______________.

b. At the dynamically efficient allocation, what is the marginal net benefit to the producers of the resource today and in one year?
   MNB(today) = _____________, MNB(next yr) = ______________.
   PV(MNB(today)) = _____________, PV(MNB(next yr)) = ______________.

c. What price in each period would lead to the dynamically efficient allocation across the two periods?
   P(today) = _____________, P(next yr) = ______________.
User Cost

- When you use a resource today, you give up the ability to use that resource in the future.
- Future benefits are discounted.
- In a dynamically efficient allocation the MNB now is equal to the marginal user cost. This is another way to state the 4th equimarginal condition.

From the video
Marginal User Cost
Marginal User Cost

If the reserves are more that $Q_{\text{max}}$

Another example using the handout

- There are 350 units to be used over 2 periods,
- What is the marginal user cost of the 1st unit used today?
- What is the marginal user cost of the 250th unit?
Team Exercise

- Explain the user cost in each of the following:
  - a landfill used by a city.
  - an aquifer with very low recharge.
  - the water in a reservoir in a region that receives all its rain in the spring.

Dynamic choices and uncertainty: Quasi-Option Value

- When new information is going to be available and decisions are irreversible, the correct question is not "do it or not," but rather, "do it now or wait."
- The value of waiting and gathering new information is called the quasi-option value.
Arctic National Wildlife Refuge

Examples of Quasi-Option value in current policy decisions

- How does the principle of quasi-option value relate to the decision to build a reservoir in Texas?
  - What information is likely to be revealed in the future?
  - Based on that information, might we make a different decision?

Quasi-option value & building a reservoir?
Examples of Quasi-Option value in every-day decisions

- How does the principle of quasi-option value relate to the decision to choose a major in college?
  - What information is likely to be revealed in the future
  - Based on that information, might we make a different decision?
  - Of course, sometimes waiting is not an option, no matter how valuable that might be.

The End

An overly complicated example

- Project to be completed in 10 years during which time the population will grow.
- Possibility #1: Population growth is strong
  - Project will generate NBs of $1 million per year
    - NPV in year 10 = $20 million, NPV in year 0 = 12.28
- Possibility #2: Population growth is weak
  - Project will generate NBs of $0.5 million per year
    - NPV in year 10 = $10 million, NPV in year 0 = 6.14
- 50-50 chance of each outcome.
Team Task: Expected NB of build now
9. What is the expected (year-10) present value of NBs?
10. What is the expected (year-0) present value of NBs?
11. The build now option: Suppose that the project costs $9 million to build. What is the expected net present value of the project? Does it pass a benefit-cost test?

Team Task: Expected NB of wait-and-see
- Suppose that instead of building now, we wait for 10 years to and then reconsider whether to build the project after knowing whether the population has grown or not.
- If the population growth is strong, would you build the project? What is the PV of NB?
- If the population growth is weak, would you build the project? What is the PV of NB?
- Since there's a 50-50 chance of each outcome, what's the expected NB of the wait and see approach?

Team Task: Dynamically Optimal Choices under Uncertainty
- Taking into account that under the wait-and-see approach the project is completed 10 years later, what is the PV of NB in year zero?
- Which is better, build now or wait-and-see?