Sector-level decisions in a sustainability-constrained economy


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Abstract: Despite two decades of debate, there remains little consensus about what sustainability is, and how it should be achieved. Economists primarily portray sustainability as a macro-level concern, but there has been less attention on the implications of this social objective for policies related to the management of individual resources. This paper derives guidance for sector-level planning and project analysis from a macro-level norm of intergenerational fairness. Optimal sustainable management involves making tradeoffs between sectors. We derive a criteria for sector-level planning and project analysis that help make those tradeoffs without losing site of the sustainability goal.
I. Introduction

The normative foundation for including sustainability in economic analyses has been discussed at length in the literature (see Pezzey 1992). A recent contribution by Asheim, Buchholz and Tungodden (2001) adds powerful support: “only sustainable paths [which we define below] are ethically acceptable whenever efficiency and equity are endorsed as ethical axioms” (p. 267). If sustainability is a societal goal, then two questions immediately follow: How should sustainability be modeled in economic analyses? And, how should policies be designed to achieve sustainability? In this paper we provide a theoretical framework that addresses both of these questions.

Most economists who have considered sustainability present it as an issue of fairness between generations. Whether or not sustainability is being achieved is, therefore, a macroeconomic issue. It is not obvious, however, how macroeconomic diagnoses should be matched with policy prescriptions that actually get things done. Most real world decisions, particularly those related to natural resources, are rather piecemeal, focusing on one economic sector at a time. Mineral reserves are considered separately from fisheries. Policies over our forests are formulated with little consideration for simultaneous developments in the nation’s human-made capital or technological infrastructure. If as a society we seek sustainability, should sustainability also be pursued in all sectors? If not, how might decisions be coordinated to encourage the appropriate tradeoffs?

In this paper we present a model of macroeconomic sustainability from which sector-level policies can be derived. We begin in the next section by providing some background on the economics of sustainability. We then summarize Woodward’s (2000)
model, which deals with sustainability of the economy as a whole and provides the foundation of our analysis. We then extend his model to consider individual sectors within the economy and the tradeoffs among sectors that will be necessary to optimally satisfy a sustainability obligation. We also derive a benefit-cost type criterion that can evaluate whether tradeoffs between current and future welfare are consistent with both efficiency and sustainability.

Requiring all economic sectors to pursue sustainability would almost certainly more than achieve the societal goal, but it would be quite costly in terms of economic efficiency. It would seem more sensible to pursue sustainability more aggressively in those sectors where it can be achieved at less cost while allowing other sectors to follow a less than sustainable path. Our analytical framework addresses this central issue, mapping out an approach that might be taken to pursue economic sustainability through sector-level planning. We conclude with a discussion of the limitations of our model, prospects for future research and policy implications.

II. The economics of sustainability

The modern literature on the economics of sustainability can be traced back to papers by Dasgupta and Heal, Solow, and Stiglitz, all published in the same volume in 1974. These papers used growth-theoretic models to explore whether per-capita consumption could be sustained in an economy with essential non-renewable resources. Solow’s model is particularly relevant here as he drew on the maximin criterion of Rawls (1971), thus combining a formal model of intergenerational equity with a model of economic growth. This motivated Hartwick’s (1977, 972) rule for social sustainability: “Invest all profits or rents from exhaustible resources in reproducible capital such as
machines.” Extensions by Calvo (1978), Dixit, Hammond and Hoel (1980), Krautkraemer (1985), and others generalized the framework of Solow and Hartwick in a number of directions. The ability of actually using Hartwick’s criterion to evaluate the sustainability of an economy, however, was diminished by Asheim (1994) and Pezzey (1994) who found that Hartwick’s rule holds only if the prices already reflect a sustainable economic path. That is, if the Hartwick criterion appears to be satisfied based on prices from a competitive present-value maximizing economy, this does not guarantee that the economy is on a path that will lead to non-declining consumption.

Bromley (1989) and Howarth and Norgaard (1990) turned the discussion in a different direction by focusing on the intergenerational allocation of resource rights. Bromley argued that because future generations are not present, intergenerational externalities cannot be addressed through a Coasean laissez faire approach to economic policies. Rather, the rights of future generations must be established, implying a duty of the current generation. Howarth and Norgaard also highlighted the importance of endowments, showing in an overlapping generations model that the implicit discount rate will depend on the intergenerational allocation of rights; if future welfare is heavily discounted this is equivalent to low allocation of rights to future generations.

An important recent contribution to the sustainability literature was made by Asheim, Buchholz and Tungodden (2001) who provided an axiomatic foundation for sustainability concerns. These authors define generation $t$ as behaving sustainably if, given its endowment, $x_t$, the choices generating utility, $u_t$, leave the next generation with an endowment $x_{t+1}$ such that the all future generations can also achieve at least $u_t$. Although this principle has been proposed for many years, (e.g., Solow 1974) the
normative support for the criterion has typically been drawn from Rawls (1971). However, Rawls himself was somewhat wary of applying his principle of justice in an intergenerational context. By showing that only sustainable paths are consistent with their axioms of equity and efficiency, therefore, Asheim and his colleagues provided a new and solid normative foundation for the consideration of policies intended to achieve sustainability.

III. The model

Our analysis builds on Woodward (2000) in which the economy is composed of an infinite stream of nonoverlapping generations. The welfare of generation $t$ is an additive function of the utility of all future generations, i.e.,

$$U(z, x) = \sum_{s=1}^{\infty} \beta^{s-t} u(z_s, x_s)$$

where $x_t \in \mathbb{R}^m$ is a vector of state variables describing generation $t$'s endowment and $z_t \in \mathbb{R}^n$ is the vector of choices made by the $t^{th}$ generation. The notation $z$ indicates the infinite series of choices, $z_t, z_{t+1}, z_{t+2}, \ldots$, and likewise for $x$. The functions $u(\cdot)$ and $U(\cdot)$ will be called the generational utility and welfare functions, respectively.

We assume that there are $m$ state equations, $x_{t+1}^i = g^i(z_t, x_t), i=1,\ldots,m$, which we will frequently refer to in vector notation,

$$x_{t+1} = g(z_t, x_t).$$

The set of feasible choices, $\Gamma(x_t) \subseteq \mathbb{R}^n$, depends on the state of the economy, $x_t$, and is determined by $K$ intratemporal feasibility equations, $f^k(z; x) \leq 0, k=1,\ldots,K$. We assume that $\Gamma(x_t)$ is nonempty throughout. The technology is assumed to be stationary in that it
does not change exogenously; endogenous progress, however, could certainly be accommodated.

The discount factor, $\beta < 1$, and corresponding utility discount rate, $r$, require careful interpretation. We take $\beta$ to be a primitive feature of the current generation’s preferences, not a variable that might be adjusted to achieve broader social goals. In the spirit of Marglin (1963), $\beta$ is best thought of as the rate of time preference that the current generation would apply in its day-to-day transactions that would include altruism toward future generations. This is equivalent to accepting Koopmans’ (1960) axioms of intertemporal preferences that lead to a discounted present value utility function. Regardless of the interpretation, although $\beta$ does reflect interest in the wellbeing of future generations, we assume that it does not include collective obligation to treat future generations fairly. Following Sen (1997), the self-imposed obligation of sustainability will be treated separately from preferences that determine each generation’s welfare. In other words, the discount factor is not the last word on sustainability.

If each generation maximizes its welfare, then (1) can be written recursively in the form of Bellman's equation,

$$\max_{z \in \Gamma(x_t)} V(x_t) = \max_{z \in \Gamma(x_t)} u(z_t, x_t) + \beta V(x_{t+1}) \text{ s.t. (2).}$$

(3)

The value function, $V(x_t)$, equals the maximum present value of utility that can be achieved given that endowment, $x_t$.

Although a concern for future generations may be reflected in the discount factor $\beta$, solving (3) does not guarantee that intergenerationally fair outcomes will result. It is well established that despite a nonzero discount factor there is no assurance that anything will be sustained (e.g., Clark 1973). Hence, if current decision-makers accept an
obligation to treat future generations fairly, the intertemporal optimization problem must be changed.

Foley (1967) defined an allocation as *fair* “if and only if each person in the society prefers his [or her] consumption bundle to the consumption bundle of every other person in the society” (p. 74). A fair outcome, therefore, is one in which no one envies the bundle of anyone else. Accordingly, Woodward (2000) arrived at the following definition:

Definition: Choices by the current generation are *intergenerationally unfair* if, given \( z_t \), the current generation either expects to envy or be envied by future generations.

This criterion is essentially the same as Riley’s (1980) principle of “intergenerational justice.”

We also are interested in policies that are efficient. Choices \( z_t \) are *intergenerationally Pareto efficient* if there are no feasible alternatives that are weakly preferred by both current and future generations. Because of the altruism imbedded in intergenerational preferences, in many economies the maximization of \( U(\cdot) \) leads to increasing welfare over time. In such economies, to reduce the current generation’s envy of the future would also require reducing the current generation’s welfare; achieving intergenerational fairness would be Pareto inefficient. It seems unlikely to us that society would desire the elimination intergenerational envy if it leads to a Pareto inferior outcome. Consistent with Asheim, Buchholz, and Tungodden (2001), therefore, our set of sustainable policies includes those that lead to increasing welfare:
Definition: A set of choices, $z$, is called sustainable or consistent with sustainability if either it is intergenerationally fair, or any feasible alternative that reduces envy is intergenerationally Pareto inferior.  

Woodward (Proposition 1) showed that fairness to all future generations is satisfied if all generations abide by the recursive sustainability constraint:

$$ u(z_t, x_t) + \beta U\left(\tau z_{t+1}, x_{t+1}\right) \leq U\left(\tau z, x\right) $$  

Adding this constraint to (3) leads to the sustainability-constrained optimization problem:

$$ V^S(x_t) = \max_{x_t} u(z_t, x_t) + \beta V^S(x_{t+1}) \quad s.t. $$  

$$ z_t \in \Gamma(x_t) \quad (5a) $$

$$ x_{t+1} = g(z_t, x_t) \quad (5b) $$

$$ V^S(x_t) \leq V^S(x_{t+1}) \quad (5c) $$

Problem (5) is similar to those proposed by Riley (1980) and Howarth (1995). As in (3), this problem is dynamically consistent since choices made in $t$ take into account the choices that will be made by future generations. The sustainability-constrained value function is denoted $V^S(x)$ to differentiate it from $V(x)$ in (3). Woodward (2000) showed that under assumptions of boundedness and free disposal, (5) can be solved numerically using a successive approximation algorithm. This sustainability constrained optimization problem provides the foundation for our analysis.
IV. Sector-level planning in a sustainability-constrained economy

A. Why care about sector-level sustainability?

We now turn our attention from the societal-level to the question of how sustainability might influence decisions at the sector level. In practice, policy makers frequently focus on pieces of the economy rather than the whole. For example, while the theory of general-equilibrium welfare analysis is well established, it is much more common for benefit-cost studies to concentrate on individual issues, disregarding all but the most obvious intersectoral effects (Boardman et al. 2001). This tendency may not be a severe blow to economic efficiency if the societal objective is to maximize social surplus. While no sector is entirely independent, it is likely that for many problems the magnitude of the external impacts is small relative to the benefits or costs internal to the sector and may tend to cancel each other out.

When a sustainability constraint is introduced into the social planning process, however, the problem of managing sectors independently is substantially more complicated. Although the objective of “sustainable resource management” is frequently invoked in policy debates, in general, management of a single sector can neither guarantee that sustainability is satisfied nor by itself cause sustainability to be violated. Hence, accepting an intergenerational obligation of sustainability does not typically have direct implications for sector-level management. Of course, there are conditions under which a sustainability obligation has immediate implications. Highly nonlinear responses to resource depletion, Leontief or lexicographic preferences, or other extreme characteristics of the economy might imply that sustainability can only be achieved if specific resource are conserved. Under such conditions, protecting the rights of future
generations might require the maintenance of resource stocks above a “safe minimum standard” (Ciriacy-Wantrup 1952) or under a “precautionary principle” (Perrings 1991, Gollier, Jullien, and Treich 2000). Accordingly, some authors have called for strong sustainability, the maintenance of aggregate natural resources without allowing the substitution of human-made for natural capital. Taken to the extreme, this leads to what Goodland and Daly (1996) call “absurdly strong sustainability,” in which no resource depreciation is allowed.

However, if the normative motivation for sustainability comes from an intergenerational compact, then whether a strong sustainability criterion should be used is an empirical question. Does society’s welfare require the maintenance of a sector’s resources? Is there a critical nonlinear juncture that must be avoided? If such extreme conditions are satisfied, then the answer is clear: sustain these resources. But if they are not, then a commitment to sustainability does not by itself provide an imperative for sector-level management. Page (1983) and Howarth (1997) have argued for a common sense approach in which most resources are sustained but substitution possibilities are also admitted. In our analysis we reach a similar conclusion, but go a step further by formally identifying the theoretical tradeoffs between sectors that should be considered when the principle of sustainability is linked with economic efficiency.

B. A sector-level model of a sustainability-constrained economy

Consider the management problem for an economy composed of \( m \) independent sectors. Social utility is assumed to be equal to the sum of the utility generated in each sector, and social welfare is the discounted present value of the infinite stream of social utility, i.e.,
\[ U(z, x) = \sum_{s=1}^{\infty} \beta^s \sum_{i=1}^{m} u^i(z^i, x^i), \]  

(6)

where \( z^i \) is the vector of period \( t \) choices made in the \( i^{th} \) sector with \( z^i = \bigcup_{j=1}^{m} z^j \), and \( z^j \cap z^i = \emptyset \) for all \( i \neq j \), and likewise for \( x \). We assume that the choices can be made in each sector independently so that \( z^i \in \Gamma^i \left( x^i \right) \) and \( x^i_{s+1} = g^i \left( z^i, x^i \right) \) for all \( i \). Under these conditions, choices that maximize the current generation’s welfare would also be those that maximize the present value of utility in each sector, i.e.,

\[ V(x_i) = \sum_i \max_{x_i} \left\{ u^i(z^i, x^i) + \beta V^i(x^i_{s+1}) \right\}. \]

In such an economy, therefore, the welfare-maximizing allocation can be achieved through a system of bureaucratically decentralized management in which each sector is given the task of maximizing its contribution to social welfare.\(^{10}\)

Now consider how such an economy might be managed if a sustainability constraint were accepted. Imposing a societal sustainability constraint,

\[ \sum_i u^i \left( z^i, x^i \right) + \left( \beta - 1 \right) V^s \left( x_{s+1} \right) \leq 0, \]  

on (6) we obtain the Lagrangian,

\[ L = \sum_i u^i \left( z^i, x^i \right) \left( 1 - \lambda \right) + \left[ \beta + \lambda \left( 1 - \beta \right) \right] V^s \left( x_{s+1} \right). \]

(7)

Assuming the feasibility constraints, (5a), do not bind, the first-order conditions of the sustainability-constrained optimization problem with respect to some \( z_k \in z^i \) would be

\[ \frac{\partial u^i}{\partial z_k} \left( 1 - \lambda \right) + \left[ \beta + \lambda \left( 1 - \beta \right) \right] \sum_j \frac{\partial V^s}{\partial x^j} \frac{\partial g^i_j}{\partial z_k} = 0, \]

(8)

where \( x^i_j \in x^i \) and \( g^i_j(\cdot) \in g^i(\cdot) \). Unlike the unconstrained case, where the social optimum can be achieved through the independent solution of sector-level optimization problems, the
sustainability-constrained problem cannot be decentralized because each of the first-order conditions share $\lambda$. That is, the optimal shadow price will affect choices throughout the economy, so that all choices are interdependent across sectors.

To see what is involved, consider a sector-level sustainability constraint that takes the form

$$u' \left( z_i', x_i' \right) + (\beta - 1) \tilde{V}^{si} \left( x_i' \right) \leq y_i',$$  \hspace{1cm} (9)$$

where $\tilde{V}^{si}$ is the value function associated with the sector-level sustainability-constrained problem, and $y_i'$ is the contribution to the societal goal of sustainability that is required of the $i^{th}$ sector. For example, if $y_i' = 0$, then the constraint requires that the sector’s value be maintained over time. The first-order conditions for such a sector problem would be

$$\frac{\partial u'}{\partial z_k} (1 - \lambda_i') + \left[ \beta + \lambda_i' (1 - \beta) \right] \sum_j \frac{\partial \tilde{V}^{si}}{\partial x_j} \frac{\partial g'_i}{\partial z_k} = 0.$$  \hspace{1cm} (10)$$

Comparing equations (8) and (10) we see that the sector-level problem will yield the socially optimal choices if $\lambda'_i = \lambda$, and \(\partial \tilde{V}^{si} / \partial x'_j = \partial V^{si} / \partial x'_j\).

If $\lambda'_i$ is not equal to $\lambda$, this indicates that the marginal welfare cost of satisfying the sector level sustainability obligation differs from the economy average and altering the constraint set could increase social welfare. For example, if $\lambda'_i < \lambda$, then the marginal cost of increasing the sector’s contribution to sustainability is less than the gain that could be achieved by allowing some other sectors to reduce their contribution. A tradeoff could be made that increases current welfare without violating the intergenerational compact. Unless the shadow prices are equated across all sectors, gains from substitution would be
possible. We can now see the problems with requiring sustainable management of all sectors.

For some sectors the present-value maximizing policies lead to economic growth. If such sectors were only required to satisfy exact sustainability, \( y^{Si} = 0 \), such a constraint would not bind and \( \lambda^i \) would equal zero. Hence, requiring exact sustainability in all sectors would tend to overshoot the sustainability goal. Furthermore, such a strict requirement would be unfair to the current generation since it suffers costs for the benefit of future generations that will be better off. Finally, requiring \( y^{Si} = 0 \) for all \( i \) is also inefficient; since the marginal costs of satisfying such constraints would probably vary widely across sectors, suggesting opportunities for gains from substitution would be abundant.

C. Sector-level policy in a sustainability-constrained economy

The requirement that \( \lambda^i = \lambda \) for all \( i \) suggests two ways that sector-level management plans might be developed to approximate the social optimum. First, a social planner could require each sector to maximize sector-level welfare subject to the constraint (9), but then vary the \( y^{Si} \)'s until the shadow prices are equal. This would push the economy in the direction of the sustainability-constrained optimum. However, since the values of \( y^{Si} \) for all \( i \) would need to be determined simultaneously, this approach would essentially render the sector-level problems redundant. Furthermore, to establish well-defined sector-level problems, not only current values of \( y^{Si} \) would be necessary, but all future values of \( y^{Si} \) as well. Hence, we do not view decentralization using \( y^{Si} \) as an attractive way to develop sector-level policies.
A more attractive approach to approximating the solution to the societal sustainability-constrained would be possible if an estimate of the societal shadow price, $\lambda$, could be obtained. This variable might be estimated by making comparisons to the $\lambda$'s from other sectors, or, as we discuss in section V below, based on information about the long-term value of economic assets. For the moment, assume that an estimate of $\lambda$, say $\hat{\lambda}$, can be obtained.\textsuperscript{11}

Using $\hat{\lambda}$ and building on the principle that at the optimum $\lambda^i=\lambda$ for all $i$, well-defined sector-level optimization problems could be defined as follows. Suppose a sector-level optimization problem were specified to maximize sector-level welfare subject to a sector-level sustainability constraint, (9). For a given value of $y^i$, the constrained optimization problem would involve choosing $z^i$ and $\lambda^i$ to solve

$$\max_{x^i} \min_{\lambda^i} u^i (z^i, x^i) (1 - \lambda^i) + \left[ \beta + \lambda^i (1 - \beta) \right] \left( \tilde{V}^s_i (x^i_{i+1}) \right) + \lambda^i y^i.$$  \hspace{1cm} (11)

If the value of $\lambda$ can is approximated by $\hat{\lambda}$, it can be substituted into (11), yielding

$$\max_{z^i} u^i (z^i, x^i) (1 - \hat{\lambda}) + \left[ \beta + \hat{\lambda} (1 - \beta) \right] \left( \tilde{V}^s_i (x^i_{i+1}) \right) + \hat{\lambda} y^i.$$  \hspace{1cm} (12)

Noting that the last term in this problem does not influence the solution, and assuming $\hat{\lambda} < 1$\textsuperscript{12}, this problem can be rewritten,

$$\max_{z^i} u^i (z^i, x^i) + \left[ \beta + \frac{\hat{\lambda}}{1 - \hat{\lambda}} \right] \tilde{V}^s_i (x^i_{i+1}).$$  \hspace{1cm} (13)

This equation suggests a simple sector-level optimization problem, differing from the present-value criterion only in the additional weight that is placed on future welfare. It must be emphasized, however, that the value function is still calculated using the
discount factor $\beta$, i.e., $\tilde{V}^{si}(x'_i) = u'(z'_i, x'_i) + \beta \tilde{V}^{si}\left(g'(z'_i, x'_i)\right)$, where $z'_i$ is the solution to (13). Hence, the sector-level problems would involve making choices as if the decision-maker’s utility discount rate were lower than it actually is. The additional weight given to future welfare reflects the premium required to push the economy in the direction of sustainability.

If sector-level managers solve (13), the resulting policies will lead to a time path for each sector along which the shadow price on the sustainability constraint equals $\hat{\lambda}$. This approach will, therefore, move sectors in the direction of the sustainability-constrained optimum, encouraging capital accumulation and discouraging capital consumption.

In general, however, using (13) to guide policy will not achieve the sustainability-constrained optimum. As noted above, the first order conditions of the decentralized problems are equivalent to those of the social problem optimum if $\lambda^i = \lambda$ for all $i$, and $\partial V^i / \partial x'_i = \partial \tilde{V}^{si} / \partial x'_i$. But this second equality will not typically be satisfied. This error occurs because there is a correlation between $x$ and $\lambda$; if $x t \neq x t+1$ then along the typically optimal path, $\lambda_t \neq \lambda_{t+1}$. Since $\lambda$ is held constant in (13), the slope of $\tilde{V}^{si}(x'_i)$ will differ from that of $V^i(x)$ so that the solution to (13) will not, in general, lead to the optimal choices. The magnitude of this error will depend on the characteristics of the economy. We speculate, however, that in most cases the error that results would be relatively small and the approach would tend to tend to push the economy in the direction of the efficient sustainable path.
Although the planning problem, (13), can be solved by sector-level managers, implementing this policy through taxes or subsidies is more problematic. The difficulty arises because solving the sector problem, (13), requires two discount factors:

\[
\left[ \beta + \frac{\lambda}{1 - \lambda} \right] \text{ used to make choices and } \beta \text{ to calculate the value of those choices.}
\]

Private decision-makers are unlikely to accept a government mandate to act in this “schizophrenic” fashion. For example, if utility were linear in income, a fully decentralized approach would need to be based on (12): current income would be taxed at the rate \( (1 - \hat{\lambda}) \), investment would be subsidized so that decision-makers put a weight of

\[
\left[ \beta + \frac{\hat{\lambda}}{1 - \beta} \right] \text{ on investments, and a lump-sum payment or tax equal to } \hat{\lambda} y_i \text{ would be imposed on the sector, with the time-path of } y_i \text{ and } \hat{\lambda} \text{ are known} \ a \ priori. \text{ Developing the correct suite of taxes and subsidies would be a significant challenge.}
\]

**D. Benefit-cost analysis in a sustainability-constrained economy**

The framework that we have developed above can now be used to present a straightforward and intuitive approach to evaluate policies that are specifically designed to address sustainability. From the Lagrangian, equation (7), and using \( \hat{\lambda} \) as an estimate of \( \lambda \), a policy that leads to a marginal increase in one of the choice variables, \( z_k \), would be included in the optimal set if

\[
\frac{\partial u^i}{\partial z_k} (1 - \hat{\lambda}) + \left[ \beta + \frac{\hat{\lambda}}{(1 - \beta)} \right] \sum_j \frac{\partial V^s_j}{\partial x^j_i} \frac{\partial g^i}{\partial z_k} > 0.
\]

For a discrete policy intervention, and using a linear approximation of the value of changes, the net present value of the benefits to the current generation is

\[
\Delta V^0 = dz_k \cdot \frac{\partial u^i}{\partial z_k} + \Delta V^1,
\]

where \( \Delta V^1 = \sum_j \left( dx^j_i \cdot \frac{\partial V^s}{\partial x^j_i} \right) (dz_k \cdot \frac{\partial g^i}{\partial z_k}) \) is the future net benefits \( i \). Assuming
that \( \lambda < 1 \), the benefit-cost test in a sustainability-constrained economy can be written succinctly,

\[
\Delta V^0 + \frac{\lambda}{1-\lambda} \Delta V^1 > 0. \tag{14}
\]

This inequality can be used to evaluate policies in a sustainability-constrained economy. The first term is the net present value of the benefits of the intervention and is the standard criterion for benefit-cost analysis. The second term is the extra weight given to future welfare in light of the societal commitment to intergenerational fairness. The coefficient, \( \frac{\hat{\lambda}}{(1-\hat{\lambda})} \), is monotonically increasing in \( \hat{\lambda} \) for \( 0 \leq \hat{\lambda} < 1 \); the higher the shadow price on the sustainability constraint, the more weight that is given to future welfare.

Since \( \lambda \geq 0 \), any policy that benefits both the current and future generation would satisfy (14). The more interesting cases occur when there is a tradeoff between current and future welfare. If the policy’s net benefits are positive, \( \Delta V^0 > 0 \), but it imposes costs on future generations, \( \Delta V^1 < 0 \), then (14) becomes

\[
-\frac{\Delta V^0}{\Delta V^1} > \frac{\hat{\lambda}}{1-\lambda}.
\]

The question asked, therefore, is whether the benefits to the current generation are sufficiently great relative to the costs to future generations. For example, suppose this inequality is not satisfied. This means that those options available in the economy to offset the intervention’s negative future consequences would be more costly than the benefits provided by the intervention. If an intervention benefits future generations, but does not pass a standard benefit-cost test, i.e., \( \Delta V^0 < 0 \) and \( \Delta V^1 > 0 \), then (14) becomes

\[
-\frac{\Delta V^0}{\Delta V^1} < \frac{\hat{\lambda}}{1-\lambda}.
\]

In this case the criterion tests whether the policy is a cost-effective means of augmenting future welfare.
The value of $\hat{\lambda}$ indicates the marginal cost of satisfying the sustainability constraint in the economy. If $\hat{\lambda}$ is high, then policies that help the future might be desirable even if their present value is substantially less than zero.

Of course, the benefit-cost test that we propose is strictly appropriate only if the economy is in the neighborhood of the sustainability-constrained optimum. Further, implementing the criterion requires an estimate of $\lambda$, which can only be exactly determined by solving the sustainability-constrained optimization problem, (5). In practice, we anticipate that sensitivity analysis would be probably conducted with respect to this parameter, just as it is routinely done with respect to alternative discount rates. However, as we discuss in the next section, even with quite limited knowledge of the economy, a bound on the value of $\lambda$ can be calculated, suggesting that the range over which sensitivity analysis would be required may be fairly narrow.

V. The shadow price of the sustainability constraint

As we have seen, an estimate of the shadow price on the sustainability constraint, (5c), is central to conducting policy analysis in sustainability-constrained economies. Woodward (2000) showed that for most economies the shadow price on the sustainability constraint is constrained to be less than one. Here we extend that analysis to show that an upper bound on $\lambda$ might be obtained with relatively little knowledge of the nature of the economy.

We begin with the simple case of Page's (1977) corn economy in which economic wellbeing is a function of a single renewable asset, $x$. Utility is strictly increasing and concave in consumption, $z$, and $x_{t+1} = x_t (1 + \gamma) - z_t$, where $\gamma$ is the corn's rate of growth.
It can easily be shown that when $\gamma<r$, the maximization of the present-value of utility will lead to monotonically decreasing consumption and declining stock. On the other hand, the optimal-sustainable choice would be to consume as much as possible while leaving $x_{t+1} \geq x_t$, i.e. $z_t^* = \gamma x_t$. If $\gamma<r$, therefore, it follows that a sustainability constraint would bind. Suppressing the intratemporal feasibility constraint, the Lagrangian for the sustainability-constrained optimization problem for this economy can be rewritten
\[
L = u(z_t)(1-\lambda) + \left[ \beta + \lambda (1-\beta) \right] V^S(x_{t+1}).
\]
Since $z_t^* = \gamma x_t$, this means that $x_{t+1} = x_t$, utility is constant at $u(\gamma x)$, and $V^S(x) = u(\gamma x)/(1-\beta)$.

The Lagrangian’s first-order condition can, therefore, be written
\[
\frac{\partial}{\partial x} u'(\gamma x)(1-\lambda) - \left[ \beta + \lambda (1-\beta) \right] \frac{\gamma}{1-\beta} u'(\gamma x) = 0,
\]
which can be simplified to $\lambda = \frac{1-\beta (1+\gamma)}{(1-\beta)(1+\gamma)}$, or $\lambda = \frac{r - \gamma}{r (1+\gamma)}$.

In this economy, therefore, the value of $\lambda$ is a simple function of $r$ and $\gamma$. At one extreme, when $\gamma=r$, then $\lambda=0$ so that the present-value maximizing policy leads to a constant consumption path and the sustainability obligation would impose no costs on current welfare. At the other extreme, $\lambda$ reaches a maximum of 1 for a cake-eating economy, when $\gamma=0$. Over intermediate values, $\lambda$ increases with the difference between $r$ and $\gamma$. For example, if $r=7\%$, then $\lambda$ would equal 0.13 if and $\gamma=6\%$, but it would increase to 0.55 if $\gamma=3\%$.

To derive a bound on $\lambda$ in a more general economy, we use what Woodward (2000) calls a sustainably-productive alternative. Given the endowment $x$, and a choice vector $z_t$, the vector $z_t^* \in \Gamma(x_t)$ is a sustainably-productive alternative to $z_t$ if for some $\theta>0$
\[ U(\mathbf{z}_{t+j}, \mathbf{x}_{t+j})|_{\mathbf{z}_{t}} \geq U(\mathbf{z}_{t+j}, \mathbf{x}_{t+j})|_{\mathbf{z}_{t}} + \theta \quad \text{for all } j = 1, 2, \ldots. \]

For example, in the corn economy, reduction in consumption period \( t \) would make it possible to increase consumption and utility in future periods, \( t+1, t+2, \ldots \). In contrast, in a cake-eating economy such alternatives do not exist.

If there exists a sustainability productive alternative, we can now show that a bound on the value of \( \lambda \).

Proposition 1: If there exists a choice variable \( z_k \) with the following characteristics satisfied in the neighborhood of the sustainability-constrained optimum, \( \mathbf{z}^*_S \): 1) \( \frac{\partial u}{\partial z_k} \) and \( \sum_j \frac{\partial V^S}{\partial x_j} \frac{\partial g_j}{\partial z_k} \) are continuous in \( z_k \); and ii) a change in \( z_k \) decreases current utility but makes possible an increase in the utility of all future generations such that

\[
0 < \theta \leq -\frac{\partial u_{t+i}}{\partial z_{t+i}^j}/\frac{\partial u_t}{\partial z_t^i} \quad \text{for all } i; \text{ then}
\]

\[
\lambda \leq \frac{r - \theta}{r(1 + \theta)}. \quad (15)
\]

The proof is provided in the appendix.

As in the corn economy, the bound on \( \lambda \) is a function of the ability of current generation to increase future welfare, reflected here in the parameter \( \theta \). There are a couple of important things to note about this result. First, the bound on \( \lambda \) must be derived along the optimal-sustainable path. The conclusion of Asheim (1994) and Pezzey (1994) that prices from a standard present-value maximizing path are not sufficient to evaluate Hartwick’s rule applies here as well; \( \theta \) must reflect a marginal change from the optimal-
sustainable path. For example, one would expect that as an unsustainable economy moves toward sustainability, some sustainably-productive alternatives would be exploited. Hence, the true value of $\theta$ may be less than it might appear if estimated from the perspective of an unsustainable path.

On the other hand, if numerous sustainably-productive alternatives exist, then the bound on $\lambda$ is determined by the most productive of these. If there exist any feasible sustainably-productive alternatives that yield a utility rate of return of $\theta \geq r$, then the sustainability constraint should not bind; both current and future welfare can be increased by exploiting such options and the present value optimal policy should lead to an intergenerationally fair outcome.

We caution that the bound on $\lambda$ appears deceivingly simple and may not represent something that could be estimated precisely in practice. In addition to the well-known difficulties in determining the appropriate discount rate (Weitzman 2001), estimating the value of $\theta$ presents other challenges. It requires estimating the relative increment to the wellbeing of all future generations that could be achieved by making sacrifices today. Further, it must reflect a marginal shift away from the optimal sustainability-constrained path, the nature of which may be difficult to anticipate. Future thought as to how $\theta$ might be estimated is needed.

Finally, it should be stressed that the true value of $\lambda$ would change over time and that policies developed on the assumption that $\lambda$ is constant will typically not achieve the social optimum. However, using an estimate of $\lambda$ would help policymakers to recognize that a sustainability obligation imposes costs on society, but that these costs are limited. We posit, therefore, that an estimate of $\lambda$ can contribute to improved policies in a
sustainability-constrained economy. To the extent that the change in $\lambda$ over time could also be predicted, policy could be further enhanced.

VI. Generalizations, extensions and potential pitfalls

We have used a number of important and restrictive assumptions, and the significance of these and the possible implications of relaxing them should be highlighted. First, the assumptions of separability and additivity that we introduced in section IV are extremely strong. Intersector relationships are widespread: externalities across sectors abound, choices in one sector are typically constrained by conditions in other sectors, and the intertemporal evolution of a sector’s resources is frequently influenced by conditions in other sectors. Because of these linkages, even without a sustainability constraint, independent sector-level planning will not typically lead to socially optimal choices.

Nonetheless, despite the ties that bind economies together, bureaucratically decentralized management is common in both private choices and public policy. The U.S. national forests provide an excellent example of such decentralized management. While the Forest Service's goals and budget are defined by Congress and numerous legal restrictions constrain the Service’s actions, each forest within the National Forest System is required to solve a specific forest-level optimization problem as part of their planning process.\textsuperscript{16} While other agencies typically have less direct control and use incentives or regulations rather than direct management, independent and decentralized planing predominates throughout government.\textsuperscript{17}

Hence, planning often ignores the interrelationships between sectors and suboptimal policies are the inevitable result, but this approach is maintained because of
the practical limitations that decision-makers face. As Weitzman (2001, 261-262) puts it, “full-blown, fully disaggregated, general-equilibrium-style analysis of the impact of environmental projects is undoable.”

When seen in light of the imperfect process through which sector-level policies are typically established, our approach for introducing sustainability concerns in sector-level decisions may not be unreasonable. The optimal sector-level sustainability-constrained policy would find the mix of policies across sectors that would maintain the opportunities of future generations at least cost. But this would require the kind of integrated modelling that is impractical. Our sector-level and benefit-cost approaches provide feasible ways for policy makers to take steps towards sustainability without loosing sight of efficiency.

A second important limitation of our model is that it is for a closed economy. Extending the analysis to a small open economy would be conceptually straightforward, though it would require predictions of future export and import prices. As shown by Asheim (1996), however, extending the notion of sustainability to a large open economy is much more difficult. Beginning to think internationally also forces us to consider the meaning of the word “generation.” Should a generation in our model be regional, national or global? The answer to this question may vary depending on who is being asked.

Finally, our assumption that the economy is deterministic would need to be relaxed. As Woodward (2000) showed, conditions of risk with well-defined probability distributions can be easily incorporated into the model if sustainability is defined as requiring that the expectation that future welfare is greater than that of the current
generation. If uncertainty or ambiguity prevails, where probability distributions are not known, then it is less clear what the meaning of sustainability might be. Howarth (1997, 576) argued for a higher standard under which the current generation would bear “the burden of proof … to demonstrate that their behavior is consistent with intergenerational fairness.” Another approach that warrants further study is the extension of the model to embrace rational decision rules that fall outside the subjected expected utility framework as in Woodward and Bishop (1997).

VII. Conclusions and implications for policy

Concerns about the future adequacy of the environment and natural resources are not new, but they certainly gained momentum during the twentieth century. Intensive use of natural resources during two world wars, the Dust Bowl, the Energy Crisis, ozone depletion, losses of biodiversity, desertification, deforestation and other such events and trends culminated in a persistent, widespread perception that current human activities may so adversely affect the earth’s resources as to compromise the wellbeing of future generations. By the end of the last century, sustainable development was firmly implanted on both academic and political agendas worldwide. But if economic sustainability is a social goal, the task in the twenty-first century will be to define practical strategies to achieve it. Our model implies some basic economic principles that may help guide policy formulation.

*All sectors with potentially large effects on future prospects need to be considered.* That scientists have repeatedly warned of the hazards associated with reduced biodiversity and global warming is a clear signal that the sectors impinging on biological and climatic resources need attention. But at the same time, capital
accumulation and progress in science and technology will be part of an efficient strategy to achieve sustainability. Failure to address technological progress is a significant oversight in much of the current sustainability literature (Weitzman 1997). Our model recognizes that future generations will inherit more than just natural resources. Appreciating both the extent and the limitations of substitutability between these sectors is important.

*Close attention to the costs of achieving sustainability goals is justified.* As our model shows, requiring exact sustainability in all sectors would likely overshoot the goal and be unnecessarily expensive. To efficiently pursue sustainability, public decision-makers should seek out opportunities to augment future welfare at relatively low sacrifice to current welfare. As a rule, they should be wary of projects with low benefit-cost ratios that are promoted on the grounds that they promote sustainability.

The benefit-cost criterion that we propose in equation (14) offers a straightforward approach to evaluating interventions in a sustainability-constrained economy. Are current costs warranted in light of the improvement in future wellbeing generated? Is it acceptable for a project to diminish future wellbeing in light of current benefits that are created? Such questions are at the heart of applied policy analysis of sustainability and can be considered through this lens.

*Manipulation of the discount rate may not be the best route to a sustainable economy.* In our model, $\beta$ from the welfare function interacts with the state equations so that at the optimum intertemporal marginal rates of substitution would equal intertemporal marginal rates of transformation. The slopes at these points of tangency imply the interest rates that would be observed in the economy. The observed interest
rate, therefore, is the result of optimization, not a variable to be manipulated in order to achieve sustainability or any other policy goal.

Farmer and Randall (1997) argued convincingly against adjusting interest rates in a misguided attempt to enhance sustainability would tend to do more harm than good by disrupting intergenerational capital markets. As Nordhaus (1999, p. 145), summarizes,

…ad hoc manipulation of discount rates is a very poor substitute for policies that focus directly on the ultimate objective. … Focusing on ultimate objectives shows trade-offs explicitly, makes the cost of violating benefit-cost rules transparent, and allows public decisionmakers to weigh options explicitly rather than allowing technicians to hide the choices in abstruse arguments.

Accordingly, if the policy goal of treating future generations is accepted, it should be incorporated in a fashion that states the goal explicitly and makes clear the costs of achieving that goal. Interest rates may be affected when sustainability constraints are imposed, but this is a result, not a cause.

*Policies in a sustainability constrained economy could take many forms.* Our model did not investigate which sorts of policy instruments might best be used to achieve economic sustainability. Our analytical approach is not intended to recommend command and control policies. Such instruments may be called for in some cases, but they have a bad name in environmental economics, and rightly so. All the work on incentive-based tools would also be relevant in designing instruments to enhance sustainability.
Sustainability is increasingly be called for as a goal for government and resource managers. While the normative arguments behind an intergenerational compact seem intuitively reasonable there has been little progress in establishing how this societal goal might impact applied public policy. Many economists see abundant opportunities for substitution, making them skeptical of sector-level rules to address societal-level concerns about sustainability. Nonetheless, if there is a societal commitment to address sustainability, as a practical matter it will require many decisions that are made at the sectoral level. The model we present here suggests how policy makers might move the economy in the direction of sustainability without ignoring economic efficiency.
VIII. Appendix: Proof of Proposition 1

By condition ii) we know the relative increase in all future generation's utility is at least \( \theta \), hence,

\[
- \sum_i \frac{\partial V^s(x_{t+i})}{\partial x^i_{t+i}} \frac{\partial x^i_{t+i}}{\partial z_k} \leq \frac{\theta}{1 - \beta}.
\] (16)

Together, conditions i and ii ensure that any intra-temporal feasibility constraints on \( z_k \) are not binding at the sustainability-constrained optimum. Therefore, the first-order condition of (5) with respect to \( z_k \) will hold with an equality so that

\[
\frac{\partial u(z_t, x_t)}{\partial z_k} (1 - \lambda) + \left[ \beta + \lambda (1 - \beta) \right] \sum_i \frac{\partial V^s(x)}{\partial x^i} \frac{\partial x^i}{\partial z_k} = 0.
\] (17)

Combining (16) and (17), we obtain

\[
- \frac{\partial u(z_t, x_t)}{\partial z_k} \left( \sum_i \frac{\partial V^s(x)}{\partial x^i} \frac{\partial x^i}{\partial z_k} \right) \leq \frac{\beta + \lambda (1 - \beta)}{1 - \lambda} \leq \frac{1 - \beta}{\theta},
\]

which, for \( \lambda < 1 \), can be simplified to (15), completing the proof.\(^18\)
References


Footnotes

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1 Our use of the Woodward model places us squarely in the “weak” sustainability camp. That is, it allows substantial substitution possibilities among natural resources and between resources, capital, and labor. It is conceivable that some resources may be quite essential. If so, then “strong” sustainability measures such as Ciriacy-Wantrup’s (1952) safe minimum standard of conservation would be called for. Farmer and Randall (1998) consider this case; we will not deal with it here.

2 Toman, Pezzey and Krautkraemer (1995) provide an excellent survey of the neoclassical sustainability literature.

3 Scott (1999) builds on this theme by evaluating the insights that can be drawn from the legal doctrine of trusts, i.e., the legal rights and obligations of those who manage assets for others.

4 We use a deterministic model for ease of presentation. As shown by Woodward (2000) risk can easily be incorporated into this framework. Extensions to conditions of uncertainty as in Woodward and Bishop (1997) are discussed in the conclusion.
See Thomson and Varian (1985) for a summary of related definitions of fairness.

Chavas (1994) represents an application of the framework of fairness to policy analysis in agriculture.

The choice $z_t$ is weakly preferred to $z_{t'}$ by generation $t$ if $U \left( \cdots, z_t, \cdots \right) \geq U \left( \cdots, z_{t'}, \cdots \right)$. 

This definition is slightly different from that used by Woodward (2000), but leads to the same outcomes except in very simple and unproductive economies.

By bounded, we mean that the state space, $X$, and the set of feasible choices, $\Gamma(x)$, are closed and bounded and the instantaneous utility function $u(x, z)$ is defined over the entire domain. These conditions are required for numerical solution, though they are not required for the existence of a solution in general.

Of course, authors that promote sustainable resource management may not be referring to a social norm of intergenerational fairness. Some authors use the terms referring to the importance of recognizing environmental externalities in policy, or reflecting a desire to protect communities or cultures. Because our focus is on sustainability as intergenerational fairness, these other issues are not addressed here.

We owe the expression “bureaucratically decentralized” to Gabriel Lozada who suggested it to distinguish our meaning from the typical use of the word “decentralized” which refers to an economy in which all decisions are made by private agents.

The value of $\lambda$ will typically change over time so that the true object of analysis would be an estimate of the series $\lambda_t, \lambda_{t+1}, \ldots$. In practice, however, just finding a single estimate of $\lambda$ for all $t$ may be a substantial challenge. Hence, our discussion focuses on
policies formulated assuming that $\lambda$ is constant over time. Relaxing this assumption would be relatively straightforward.

12 As will be seen in section V, $\lambda < 1$ in all but the most unproductive economies.

13 It would be erroneous to simply use $\left[ \beta + \lambda / (1 - \lambda) \right]$ in place of $\beta$ as this would ignore the term $\lambda_y^i$ in (12). From a practical perspective, for high values of $\lambda$ this augmented discount factor could exceed one, leading to an unbounded dynamic optimization problem.

14 Pezzy and Toman (forthcoming) use the term “schizophrenic” to describe the apparent contradiction between an interest in maximizing welfare and a concern for sustainability. We use the term here to highlights how such schizophrenia might express itself in practice.

15 The implication of this assumption is quite similar to that achieved by Asheim’s (1991) notion of productivity, although he considers contributions to a consumption stream in a one-dimensional economy, rather than utility in a multi-dimensional case.

16 This process is not just figurative; a large mathematical programming model with a present-value objective calibrated for each forest plays a central role Forest Service planning (Iverson and Alston 1986).

17 There is a vast literature that seeks to understand the reasons for and consequences of decentralized government planning (e.g., Oates 1972, Tiebout 1956). We do not attempt to develop a formal model of why government policy might be decentralized. Rather, we assume that for practical reasons, a decentralized program is used despite the existence of a well-defined social planner’s problem.
Note that an interior solution to (5) is not possible if $r < \theta$ since under this condition both current and future welfare could be improved by changing $z_n$. 