Optimal-Sustainable Management of Multi-Species Fisheries: Lessons from a Predator-Prey Model

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Abstract: In this paper we define fisheries management as sustainable if it does not lead to a decline in net present value of the fishery. This definition is based on the principle of intergenerational fairness. If the sustainability, or intergenerational fairness, is held as an obligation by fishery managers, then the traditional present-value maximization objective would be constrained. Using numerical solutions to a simple predator-prey model, we explore how the optimal-sustainable management of this fishery would differ from management that seeks to maximize the present value of the benefits. General lessons for fishery management are discussed.

keywords: Sustainability, Fisheries management, Dynamic optimization
Theories of optimal management have played a useful role in applied fisheries management. While managers rarely attempt to apply such criteria exactly, these theoretical notions provide a useful guideline. Certainly the notion of maximum sustainable yield played this role for most of this century and optimal sustainable yield, in which social, economic and biological benefits are recognized, is the guiding principle today (Roedel [1975]).

In most economic models, the criterion that lies behind prescriptions for optimal management is maximization of the present value of net benefits. Anderson [1986, 32] sums up the standard perspective, "Put succinctly, proper use of a fish stock requires that resources be utilized to exploit it such that the present value of future net returns is maximized." This perspective is reflected in virtually all economic models of optimal fishery management (e.g., Clark [1976], Conrad [1995]).

In contrast to the economic models, the maximization of net benefits does not seem to be the only policy objective that motivates most fisheries management (Charles [1994]). The terms that are used to describe appropriate fisheries management today de-emphasize the efficient management of a stream of economic benefits and focus on the issue of sustainability of the fishery. For example, consider the current interest in ecosystem management. Definitions of ecosystem management almost uniformly include sustainability among the array
of policy objectives\(^1\) and Christensen et al. [1996, 666] state, "sustainability must be the primary objective" of ecosystem management.

When sustainability is introduced as a policy objective, however, the standard models used by economists are no longer directly applicable. For example, it is well established that under extreme circumstances the "optimal" fishery management can involve driving resource to extinction (Clark [1973]). More generally, there is nothing in the present-value criterion that ensures that anything will be sustained.

In this paper we propose a definition of sustainable fisheries management that is appropriate in the context of a multi-dimensional resource. We will argue that this framework provides useful conceptual guidance for fisheries managers seeking to achieve both efficiency and sustainability. We apply the model to the theoretical problem of the management of a two-species fishery involving a predator-prey relationship. We then discuss how management strategies would be altered by imposing a sustainability constraint on the present value objective.

Results of this theoretical exercise will help us to address two questions about sustainable fisheries management: What is it that should be sustained? and, What sorts of tradeoffs between the species in the fishery are admissible? We close with a discussion of how the model sheds light on the applied problems of fisheries management.

\(^1\) Schramm and Hubert [1996, 6] list four definitions of ecosystem management, two of which include the word, "sustainable," and a third that calls for protection the long-term integrity of the system.
An economic definition of sustainability

The most basic question that any definition of sustainability must answer is, What is that should be sustained? The fact that this question has no immediately obvious answer is apparent in the breadth of definitions of sustainability and sustainable fisheries management (e.g., WCED [1987], Charles [1994]). Still, at the heart of virtually all concerns about sustainability is a notion of intergenerational fairness or equity (see Pezzey [1989]). The definition of sustainable fisheries management that we use is based on this principle.

The notion of fairness has been studied extensively by economists (Foley [1967], Varian [1974], Baumol [1986])². The pervasive principle in this literature is that an allocation is fair "if and only if each person in the society prefers his [or her] consumption bundle to the consumption bundle of every other person in the society" (Foley [1967], 74). There are two important components to this definition. First, is that fairness implies a lack of envy -- if an allocation is completely fair then no agent would be envious of the allocation of any other agent. Second, the definition allows for the possibility that that some elements of the endowment might be substitutes for others. Hence, even if two agents have very different allocations it still may hold that neither agent envies the other.

We now consider how the principle of fairness might be applied in the intertemporal context of a fishery. Fisheries are valued because they make possible a stream of benefits to society. These benefits include fish, recreation and a range of ecosystem services. We value fisheries not only because of the benefits that they can generate immediately but because of the promise of what they will provide us in the future. The value of a fishery at a particular point in time, therefore, is a measure of the stream of benefits that can be generated given the state
of the fishery at that time. If this value declines over time, then stakeholders that come later in
time will be envious of earlier stakeholders. Hence, if current management leads to a fall in the
value of the fishery, then it follows that management is unfair to future stakeholders in the
fishery or, as we will use the term, is inconsistent with sustainability.

The notion of value used above is general but is not sufficiently precise to be directly
operational. In this paper we employ the standard economic interpretation of value based on
the discounted present value of the benefits (Anderson [1986]). That is, for the exploratory
purposes, we define the value of a fishery, $V(x)$, as equal to the present value of the stream of
surplus generated by the fishery,

$$V(x_0) = \max_{\{x_t\}_{t=0}^\infty} \sum_{t=0}^\infty \beta^t U(x_t, z_t)$$

where $x_t$ is the state of the fishery in period $t$, $z_t$ is the vector of management decisions made in
period $t$, $\beta$ is the discount factor$^3$ and $U(\cdot)$ is a function that captures economic benefits
achieved in the fishery during the period $t$. In the examples below we consider only the
producers’ surplus generated by commercial harvests from the fishery. Other market and non-
market benefits could, in principle, be incorporated into the valuation of the intratemporal
benefits.

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$^2$ See Thomson and Varian [1985] for a review of this literature.

$^3$ If $\rho$ is the discount rate, the $\beta=1/(1+\rho)$.

$^4$ This framework can easily be adopted to situations of risk, in which $x$ develops stochastically
over time. In this case the value function would be

$$V(x_0) = \max_{\{x_t\}_{t=0}^\infty} E_t [\sum_{t=0}^\infty \beta^t U(x_t, z_t, \epsilon_t) = \max_{\epsilon_t} E_t [U(x_t, z_t, \epsilon_t) + \beta V(x_{t+1})]]$$
Given the interpretation of sustainability that we have provided above and our choice of a measure of value, an operational definition of sustainable fisheries management follows. Namely, the choices in period $t$ are said to be consistent with sustainability if

$$V(x_t)_{t_i} \leq V(x_{t+1})_{t_i}.$$  

(1)

If a fishery manager adheres to a binding obligation of sustainability, then management goal would be to maximize the present value of surplus without violating (1). The resulting constrained optimization problem can be written in the recursive form,

$$V^S(x_t) = \max_{z_t} \left[ U(x_t, z_t) + \beta V^S(x_{t+1}) \right] \text{ s.t. } V^S(x_t)_{t_i} \leq V^S(x_{t+1})_{t_i},$$  

(2)

where the value function is written $V^S(\cdot)$ to differentiate it from the value function that arises from the unconstrained maximization of the present value of the benefits.

The sustainability constraint in (2) is intended to ensure that fairness is achieved from one generation to the next. To apply our sustainability criterion, a decision would need to be made regarding the time lapse from $t$ to $t+1$. In simple "well-behaved" systems, a time lapse of a single year may be appropriate as use of the resource would be fair to future generations only if it satisfied (1) from one year to the next. In more complicated systems a longer time horizon might be appropriate. For example, in cyclical systems or fisheries in which pulse fishing is optimal, a longer perspective would be required. In general, decisions about what length of time that is considered to be a "generation" would be made prior to the formulation of the model.

Note that this sustainability criterion is not equivalent to non-declining utility. A time-path which leads to a decline in income for a short period followed by increases in utility later can be consistent with (1).
The solution of dynamic optimization problems are difficult in general and the sustainability-constrained optimization problem, \((2)\), is no exception. When a closed-form analytical solution for most dynamic optimization problems is not possible, numerical methods are required to approximate their solutions (Rust [1996], Judd [1998]). This also holds for the sustainability-constrained optimization problem considered here. In an appendix we discuss the details of an algorithm that can be used to solve such problems and the restrictions that must be imposed in order to allow their solution.

In the remainder of this paper we apply this model of sustainability-constrained optimization to the case of a simple predator-prey fishery. While the fisheries model is intended to be heuristic and is not empirically based, it is most representative of fisheries in which two commercial species dominate. Nonetheless, as we will discuss in the conclusion, some general lessons for fisheries management can be obtained from this stylized example.

**Description of the Predator-Prey model**

The model that will serve as the basis for our discussion is composed of two species that interact in a predator-prey relationship. Both the prey species, \(x_1\) and the predator, \(x_2\), are harvested commercially.\(^6\) Effort expended on the \(j^{th}\) species in period \(t\), \(z_{jt}\), produces harvests, \(h_{jt} = x_{jt} \left(1 - e^{-q_j z_{jt}}\right)\) where the parameter \(q_j\) indicates the effectiveness of effort in capturing \(j^{th}\) species. We assume that indirect harvests or bycatch are negligible. The stocks that remain after harvesting evolve according to the predator-prey model of Clark [1985, 195]:

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\(^6\) While no specific units are used here, \(x_1\) and \(x_2\) are measures of the biomass of the two species.
\[ x_{1t+1} = (x_{1t} - h_{1t}) \cdot [1 - m_1 - \alpha (x_{2t} - h_{2t})] + r \] (3)

and

\[ x_{2t+1} = (x_{2t} - h_{2t}) \cdot [1 - m_2 + \phi \alpha (x_{1t} - h_{1t})] . \] (4)

The parameters \( m_1 \) and \( m_2 \) are the autonomous mortality rates of each species while \( r \) is the rate of density-independent recruitment for the prey species. The predator-prey interactions are determined by the parameters \( \alpha \) and \( \phi \) with \( \alpha (x_{2t} - h_{2t}) \) being the percent of the predator population that is consumed in any given period and \( \phi \) is the rate at which the predator converts the prey into its own biomass.\(^7\)

For the unexploited fishery there are two equilibria:

\[ x_1 = r/m_1, \; x_2 = 0 \quad \text{and} \]

\[ x_1 = m_2/\phi \alpha, \; x_2 = r\phi/m_2 - m_1/\alpha. \] (5)

When fishing effort is introduced, \( h_i > 0 \), the steady-state values become

\[ x_1 = \frac{r \cdot e^{qz_1}}{m_1 + e^{qz_1} - 1}, \; x_2 = 0 \]  

and, for interior solutions,

\[ x_1 = x_1^{ss}(z) = \frac{e^{qz_1}(-1 + e^{qz_2} + m_2)}{\alpha \phi}, \; \text{and} \] (6)

\(^7\) This model has numerous important shortcomings and is used only for illustrative purposes. Recruitment to both species, for example, is inconsistent with reasonable biological models. Hence, the specific results of the model are, at best, indicative of patterns that might be found in more realistic models.

\(^8\) The values of the parameters used in the numerical simulations below and presented in Table 1 were chosen specifically to yield steady-state unexploited stocks of 4.0 and 2.0 for \( x_1 \) and \( x_2 \) respectively.
\[ x_2 = x_2^{SS}(z) = \frac{e^{\theta z}}{\alpha\left(1 + e^{\theta z} + m_2\right)}. \]

\[ \left(-1 + e^{\theta z} + e^{2\theta z} - e^{2\theta z_1 + 2\theta z_2} + m_1 - m_1 e^{2\theta z_1} + m_2 - m_2 e^{2\theta z_2} - m_1 m_2 + \alpha \phi r\right). \] (7)

For reasons of simplicity, we consider the simple case in which the fishery is valued only for the profits that it is able to generate in each period, i.e.,

\[ U(\cdot) = \pi(x_1, z_1, x_2, z_2) = p_1 h_1 + p_2 h_2 - c_1 z_1 - c_2 z_2, \]

where \( p_1 \) and \( p_2 \) are fixed prices of the two species per unit of output and \( c_1 \) and \( c_2 \) are the unit costs of effort on expended on each species.

Using the standard present-value criterion, the manager's objective is

\[ \max_{x_1, z_1, x_2, z_2} \sum_{t=0}^{\infty} \beta^t \pi(x_1, z_1, x_2, z_2) \text{ s.t. } (3) \text{ and } (4). \]

The value of the stock at time \( t=0 \) can, therefore, be written recursively in the form of Bellman's equation,

\[ V(x_1, x_2) = \max_{z_1, z_2} \left[ \pi(x_1, z_1, x_2, z_2) + \beta V(x_{1r+1}, x_{2r+1}) \right] \text{ s.t. } (3) \text{ and } (4). \] (8)

Policies that satisfy this objective will be called PV-optimal.

Our eventual goal is to identify the full set of policies that would arise across the state space. However, we first solve for the PV-optimal steady state as a useful first step in identifying the nature of those policies. Making standard regularity assumptions, the optimal choices, \( z_1 \) and \( z_2 \), can be obtained with the first order conditions of (8)

\[ \partial V / \partial z_i = \partial U / \partial z_i + \beta \sum_j \left( \partial V / \partial x_{j+1} \cdot \partial x_{j+1} / \partial z_i \right) = 0, \quad i = 1, 2. \] (9)

Let \( \lambda_{zt} = \partial V / \partial x_{zt} \), so that

\[ \lambda_{zt} = \partial U / \partial x_{zt} + \beta \sum_j \left( \lambda_{zj+1} \cdot \partial x_{j+1} / \partial x_{zt} \right), \quad i = 1, 2. \] (10)
At a steady state $\lambda_j = \lambda_{j+1} = \lambda_j$ so that (9) and (10) can be rewritten,

$$
\frac{\partial V}{\partial z_i} = \frac{\partial U}{\partial z_i} + \beta \sum_j \left( \lambda_j \cdot \frac{\partial x_{j+1}}{\partial z_i} \right) = 0, \quad i = 1, 2
$$

(11)

and

$$
\lambda_i = \frac{\partial U}{\partial x_{it}} + \beta \sum_j \left( \lambda_j \cdot \frac{\partial x_{j+1}}{\partial x_{it}} \right), \quad i = 1, 2.
$$

(12)

Solving (6), (7), (11) and (12) simultaneously for $x_1$, $x_2$, $z_1$, $z_2$, $\lambda_1$, and $\lambda_2$, it is possible to identify the steady state equilibrium.\(^9\) When no interior solution is optimal, then the equations in (5) would be substituted for (6) and (7).\(^10\)

In Table 1, we present the steady-state PV-optimal values of $x_1$, $x_2$, $z_1$, $z_2$, $h_1$ and $h_2$ given the parameter values listed in the table. That table also presents the elasticities of each of these variables with respect to changes in the parameters. The most striking result in this table is that the elasticities are quite high -- changes in the parameters' values would lead to significantly different results at the steady state. This is particular true of harvest rates and, at the extreme, a one-percent increase in the price of the prey would lead to a 7% increase in steady-state harvests of that species and 9% decrease in harvests of the predator. The high elasticities with respect to the biological parameters are particularly disturbing since the uncertainty surrounding these parameters is likely to be significant and they are likely to change

\(^9\) Since the equations in this system are nonlinear, it is not immediately clear that there is a unique steady state. However, as will be seen in the numerical simulations below, the system converged to a single steady state from a wide range of starting values.

\(^10\) In conducting sensitivity analysis it was found that for some parameter values the solution to this system of equations yields values with $z_1$ or $z_2$ less than zero. To avoid this, Kuhn-Tucker conditions would need to be included in the problem. The additional complication required, however, would add little to our understanding of the questions under consideration in this paper so only parameter values that lead to interior solutions were considered.
over time (Parma [1990]). While the magnitudes of the elasticities are quite sensitive to the parameter values, their signs were found to be consistent over a wide range.

Table 1
Numerically calculated elasticities of steady state stock, effort and harvest levels of the prey (1) and predator (2) with respect to model parameters

<table>
<thead>
<tr>
<th>Parameter values</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$z_1$</th>
<th>$z_2$</th>
<th>$h_1$</th>
<th>$h_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>steady state</td>
<td>1.24</td>
<td>0.62</td>
<td>0.12</td>
<td>0.04</td>
<td>0.14</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Elasticities $^{11}$ (%Δ in variable/%Δ in parameter)

<table>
<thead>
<tr>
<th>Parameter values</th>
<th>$\varepsilon_{x1}$</th>
<th>$\varepsilon_{x2}$</th>
<th>$\varepsilon_{z1}$</th>
<th>$\varepsilon_{z2}$</th>
<th>$\varepsilon_{h1}$</th>
<th>$\varepsilon_{h2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ecological</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m_1$</td>
<td>0.48</td>
<td>-0.52</td>
<td>0.00</td>
<td>-2.16</td>
<td>-1.40</td>
<td>-2.55</td>
</tr>
<tr>
<td>$m_2$</td>
<td>0.11</td>
<td>0.05</td>
<td>-1.52</td>
<td>2.68</td>
<td>-6.27</td>
<td>2.57</td>
</tr>
<tr>
<td>$r$</td>
<td>0.80</td>
<td>0.74</td>
<td>-0.17</td>
<td>4.40</td>
<td>1.17</td>
<td>4.88</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.20</td>
<td>-0.18</td>
<td>1.69</td>
<td>-3.62</td>
<td>7.20</td>
<td>-3.58</td>
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<tr>
<td>$\phi$</td>
<td>0.60</td>
<td>-0.06</td>
<td>1.91</td>
<td>-3.49</td>
<td>7.89</td>
<td>-3.34</td>
</tr>
<tr>
<td>$q_1$</td>
<td>1.00</td>
<td>-0.21</td>
<td>-1.86</td>
<td>6.38</td>
<td>-6.36</td>
<td>5.79</td>
</tr>
<tr>
<td>$q_2$</td>
<td>0.60</td>
<td>0.12</td>
<td>-1.02</td>
<td>0.71</td>
<td>-0.79</td>
<td>0.79</td>
</tr>
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<td>economic</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.90</td>
<td>-0.01</td>
<td>1.30</td>
<td>-2.91</td>
<td>2.05</td>
<td>-2.75</td>
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<tr>
<td>$p_1$</td>
<td>1.00</td>
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<td>7.89</td>
<td>-6.71</td>
<td>7.22</td>
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<tr>
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<td>0.20</td>
<td>0.57</td>
<td>0.31</td>
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<tr>
<td>$c_1$</td>
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<td>1.86</td>
<td>-7.38</td>
<td>6.36</td>
<td>-6.73</td>
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<tr>
<td>$c_2$</td>
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<td>-0.12</td>
<td>1.06</td>
<td>-0.77</td>
<td>-0.09</td>
<td>-0.85</td>
</tr>
</tbody>
</table>

PV-optimal management of the Predator-Prey fishery

The steady state values presented in Table 1 do not tell the whole story. Once the fishery is at the steady state, the PV-optimal policy presented in the table leave it unchanged

$^{11}$ The elasticities were obtained based on the solutions to the system of steady state equations at the base-case parameter values and at values both 0.1% below and 0.1% above. The average elasticity over both the positive and negative shocks is presented here.
from period to period. Hence, the sustainability criterion is satisfied at the steady state. Since our principal concern in this paper is the consideration of the sustainability of policies, we will be more interested in policies at points away from the steady state.

To evaluate policies away from the steady state, we solved for the full array of policies using a successive approximation algorithm. The details of the numerical methods used to solve for the policies are elaborated in an appendix. Figure 1 presents a "phase portrait" of numerically generated PV-optimal paths beginning at the points indicated with diamonds. From all points in the state space PV-optimal management leads to the steady-state levels indicated in Table 1. Adjustment to the steady state is quite rapid with most paths reaching within one-percent of the steady state in less than ten periods, some in as little as one period. Adjustment toward the steady state is significantly slower if the initial stocks are quite low.

\[\text{12 The base-case parameter values were chosen only because their relative magnitudes seemed plausible and because the led to interior solutions for the variables } x_1, x_2, z_1 \text{ and } z_2. \text{ They are not intended to be parameters of any true fishery.}\]
When the fishery is heavily stocked with both species (the northeast corner of the figure) the PV-optimal strategy leads to rapid reductions in both the predator and the prey. At the other extreme, in the southwest corner of the figure, the optimal policy leads to accumulation in both stocks, with the growth in $x_1$ taking place first. When the initial stocks are located either in the northwest or southeast corners then, relative to the steady state, one of the species is overstocked and the other is understocked. The optimal policies from such points lead to reductions in the one species and increases in the other. It is at these points where the question of sustainability is most interesting—when is this substitution of one element of the asset vector for the other consistent with sustainability?

We can begin to answer this question by looking at the PV-optimal value function, $V(x_1,x_2)$, in Figure 2. This surface shows the present value of the stream of net benefits that can be obtained starting at each point in the state space. The value function is monotonically...
increasing in $x_1$ so that, holding $x_2$ constant, policies that lead to positive changes in $x_1$ are consistent with sustainability, while those that lead to reductions in $x_1$ will be inconsistent with sustainability. Due to the predator's dual asset-nuisance characteristic, however, $V(x_1,x_2)$ is not monotonic in $x_2$. When predator stocks are low, the total value of the fishery actually declines as the biomass of the predators increases because the deleterious effect on the prey species outweighs the benefits to the predator fishery. Only after the stock reaches a level above about 0.6 do increments to that stock actually make the aggregate fishery more valuable.

Figure 2 is helpful in beginning to determine which of the paths in Figure 1 are consistent with sustainability. As long as $x_2$ is above 0.6, any path that leads to increments in both species will be consistent with sustainability. As seen by the contours of the value function in the figure, if $x_2$ is above 0.6 then slight reductions in the predator stock can be sustainable if there is a coincident increase in the stock of the prey. Likewise, we see that significant reductions in $x_1$ can be consistent with sustainability if $x_2$ grows slightly. When the
predator stock is below 0.6, however, then the predator's nuisance quality dominates and growth in the stock of this species actually leads to a decline in the value of the fishery. Hence, some of the paths in Figure 1 that lead to increments to both stocks actually reduce the value of the fishery along their approach to the steady state.

**Optimal-sustainable management of the Predator-Prey fishery**

We now turn to the discussion of policies when the fishery agency chooses to obligate itself to satisfy the sustainability criterion. In this case the management agency would seek to maximize the present value of the harvests subject to the sustainability constraint,

$$V(x_t)_{L_z} \leq V(x_{t+1})_{L_z}. \quad (1)$$

Policies that solve the sustainability-constrained optimization problem will be referred to as S-optimal.
There is no a unique steady state to the sustainability-constrained problem for this fishery and it is not possible to find an analytical solution to this problem. Hence, as discussed above, this problem must be solved numerically. In Figure 3 we present a variety of time paths that follow from the S-optimal policy rule. Comparing this figure with Figure 1 we see how the introduction of the sustainability constraint alters the management of the fishery. The S-optimal paths that begin with $x_2$ below the PV-optimal steady state end at essentially the same point, though the route taken differs somewhat because of the non-monotonic portion of $V$ noted above. When $x_2$ starts above 0.6, however, the S-optimal paths follow along iso-value function contours until a steady state is reached. The actual steady state that is reached is a function of the initial stocks and the locus of steady states slopes from the PV-optimal steady state in a northwesterly direction as indicated by the dotted line. Hence, by identifying where tradeoffs are possible and can be exploited without reducing its value, Figure 3 clarifies the meaning of sustainability for this multidimensional resource.
In figures 4 and 5 we present the PV- and S-optimal harvest levels as functions of the stocks, $x_1$ and $x_2$. We see that the introduction of the sustainability constraint has significant effect on the policy choices for both species. As seen in Figure 4, harvests of the prey stock do not change significantly when the predator is relatively scarce. However, as the predator stock increases, the maintenance of its food source, $x_1$, grows so that sustainability is achieved by reducing prey harvests substantially. Not until the predator stocks are substantially above the PV-optimal steady state do prey harvests begin to increase again.
Figure 5
PV- and S-optimal harvests of the predator

The PV- and S-optimal harvests of the predator, presented Figure 5, demonstrate that the introduction of the sustainability constraint has a dramatic affect on the harvests of this species. In the unconstrained model the optimal policy involved harvests only if $x_2$ was greater than 0.6, the point at which the marginal value of increased predator stocks becomes positive. In the sustainability-constrained model, however, it is in this range that predator harvests are most significant. Below 0.6, harvests of the predator are used to keep that stock low until sufficient growth in $x_1$ can occur so that the value of the stock can be maintained. On the other hand, if the predator stock exceeds 0.6, then its value to the fishery becomes positive so that harvests are sharply reduced to avoid total reducing the value of the fishery.
Particularly given the harvests levels presented in Figure 5, it is not surprising to find that the introduction of the sustainability constraint comes at a significant cost. This cost is visible in Figure 6 where we present the total value of the fishery in the unconstrained and constrained models, $V(x_1,x_2)$ and $V^s(x_1,x_2)$ respectively. As expected, near the PV-optimal steady state, the sustainability constraint has no impact on the value of the fishery. However, for stocks away from the steady state, sustainability comes at substantial costs. As seen in Figure 3, imposing the sustainability constraint leads to a result that for low $x_2$ and high $x_1$ leads to a much slower and roundabout approach to the steady state. This detour leads to a substantial reduction in the present value of harvests -- up to 43%. At the other extreme, where $x_2$ is high, the constraint leads to sharp reductions in harvests of both species and a 40% decline in the total value of the fishery. Hence, we see that if managers treat both current and future stakeholders equally (Figure 6B), this can force them to forego significant rents that can be obtained in the short run.
**Discussion of the numerical results**

The exact patterns exhibited in the figures above are specific to the model under consideration and the parameter values used. Changes in relative prices or biological parameters can significantly alter the results. There are, however, some important lessons that can be drawn from this model that are applicable more broadly.

First, as is seen in the policies presented in figures 4 and 5, the sustainability constraint can significantly complicate the problem. While interactions need to be taken into account, the PV-optimal policies are largely a function of one species or the other; single-species models may not be too far off the mark in determining the appropriate policy in this model. When the sustainability constraint binds, however, the S-optimal policy for each species is intimately related to stock of both species. Figures 4 and 5 demonstrate that the S-optimal policy must take into account not only the direct effect on the species being managed but their dynamic interaction and how harvests of the two species jointly affect the value of the fishery in the future.

The second principal conclusion that one can draw from the analysis is that sustainability is not simply about the preservation of each resource. For virtually all the paths presented in Figure 3, the optimal sustainable policies led to a decline in one or the other of the stocks. Although generalizations from this specific model cannot be made, the results presented here suggest that optimal sustainability will often involve tradeoffs between economic assets. Furthermore, as is seen in the comparisons of the constrained and unconstrained policies, a sustainability constraint can lead to significant and surprising results. Identifying the optimal-sustainable policy will likely involve more than simple rules of thumb.
Finally, we find that the S-optimal paths do not lead to a unique steady state as do the PV-optimal paths in Figure 1. Where a sustainability-constrained path ends depends critically upon where it started. This draws an important distinction between management in a sustainability-constrained economy and the unconstrained PV-maximizing management of a fishery. In PV-optimal management the principal concern is to find the unique equilibrium and then "any 'reasonable' method of reaching [that point] would probably be close to the optimum" (Clark [1985], 198). Since sustainability-constrained fishery management does not lead to a unique steady state, the policy objectives in this case are fundamentally altered.

Conclusions

We began by suggesting that the fundamental value of theoretical fisheries models is that they provide some general guidance to policy makers. We should stop, therefore, to consider what guidance is provided by the sustainability-constrained model introduced here and how this differs from that of the standard present-value maximization model.

First, we provide a new interpretation of the term sustainability that is established based on the normative foundation of intergenerational fairness. Whether one agrees that this is the appropriate goal for fisheries management or not, we hope that it will at least clarify what is sought when sustainability is put forth as a policy objective. Hence, this interpretation of the meaning of sustainability provides one answer to the questions: What should be sustained and what tradeoffs are admissible? At a minimum, therefore, we provide a benchmark against which alternative criteria of sustainable fisheries management can be evaluated.
How might fisheries policies in a sustainability-constrained management model differ from management intended to maximize the present value of the resource? First, the sustainability constraint adds an important additional interaction between the two species. This fundamentally alters policy. S-optimal policies must take into account not only the biological interactions between the two species, but also how the two species work jointly to sustain the value of the fishery. Sustainability requires, therefore, that policy makers avoid piecemeal approaches to resource management problems. This adds an additional motivation for system-wide approaches to resource management.

We also found that S-optimal policies might differ in surprising ways from those that seek only to maximize the present value of the resource. For example, PV-optimal harvests of the predator led to rapid depletion of these stocks. Such seemingly myopic behavior has long been subject of criticism from those concerned with sustainability and such policies were not found to be optimal in the sustainability-constrained model. However, we also found that the S-optimal harvests of the predator actually exceeded the PV-optimal harvests in a region of the state space. If economic sustainability is proposed as a policy goal, therefore, its implications will not always be transparent.

Sustainability has long been put forth as an objective of managers of fisheries and other natural resources. However, when the resource in question is multi-dimensional, the meaning of sustainability is not always clear. What should be sustained and why? The framework in this paper provides one possible set of answers to this question based on the principle of intergenerational fairness. While the fishery model that we use here is extremely simplistic, it would be conceptually straightforward to include additional elements in the endowment vector, non-market benefits from the species and/or uncertainty. The incorporation of general-
equilibrium price effects would also be a worthwhile extension. Alternative natural resources could similarly be analyzed. Hence, the general framework and methods in this paper represent a quite flexible structure in which to study the issue of sustainable resource management.
Appendix

Numerical methods used to solve for the PV-optimal and S-optimal policies in the predator-prey model

The results presented in this paper were generated by a numerical algorithm that approximately finds the PV- and S-optimal policies. In this appendix we briefly describe the algorithm used to solve both problems and the numerical methods used.

The PV-optimal policies are found by the process of successive approximation of the value function (Bertsekas [1976] 190). That is, using an arbitrary initial estimate of the value function \( V^1(x) \), the \( k \)th stage of the algorithm is to solve the problem

\[
V^{k+1}(x) = \max_{\zeta} \left[ U(x, \zeta) + \beta E_i V^k(x_{i+1}) \right]
\]

at a finite set of points in the state space, say \( X \). The bounds and the number of points in \( X \) are set in order to ensure the greatest accuracy possible within a reasonable amount of computational time. This problem is then solved repeatedly until a convergence criterion is satisfied.

Since, the set of points in the state space is necessarily finite, some approximation method must be used to estimate the value function at points not included in the set. That is, for most choices \( x_{i+1} = g(x_i, \zeta) \notin X \) so that the values obtained in the first stage can not be directly used to directly calculate \( V^k(x_{i+1}) \). A number of alternatives exist for estimating \( V(x) \) over the compete range of \( x \). These include rounding, interpolation and parametric approximation. In this case we used Chebyshev polynomials. Press et al. [1989] provide a careful discussion of the use of these polynomials and their advantages over other approximation methods. In all the cases presented in this paper, a two-dimensional Chebyshev polynomial with 17 nodes in each dimension is used. The optimization problem at each point in the state space is solved
using the NPSOL constrained optimization algorithm [Gill et al. 1986]. Once the successive approximation converges, the approximations of the unconstrained value function, say \( V'(x) \), and the set of PV-optimal policies are obtained from the last iteration of the algorithm.

The S-optimal policies and value function are obtained in a similar manner. Using \( V'(x) \) as an initial approximation of the sustainability-constrained value function, \( V(x) \) is approximated by successively solving the sustainability-constrained optimization problem,

\[
V^{n+1}(x) = \max_{\tilde{z}_t} \left[ U(x, z_t) + \beta E_t V^n(x_{t+1}) \right] \text{ s.t. }
U(x, z_t) + \beta E_t V^n(x_{t+1}) \leq V^n(x_{t+1})
\]

with \( V^0(x) = V'(x) \). This algorithm leads to a monotonically declining value function, i.e., \( V^{n+1}(x) \leq V'(x) \) for all \( x \in X \). This property, along with assumptions that lead to bounds on \( V'(\cdot) \) imply that the successive approximation algorithm will converge to the true sustainability-constrained value function.\(^\text{13}\) As in the unconstrained portion of the algorithm, the value function at each stage is approximated using Chebyshev polynomials. Unlike the unconstrained problem, the sustainability-constrained portion of the algorithm is not a contraction mapping. Hence, it is necessary to use a stricter conversion criterion in this portion of the algorithm to increase the likelihood that the algorithm is sufficiently close to the true value function. As with the PV-optimal case, the S-optimal policies are then taken from the solution of the last iteration of the successive approximation algorithm.

Once the PV- and S-optimal problems are solved, it is possible to use the approximated value functions, \( V'(\cdot) \) and \( V(\cdot) \) to solve for the optimal policy at any point in the state space. The paths in figures 1 and 3 are then obtained by first selecting arbitrary starting points near

\(^\text{13}\) A formal proof of convergence can be obtained from the authors.
the perimeter of the state space and then tracing out the PV-optimal and S-optimal paths by solving the appropriate optimization problem at each point in the path. The bounds on the state space were chosen so that the qualitatively interesting patterns are observed and at a point where any expansion in the bounds does not alter the nature of the results.

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