1. [30 Points] Consider the following panel data model

\[ Y_{it} = X_{it} \beta + c_i + u_{it}, i = 1, \ldots, N, t = 1, \ldots, T, \]

where \( c_i \) is a time invariant individual effect and \( u_{it} \) is an iid idiosyncratic error.

(a) The Random Effects (RE) estimator requires that

\[ E[c_i | x_{i1}, \ldots, x_{iT}] = 0, i = 1, \ldots, N. \] (1)

What are the statistical properties of the RE estimator if this condition does not hold?

(b) The Fixed Effects (FE) estimator requires that

\[ E[u_{it} | x_{it}, c_i] = 0, i = 1, \ldots, N, t = 1, \ldots, T. \] (2)

Explain why \( c_i \) is included in the conditioning variables for the conditional expectation of \( u_{it} \) for the FE estimator to be valid, i.e., why \( E[u_{it} | x_{it}] = 0 \) is not sufficient for the consistency of the FE estimator?

(c) If conditions (1) and (2) are satisfied. Would you prefer the RE or FE estimator? Explain your answer.

2. [40 Points] Consider the simple linear model

\[ Y_i = X_i \beta + u_i, i = 1, \ldots, N. \]

(a) Suppose \( u_i \)'s are iid normal with mean zero and variance \( \sigma \). Recall that the normal
density function with mean $\mu$ and variance $\sigma$ are given by $f(x) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$.

Write down the Maximum Likelihood Estimator (MLE) of this model.

(b) For simplicity, suppose the variance $\sigma$ is known. Show that the MLE estimate of the model is identical to that of the Ordinary Least Squares (OLS).

(c) Suppose now that the normality distributional assumption is incorrect. Under this mis-specified distributional assumption, the point estimates of the MLE and OLS remain the same, but not their inferences. What would be the proper inference (variance covariance matrix) of the OLS estimate in this case? What test can be used to examine the validity of the normal distributional assumption of the MLE?

3. [30 Points] Consider the following model

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + u_i, i = 1, \ldots, N.$$ 

Suppose we are interested in the following hypothesis

$$H_0 : \beta_1 \beta_2 = 1.$$ 

(a) Describe how to construct a Wald test for the null hypothesis.

(b) Describe how to construct a Score test for the null hypothesis.

(c) Which test would you recommend? Explain your answer.