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Hedges or safe havens—revisit the role of gold and USD against stock: a multivariate extended skew-t copula approach

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Our paper concerns the question of whether there exist hedge assets during extreme market conditions, which has become increasingly important since the recent financial crisis. This paper develops a novel extended skew-t copula model to examine the effectiveness of gold and USD dollar (USD) as hedge or safe haven asset against stock prices for seven developed markets over the 2000–2013 period. Our results indicate the existence of skewness and heavy/thin tails in the distributions of all three types of assets in most of the developed markets, lending support to the employment of flexible distributions to evaluate the tail dependences among assets. We find that USD is preferred to gold as a hedge asset during normal market conditions, while both assets can serve as safe haven assets for most countries when stock markets crash. Our simultaneous analysis of the three assets advises against a joint hedge strategy of gold and USD due to the high tail dependence between them during extreme market conditions. This result highlights the importance of simultaneous modelling of multiple assets in financial risk analysis.

Keywords: Gold; USD; Extended skew-t copula; Hedge; Safe haven

JEL Classification: C13, G11, G14

1. Introduction

The past two decades have seen a tremendous growth in financial markets and financial instruments in terms of their volume, value and scope. At the same time, many extreme market conditions, such as the Internet bubble, the subprime crisis and the European debt crisis, have reminded investors of the importance of risk management. Traditionally, scholars in the asset-pricing or other finance areas tend to think of risk as systematic, given a well-diversified portfolio, due to co-movement with the general market conditions. Thus, in practice, the most common tools to hedge for these co-movements are derivatives, such as options and futures, and some Greek hedging strategies, such as delta and gamma hedging, have been widely applied to control for market risk.

However, the recent relatively high frequency of extreme events highlights the need for tools to hedge the risk under these extremely severe market conditions. According to Baur and Lucey (2010), an asset is classified as a hedge asset if it is unrelated or negatively related to another asset during normal market movements, while it is called a safe haven if it is unrelated or negatively related to another asset in times of extreme market movements. Their definitions, which we follow throughout this paper, clearly distinguish the hedging roles under normal market conditions and safe haven roles under extreme market conditions. Because regional and global financial crises have occurred more frequently in recent times, the search for safe haven assets has become an important topic in international finance. Understanding the value changes of the overall portfolios by including safe haven assets during extreme financial situations is especially important for investors and financial institutions to protect their total investment values and for

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scholars to understand the comovement of asset returns during extreme market conditions.

To further investigate this problem, we develop a multivariate extended skew-\(t\) (MEST) copula model to examine the joint dependence and asymmetric tail dependence among different assets. We use the proposed model to examine the effectiveness of gold and its competitor, US dollar (USD), as a hedge or safe haven against stock returns for seven developed markets over the 2000–2013 period, when Euros have been adopted as the unified currency in the European Union (EU) and when many financial crises were experienced. Although the commonly used normality assumption of asset return distributions is simple, this assumption has empirically been shown to be unrealistic because the skewness and excess kurtosis of return distributions are usually non-zero (Campbell et al. 1997). This problem will be especially crucial when we want to investigate asset performance during extreme market conditions.

Historically, many non-normal distributions have been examined to better describe the dynamics of asset returns.† Copula functions with non-normal distribution assumptions are commonly used in finance, especially in the risk management area.‡ However, so far, none of the previously proposed models can allow for a thin-tailed distribution or preserve closed-form solutions under conditioning. The advantages of preserving a closed-form under conditioning are bifold. First, this property can facilitate theoretical development of multivariate analysis such as the vine copula and multivariate tail dependence functions. Second, this property can mitigate the complexity of evaluation of conditional distributions. From our empirical analysis, we find that the MEST copula is able to capture asymmetric dependence in a parsimonious way. Importantly, it accommodates strong asymmetric tail dependence among the assets. The significant shape and extension parameters in the MEST copula indicate the existence of skewness and heavy/thin tails in most of the markets, justifying the merits of the proposed model. When estimated together with the skew-\(t\) (ST) copula model in terms of evaluating the lower tail dependence coefficients, our results demonstrate the advantages of our proposed model over the ST model because the lower tail dependence coefficients by the ST copula model are systematically overestimated. Moreover, we extend the bivariate copula analysis on the effectiveness of hedges and safe havens to a multivariate case. Previous studies analysed two assets at a time without considering the structure of the joint distribution and the influence of other assets, and we have shown that without considering the effects of other assets, the estimated tail dependences can be misinterpreted. To the best of our knowledge, this is the first study to model three assets jointly and investigate the hedge and safe haven issue through non-normal distribution estimation at the same time.

Our empirical results demonstrate that the fundamental factors that drive hedging during normal times and those that drive hedging during extreme market periods can be different. Specifically, we find that gold serves, at best, as a weak hedge asset against stock market performance for all seven studied countries, but USD can possibly serve as a strong hedge asset for Canada and Australia under normal market conditions. This set of results is generally consistent with the Pearson’s correlation commonly used in the literature. During a stock market crisis, gold can serve as a weak safe haven asset for all countries except Switzerland and as a strong safe haven asset for the USA, the UK and Germany. On the other hand, USD can serve as a weak safe haven asset for almost all countries, with the exception of Switzerland and Australia, and as a strong safe haven asset for the UK, Germany, Japan and Australia. When an alternative investment such as gold and USD serves as a hedge asset for investors, it is not necessary that it also serve as a safe haven asset for the same group of investors, and vice versa. We attribute this finding to the theoretical foundation that both assets can serve as a hedge against inflation fluctuations under normal market conditions, as suggested by Jaffe (1989) and McCown and Zimmerman (2007), but the flight-to-quality hypothesis suggested by Caballero and Krishnamurthy (2008) may drive the safe haven effects of these investments under extreme market conditions.

To further test whether the safe haven roles of these assets are jointly or independently determined, we use our multivariate model to analyse whether gold (USD) can act as a hedge or safe haven against stocks when USD (gold) returns are found under extreme market conditions. By testing the joint lower tail probability and conditional tail probability, we conclude that USD can be a weak safe haven for the UK and Germany conditional on large drops in gold prices, and gold can be a weak safe haven for the USA and Australia conditional on the poor performance of USD. Generally, based on the analysis of both overall and extreme dependence measures, our results confirm that the USD is a more likely hedge against stocks than gold, but the safe haven effects of USD and gold will not increase when both assets are used as safe haven assets. We find that neither of the assets dominates the other during stock market crashes, and this finding implies that the joint safe haven strategy does not outperform the single safe haven strategy. Previous studies with no consideration of the joint properties of USD and gold may lead to a naive conclusion about the usefulness of gold and USD as safe haven assets.

Our work contributes to the rich literature on risk management through alternative investments in the following ways. Regarding methodology, we generalize the extended skew-\(t\) (EST) copula function to a multivariate case and provide a new econometric framework to jointly analyse the hedge and safe haven effects of alternative investments to solve the problem that the traditional methodologies cannot allow for fat/thin tail generalization, and we apply this model to test multiple alternative investments at the same time. In terms of the aspects of empirical results and economic interpretations, we reexamine the hedge and safe havens of stock–USD–gold from an investor’s point of view, while most previous studies simply combine these data series without considering whether the implied investment strategies can be obtained by domestic investors. Our

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†A comprehensive review of the asymmetric distributions used in finance is provided in Adcock et al. (2012).
‡See Demarta and McNeil (2005), Kollo and Pettete (2010), Azzalini and Capitanio (2003), Christoffersen and Langlois (2013), etc.
results indicate that the fundamental factors that drive the hedge and safe haven effects can be different and provide some indirect evidence supporting the inflation hedge hypothesis and the flight-to-quality safe haven hypothesis. Moreover, by simultaneously estimating stock, gold and USD, we find that some of the findings that exist under the stock–USD and stock–gold unconditional analysis would not hold when the third asset is added to investors’ portfolios. This finding questions whether gold and USD can serve as independent safe haven assets for investors’ risk management strategies.

The paper proceeds as follows. In Section 2, we provide a literature review on the relation between gold and the USD on the stock market. Section 3 reviews the use of non-normal distribution and copula functions in the finance literature, and Sections 4 and 5 derive our proposed MEST copula model and the lower tail dependence coefficient of the MEST copula model. In Section 6, we present the data, model, estimation results and economic interpretations. Sections 7 and 8 provide some robustness checks and comparisons between our proposed model and models used in previous studies. Finally, Section 9 concludes. Some technical details are gathered and presented in the Appendix.

2. Hedging the effectiveness of gold and the USD in the international capital market

Both gold and USD have been widely used by international investors for different purposes, such as hedging stock market movement and other macroeconomic variables. According to a recent Gallup poll, some 30% of respondents considered gold to be the best long-term investment, making it a more popular investment than real estate, stocks and bonds (Saad 2012). On the other hand, USD has been deemed the most dominant currency worldwide for some time, and it is not surprising that foreign investors hold some assets in the form of US currencies. However, the reason for the popularity of these alternative investments is still debated, and here, we focus our research on the hedging role of these assets. The role of these alternative investments in investors’ portfolios has been tested in different forms and is sometimes modelled in the finance and economic literature. In this section, we will first review the current studies in the hedging effects of these two commonly deemed safe assets. Later, we will develop the hypotheses that will be tested under our empirical framework.

2.1. Literature review

Gold has been traditionally deemed as a good hedging tool against macroeconomic risk, including inflation and interest rate risk. In academia, there are some theoretical and empirical results regarding the hedging role of gold. Gorton and Rouwenhorst (2006) document that commodities can offer positive returns during financial downturns. Gold is generally considered one of the most popular assets in the commodity market. As shown by Batten et al. (2015), the Over-the-counter (OTC) derivatives outstanding in gold account for one-fifth of all commodity derivatives in terms of the gross figure in 2013. According to Jaffe (1989), gold is a hedge against both stock losses and inflation; therefore, including gold in financial portfolios can reduce their variance and slightly improve returns. McCown and Zimmerman (2007), based on the capital asset-pricing model, demonstrate that gold shows hedging ability on stock portfolios and inflation during the period from 1970 to 2006. However, negative Capital-Asset-Pricing-Model (CAPM) betas are only observed during the 1970s, suggesting that stock-hedging ability may be attributed to inflation-hedging ability. Gold can hedge other macroeconomic state variables as well. For example, Ruff and Childers (2013) show that the gold price and the Consumer Confidence Index are negatively associated, implying that gold has characteristics of hedging against economic instability. However, not all studies agree on the hedging role of gold against stock market movement. A recent paper by Erb and Harvey (2013) critically examines the role of gold in asset allocations, including inflation hedging, currency hedging and disaster protection against stock. Little evidence is found for gold to be an effective hedge against inflation, and the gold-as-currency hedge argument does not seem to be supported by the data. Other studies that examine the financial characteristics of gold include Faugere and Van Erlach (2006), Lucey et al. (2006) and Sherman (1982). While the majority of prior research agrees on the diversification benefits of gold under certain conditions, there are also contradictory results. For example, Anderson et al. (2014) examine the so-called ‘permanent portfolio’, which consists of stocks, bonds, gold and cash, and this permanent portfolio only outperforms buy-and-hold or stock and bond portfolios on a risk-adjusted basis.†

The USD is another candidate competing for hedging against stock market fluctuations in foreign countries. Giovannini and Jorion (1989) show that the co-movements and the dependence structure between equities and USD would have important implications for their cross-market risk management for global portfolio diversification. In the literature, many theoretical models have been constructed to explain the relationship and co-movements between these two markets; see e.g. Dornbusch and Fischer (1980), Branson (1983) and Frankel (1983). All these models argue that the stock market impacts the exchange rate and vice versa. Hau and Rey (2006) suggest that the relations between stock and USD can be either positive or negative due to the return-chasing, portfolio rebalancing and exposure effects. To sum up, empirical studies of the interaction or causal relationship between these two assets report mixed results, and thus, the hedging effect of the USD is questionable.

Although these two assets are used to hedge risk in the stock market, there are few theoretical foundations for how

†There is also indirect evidence of the hedging effect of commodities. The results are also mixed (Izorek 2007, Woodard 2008, Cao et al. 2010, Conover et al. 2010, Daskalakis and Skiadopoulos 2011, You and Daigler 2013). Recently, the financialization of commodities has been proposed and generally accepted as one reason why the correlation between stocks and commodities has declined in recent years. See Kyle and Xiong (2001), Tang and Xiong (2012) and Silvennoinen and Thorp (2013).
these assets can hedge stock market risk during extreme economic conditions. There is some indirect evidence that these assets can theoretically serve as safe havens when markets crash. For example, when the prices of risky financial assets drop dramatically, investors may decide to sell risky assets and rush into buying safe assets to alleviate losses. Thus, under the flight-to-quality hypothesis of Caballero and Krishnamurthy (2008), the prices of these alternative assets may surge. However, to our knowledge, there is no theoretical model explaining why gold or the USD is usually referred to as safe haven assets. McCauley and McGuire (2009) suggest that the appreciation of USD during the period can be attributed to the following four reasons: safe haven, unwinding of carry trades, dollar short-age and overhedging by non-US investors. The main theme of the assertion is that because USD is generally deemed as a high-quality currency and used by local and foreign financial market participators, the crisis increases the demand of USD, causing USD appreciation. This argument implies that USD will appreciate under financial crisis, which is consistent with our definition of safe haven.

Empirical studies provide mixed results regarding whether gold is a safe haven asset. For example, Baur and Lucey (2010) show that gold is a safe haven only in the very short term: on average, gold holders earn a positive return on the day of an extreme negative stock return, but the return on gold is likely to be negative on average in the following two weeks. Ciner et al. (2013) employ the dynamic conditional correlation model to measure the correlation structure and find that gold consistently acts as a safe haven in times of market crashes. On the other hand, Baur and McDermott (2010) find that gold is a safe haven asset for major European stock markets as well as the US stock market but not for Australia, Canada, Japan and large emerging markets. In addition, Lucey and Li (2015) provide evidence of the strength of the safe haven, finding that, at times, gold is not the strongest or the safest haven based on US data. However, these studies (Baur and McDermott 2010, Baur and Lucey 2010, Ciner et al. 2013) only examine the marginal effects of stock prices on gold prices based on the estimated coefficients from linear regressions. Threshold dummy variables are included and given by a specific quantile of the stock return distribution to capture extreme stock market movements. The conventional correlation coefficient may not be an appropriate measure of the dependence among financial assets when the bivariate normality assumption on the joint distribution of stock and gold does not hold. Therefore, results obtained from linear regressions may not fully account for extreme market co-movements.

On the empirical investigation of the safe haven role of dominant currencies on stock market crashes, Kaul and Sapp (2006), Ranaldo and Soderlind (2010) analyse the safe haven status of various currencies. Beck and Rahbari (2008) find that dollar bonds act as ‘safe haven currencies’ during sudden stops, and they are a better hedge for global sudden stops and for regional sudden stops in Asia and Latin America. Ning (2010) uses copula models to examine the extreme co-movements of USD and stock markets in G4 countries, finding significant symmetric tail dependence in all stock–USD return pairs for both pre- and post-Euro periods. Wang et al. (2013) present a dependence-switching copula model to describe the dependence structure of these two assets in six industrial countries between 1990 and 2010. Their results demonstrate that the general dependence and tail dependence are asymmetric for most countries when the stock market and exchange rate market move in opposite directions, but they are symmetric for all countries when these two markets move in the same direction.

To summarize the current studies in the field, there is some evidence that gold and USD can be used as hedging tools through hedging inflation or other macroeconomic variables, although the empirical results are somewhat mixed. Theoretically, few studies have examined why or whether these assets can serve as safe havens except for the flight-to-quality hypothesis. There are some empirical studies that investigate the roles of gold and USD as safe havens, but these studies often either examine two assets at a time without taking into account the possible effect of the existence of other alternative assets, or investigate the average dynamic correlation among multiple assets without considering their extreme co-movements. For example, Ning (2010) and Wang et al. (2013) examine the extreme dependence between stocks and USD, but use only one alternative investment, USD, as the safe haven. Zagaglia and Marzo (2013) investigate how the relation between gold and the USD has been affected by the financial turmoil. Other studies, such as Baur and McDermott (2010) and Ciner et al. (2013), investigate the safe haven property between stocks and gold or focus on pairwise correlation structures for multiple assets, but do not consider extreme market situations. Moreover, some studies analyse the relationships between gold and exchange rate. An early work by Sjaastad and Scacciavillani (1996) show theoretically and empirically gold price can be linked to the exchange rate, and among different currencies, the European currencies, which possess high market power in the gold market and inflations, cause the gold price fluctuating. On the other hand, Capie et al. (2005) use Yen and Sterling as examples to show that empirically, gold can serve as a hedge against dollar only for some periods, and the relation is not guaranteed. In a more recent work, Joy (2011) finds that gold can be served as a hedge against exchange rates, but cannot be served as a safe haven asset for exchange rates. Reboredo and Rivera-Castro (2014b) use a sample period similar to ours (2000/01 to 2013/03) and find that the gold price is positively correlated with the USD depreciation for most currencies. Furthermore, they apply the Value-at-Risk (VaR) approach, which also can be used to measure the downward risk, and find that including gold into the portfolio of pure currencies can provide some safe haven effects, especially in a shorter time horizon, such as days and weeks. In another work, Reboredo and Rivera-Castro (2014a) test the tail dependence between gold and currencies and find that gold can serve as a hedge against USD, but it is a weak safe haven asset against extreme USD price movements. To the best of our knowledge, none of the existing studies investigates the joint relationship beyond two dimensions and accounts for the joint extreme dependence at the same time. This motivates our attempts to use innovative econometric tools to fill this gap in the literature by considering
the effectiveness of gold and USD simultaneously as hedges or safe havens on stock.

2.2. Hypothesis testing

In this section, we develop our hypothesis based on the previous literature on a qualitative base. We defer the more detailed and quantitative descriptions of these hypotheses to the Empirical Analysis section, after we introduce our econometric model to help readers better understand the economic reasons and econometric methods in this paper.

From the previous literature, we can observe that both Gold and USD have been used and tested as hedging tools or safe havens, but the reasons they can serve these roles are still debated. Jaffe (1989) and McCown and Zimmerman (2007), among others, suggest that the inflation-hedging ability of gold is the reason gold can serve as a hedging tool against stock market movement. Gold, as one kind of precious metal, is deemed to have a more stable real value due to the limits of its supply, and the fluctuation of price levels can be consistent with changes to the gold price. On the other hand, the change in USD can also be correlated with inflation or price level changes if imports or exports are important for domestic countries. For example, an appreciation of USD (for residents in other countries) can increase the price level of import goods and decrease local consumers’ real consumption but stimulate foreign consumption of export goods therefore increase the profitability of production.

However, although both assets can theoretically be used to hedge inflation changes, they may not be good hedges against stock market movement. As argued by Boons et al. (2014), the performance of these inflation-hedged alternative investments can be affected by the short-hedged demand by producers, the long-hedged demand by consumers and the speculative demand of investors.‡ Thus, if the forces from the two assets other than inflation hedging by consumers are strong, the hedging ability of these alternative assets can be largely weakened. For example, if these two assets are used as pure speculation investment vehicles, the movement of the prices of these assets can deviate from the inflation; consequently, gold and USD cannot be used as hedge assets. To test whether these alternative investments can be used to hedge the risk in the stock market, we will test the following hypotheses:

Hypothesis 1A: Gold is a hedge asset against the domestic stock market movement.

Hypothesis 1B: USD is a hedge asset against the domestic stock market movement.

According to the definition used by Baur and Lucey (2010), if an asset can be used as a strong hedging tool against the stock market, it must have a significant, and often negative, co-movement with the stock market.‡ This is probably the most commonly seen measure of hedging. If the hedge asset and the underlying assets are correlated significantly, we can easily find the degree of co-movement of these two assets by finding the linear relation between these two variables, which is similar to the concept of the commonly used delta hedge. In the empirical testing, the general co-movement of two assets can be measured by Pearson’s correlation, Kendall’s $\tau$ coefficients or by linear regressions, as commonly seen in the empirical literature.

When extreme market conditions are observed, the co-movements of stock markets and alternative investments may not be solely caused by an inflation-hedging effect. Caballero and Krishnamurthy (2008) argue that the flight-to-quality hypothesis can explain the surge of the prices of alternative investments under stock market crashes. Gold, as a store of value rather than an earning asset, can maintain the same real value when the prices of other assets drop and are thus traditionally deemed as a high-quality asset under extreme market conditions.§ Alternatively, when the domestic stock market crashes, the local and foreign investors may decide to switch their investments from the local stock market to stock markets in other countries, and this action can cause the depreciation of local currencies or the appreciation of USD. In this sense, USD should serve as a safe asset during market crashes. Note that the explanation provided here cannot be explained solely by the changes in inflation unless sudden changes in the inflation level constitute the main reason for stock market crashes. Thus, common significant co-movements between these alternative investments and the domestic stock market do not guarantee a similarly significant relation observed in extreme market conditions. To test whether the hedging and safe haven effects are consistent for gold and USD, we test the following hypotheses:

Hypothesis 2A: Gold is a weak safe haven against domestic stock market crashes.

Hypothesis 2B: USD is a weak safe haven against domestic stock market crashes.

Hypothesis 3A: Gold is a strong safe haven against domestic stock market crashes.

Hypothesis 3B: USD is a strong safe haven against domestic stock market crashes.

Baur and Lucey (2010) call an asset a safe haven if it is unrelated or negatively related to another asset in times of extreme market movements, and we follow their definition throughout this paper. Baur and McDermott (2010) further

‡Strictly speaking, Boons et al.’s paper discusses the systematic risk associated with commodity futures rather than the co-movement of these assets and stock markets. However, their model assumes that the commodity fully represents the consumption price level, which is directly linked to inflation. Thus, their arguments can also be used in the context here.

‡On the other hand, if the co-movement between the two assets is insignificant, the chosen alternative asset can serve at best only as a weak safe haven asset. In the other words, investors can include this asset in their portfolios for diversification purposes rather than as the strong hedge, as it is commonly used.

§The characteristics of gold as a financial asset are explained in greater detail by Baur and McDermott (2010).
investigate the difference between weak and strong safe haven assets. According to their definition, a strong (weak) safe haven is defined as an asset that is negatively correlated (uncorrelated) with another asset or portfolio in certain periods only, e.g. in times of falling stock markets. Simply speaking, a weak safe haven would perform normally regardless of whether the stock market crashes, but a strong safe haven would perform extremely well during the market downturn. In econometrics tests, the tail dependences can be used to test the co-movement of assets under extreme situations. Based on the tail dependence function developed in Section 5, we use the low (in stock)–low (in gold or USD) tail dependence to test whether gold and USD are weak safe havens, and we use the low/up tail dependence to test whether gold and USD are strong safe havens.†

Thus far, we have focused on the hedging or safe haven roles of gold and USD on the domestic stock market, but some of the findings have already been presented in previous studies, although usually for only one asset at a time. Under the flight-to-quality hypothesis, investors will choose to withdraw funds from low-quality assets such as stocks during domestic stock market crashes and switch to high-quality assets such as gold or USD. However, these high-quality assets can be substitutes during market crash periods, and which high-quality asset investors will choose to flee to be unclear, surely presenting an interesting question. Hence, we later control for the performance of one asset and test the hypotheses of whether safe haven effects still exist for the other asset.

Hypothesis 4A: Gold is a weak safe haven against domestic stock market crashes after controlling for the extreme poor performance of USD.

Hypothesis 4B: USD is a weak safe haven against domestic stock market crashes after controlling for the extreme poor performance of gold.

To understand the co-movements among these three assets (stocks, gold and USD), we need to estimate a multivariate distribution with these three random variables. The econometric method used to estimate the multivariate distributions will be discussed below, and the explicit form of the tail dependences, which are used to test the extreme conditions’ co-movements, will be derived in the next section. These newly developed econometric techniques should allow us to further understand of the hedging and safe haven roles of alternative investments.

3. Non-normality distributions with copula function methodology reviews

In our later empirical studies, we model the entire joint distribution of returns among these three assets without assuming multivariate normality. Our newly developed MEST copula model allows for asymmetric dependence and tail dependence (fat-tailed as well as thin-tailed dependence), which cannot be obtained with previously proposed econometric methods.

Theory that concerns multivariate joint distributions that are elliptically symmetric (Fang et al. 1990) has emerged in the literature in the past two decades. However, elliptically symmetric distributions cannot capture the skewness of return distributions. In many cases, asymmetric distributions are more adequate to describe asset returns. For example, Campbell et al. (1997) present the moments of daily and monthly individual and aggregated stock from 1962 to 1994 and find that excess kurtosis is seldom statistically insignificantly different from zero and that the skewness of stock is also often significantly different from zero, especially in the daily return samples.

A number of recent papers consider skewness in multivariate distributions. For example, Sahu et al. (2003) introduce skewness into multivariate elliptically symmetric distributions. Azzalini and Capitanio (2003) study a general procedure to perturbate a multivariate density and to generate a family of non-symmetric densities. Arellano-Valle and Genton (2010) propose a class of MEST distributions to allow for asymmetric and thin-tailed distributions. For the application of these distributions in asset pricing and portfolio selection, see Adcock (2010). A comprehensive review of asymmetric distributions used in finance is provided by Adcock et al. (2012).

The past two decades have seen the increased usage of the copula methods in the finance literature. Applications of copulas in finance are typically restricted to bivariate (either symmetric or asymmetric) or multivariate symmetric cases. Some recent studies have begun to employ asymmetric distributions to construct multivariate asymmetric copulas to provide better flexibility. For instance, Demart and McNeil (2005) propose a ST copula based on a Gaussian mixture representation. Kollo and Pettere (2010) introduce a multivariate ST copula based on the multivariate ST distribution of Azzalini and Capitanio (2003). Smith et al. (2012) establish ST copulas implied by Sahu et al. (2003). These methods have been adopted in the finance literature. Christoffersen et al. (2012) propose a new dynamic asymmetric copula model based on the ST copula detailed by Demart and McNeil (2005) to capture long-run and short-run dynamic dependence, multivariate non-normality and asymmetries in large cross-sectional data. Christoffersen and Langlois (2013) apply the dynamic asymmetric copula on the four-factor CAPM model, and Christoffersen et al. (2013) use it to investigate the diversification in corporate credit. González-Pedraz et al. (2015) propose a conditional ST copula to capture the impact on optimal portfolios. However, these aforementioned copulas can only capture asymmetric and heavy-tailed dependence; they fail to allow for thin tails of multivariate distributions and preserve the closed functional form under conditioning. All of these problems can be mitigated in our proposed model, which will be derived in detail in the next section.

†Note that if the joint distribution between alternative investments and the stock market is symmetric, there is no need to test both weak and strong safe haven hypotheses. However, the significant skewness observed in later empirical results suggests that the symmetric assumption is violated. This is another factor that motivates us to apply a more complicated MEST in our empirical tests.
4. MEST copula model

The copula function is a method to join or couple multivariate distribution functions to their one-dimensional marginal distribution functions. Due to the separability of dependence structures and marginal distributions, a copula-based estimator can accommodate rich dependence structures with just a few parameters. In addition, such an estimator facilitates the modelling of tail dependence to capture the stylized properties of the tails. This section outlines the MEST copula model we develop to capture the dependence between variables. Our model can be implemented by a two-stage process. We first fit each return series with a GARCH model and then calculate the implied standardized residuals. The empirical cumulative distribution functions of the standardized residuals are used as inputs in the second-stage copula density estimation, which features a MEST copula.

According to Sklar (1973), a multivariate distribution function can be decomposed into marginal distributions and a copula function that incorporates all marginal distributions. For a continuous \( p \)-dimensional random vector \( X = (X_1, \ldots, X_p) \) with marginal densities \( f_1(x_1), \ldots, f_p(x_p) \) and marginal distributions \( \{F_1(x_1), \ldots, F_p(x_p)\} \), a unique copula function exists that satisfies

\[
f(x_1, \ldots, x_p; a_1, \ldots, a_p, \theta) = \prod_{i=1}^{p} f_i(x_i; a_i) c(F_1(x_1; a_1), \ldots, F_p(x_p; a_p); \theta),
\]

where \( c(\cdot) \) is the copula density function. In practice, the dependence parameter \( \theta \) is often estimated by the maximum likelihood method. Due to the non-i.i.d. nature of financial data, a filtration is necessary to obtain i.i.d. series, which are required by the classical maximum likelihood theory. A common practice of filtration is to fit a univariate GARCH-type model to each marginal series. Denote the resulting standardized residuals by \( \hat{\mu}_t, t = 1, \ldots, p \). Their corresponding ranks \( u_t \), or empirical cumulative distributions, are used in the subsequent copula estimation. Below, we shall propose a MEST copula model that will be used in our investigation of the dependence among the returns of assets.

4.1. Estimation of marginal distributions

The first stage of copula models consists of the estimation of individual marginal distributions. Because asset returns may exhibit autocorrelation and autoregressive conditional heteroscedasticity (e.g. Patton 2004, 2006, Reboredo 2013), a filtration is necessary to obtain i.i.d. marginal series. In this study, we consider an AR(p)-GJR(1,1) model (Glosten et al. 1993), using the quasi-maximum likelihood estimation.

The error terms are assumed to follow the EST distribution used by Arellano-Valle and Genton (2010) to allow for asymmetric and heavy/thin tails.

Denote the standardized error terms for the \( i \)-th series by

\[
\mu_i = \left( \frac{z_{1i}}{\sqrt{h_{1i}}}, \ldots, \frac{z_{pi}}{\sqrt{h_{pi}}} \right)^T.
\]

Its density function is given by:

\[
\frac{1}{T_1(T_1/\sqrt{1 + \lambda_i^2}; v_t)} T_i(\mu_i; v_t) T_1 \times \left\{ \left( \lambda_i \mu_i + \tau_i \right)^{v_t - 1} \left( \frac{v_t + 1}{v_t + \mu_i^2} \right) ; v_t + 1 \right\},
\]

where \( \lambda_i \in \mathbb{R} \) is a shape parameter and \( \tau_i \in \mathbb{R} \) is an extension parameter. \( T_i(\cdot; v_t) \) is the density function of the univariate Student’s \( t \) distribution with a zero mean, unit scale and degree of freedom \( v_t \) and \( T_1(\cdot; v_t + 1) \) is the univariate Student’s \( t \) distribution function with a degree of freedom \( v_t + 1 \).

We fit each marginal series to the EST distributions using the maximum likelihood estimator. The resulting standardized residuals are then transformed to \( u_t, t = 1, \ldots, T \) and = \( \frac{1}{T} \sum_{t=1}^{T} I(u_t \leq \mu_t) \) is the rescaled empirical cumulative distribution function.

4.2. EST copula model

To capture the dependence structure among the various asset returns that accommodates features of skewness and kurtosis, this study constructs a MEST copula based on the MEST distribution described by Arellano-Valle and Genton (2010).

Let \( Y = \{Y_1, \ldots, Y_p\} \) denote the filtered returns of \( p \) assets. Suppose that \( Y \) has a MEST distribution, denoted by \( Y \sim \text{MEST}_p(\Phi, 0, v, \tau) \). Its density function evaluated at \( y = \{y_1, \ldots, y_p\} \in \mathbb{R}^p \) is given by:

\[
\frac{1}{T_1(T_1/\sqrt{1 + \theta^T \Phi; v})} t_{p}(y; \Phi, v) T_1 \times \left\{ \left( \theta^T y + \tau \right)^{\frac{v + p}{v + \theta^T \Phi \theta}} ; v + p \right\},
\]

where \( \theta = [\theta_1, \ldots, \theta_p]^T \in \mathbb{R}^p \) are the shape parameters, \( \tau \in \mathbb{R} \) is an extension parameter and \( v > 0 \) is the degree of freedom. \( t_{p} \) is a \( p \)-variate Student’s \( t \) density function with a correlation matrix \( \Phi \), whose density is given by:

†There are two reasons for us to employ the empirical distribution of the standardized residual. First, Chen et al. (2006) note that the canonical maximum likelihood estimator via the empirical cumulative distribution function (CDF) has the added benefit of that the asymptotic distribution of copula estimations in the second stage is not affected by the sampling variations of first-stage estimation of the marginal distributions. In addition, we have conducted our estimation using either both inference functions for margins and canonical maximum likelihood methods, and the results of multivariate copula estimation from both methods are very similar. Therefore, we choose the more popular approach, which is the canonical maximum likelihood, when computing the cumulative probability of the standardized residuals.
\[ \mu_y = \frac{\Gamma\left(\frac{\nu}{2}\right)}{\Gamma^2\left(\frac{\nu}{2}\right)\Gamma\left(\frac{\nu}{2}+1\right)} \left(1 + \frac{v + \gamma^T\Phi^{-1}v}{v}\right)^{-\frac{\nu}{2}} \]  

(4)

\( T_1(\mu; \nu) \) denotes a standard univariate Student’s \( \nu \) distribution function with degrees of freedom \( \nu \).

The cumulative distribution function of \( Y \) is

\[ F(y) = P(Y \leq y) = \frac{1}{T_1(\nu; \nu)} T_{p+1}\left(\left[ y \mu \right]; -\delta, 1\right), y \in \mathbb{R}^p \]  

(5)

where \( \delta = \frac{\mu^T}{\sqrt{1+\gamma^T\Phi_0\gamma}} \) and \( T_{p+1}\left(\cdot; \nu\right) \) denotes a \( p+1 \)-variante Student’s \( \nu \) distribution function with a degree of freedom \( \nu + 1 \).

Recall that the marginal density function of \( Y_i \) takes the form

\[ f_i(y_i) = \frac{1}{T_1(\nu; \nu)} t_{1}(y_i; \nu) T_1 \]  

(6)

\[ \times \left\{ \lambda_i y_i + \tau_i \right\}^{\frac{v + 1 - t_i}{2}} (v + 1) \], \quad i = 1, \ldots, p

Let \( \lambda = [\lambda_1, \ldots, \lambda_p]^T \). The relation of the shape parameters between the joint and marginal distributions is given by

\[ \Theta = \frac{\Delta (\Delta \Theta - \lambda \lambda^T)^{-1} \lambda}{\sqrt{1+\gamma^T(\Delta \Theta - \lambda \lambda^T)^{-1} \gamma}} \]

where

\[ \Delta = diag\left(\sqrt{1+\lambda_1^2}, \ldots, \sqrt{1+\lambda_p^2}\right) \]

As shown in (1), the joint density function can be interpreted as the product of univariate marginal densities and a copula density function. It implies that the copula density function is the joint density function divided by the product of univariate marginal densities. Therefore, the \( p \)-variante EST copula† is given by:

\[ c(u_1, \ldots, u_p; \Phi, \theta(\lambda), \lambda, \tau, v) = \frac{f\left(F_{i}^{-1}(u_i), \ldots, F_{p}^{-1}(u_p); \Phi, \theta(\lambda), \lambda, \tau, v + p\right)}{\prod_{i=1}^{p} f_i\left(F_{i}^{-1}(u_i); \lambda_i, \tau, v + 1\right)} \]  

(7)

where \( F_{i}^{-1}(u_i) \) denotes the inverse distribution function of univariate EST for the variable \( i \). The copula function of (7) is then given by:

\[ C(u_1, \ldots, u_p; \Phi, \theta(\lambda), \lambda, \tau, v) \]

\[ = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} c(k_1, \ldots, k_p; \Phi, \theta(\lambda), \lambda, \tau, v) dk_1, \ldots, dk_p. \]  

(8)

Subsequently, the MEST density function is expressed as follows:

\[ f(y_1, \ldots, y_p; \Phi, \theta(\lambda), \lambda, \tau, v) \]

\[ = \prod_{i=1}^{p} f_i(y_i; \lambda_i, \tau, v)c(u_1, \ldots, u_p; \Phi, \theta(\lambda), \lambda, \tau, v). \]  

(9)

The log-likelihood function of (9) is:

\[ L(\theta) = \sum_{i=1}^{p} \sum_{t=1}^{T} L_{i,t}(\Psi_i) + \sum_{t=1}^{T} L_{c,t}(\Psi_c) \]  

(10)

where \( \Theta = (\theta(\lambda), \Phi, \lambda, \tau, v) \), \( \Psi_i = (\lambda_i, \tau, v) \), \( L_{i,t}(\Psi_i) \), and \( L_{c,t}(\Psi_c) \) are the log-likelihood function of the marginal density of \( \lambda_i, \tau, v \), and the log-likelihood function of the copula density, respectively.

The copula density can be estimated separately from the marginal densities using the method used by Li (2005). In particular, we can estimate the parameters \( \Psi_c = (\Phi, \theta(\lambda), \lambda, \tau, v) \) by maximizing \( \sum_{t=1}^{T} L_{c,t}(\Psi_c) \), the log-likelihood function of (10), based on a quasi-maximum likelihood theory.

### 4.3. Interpretation of the coefficients of the EST model

Because the EST is more flexible than the general family of elliptical distributions, such as Gaussian and Student’s \( \nu \) distributions, it is difficult to provide statistical meanings for a particular coefficient of the EST. However, in some extreme cases, the EST distribution can be transformed into some types of simple distributions. For example, when the shape and extension parameters equal zero (\( \lambda_i = \tau_i = 0 \)), the EST distribution degenerates to the conventional Student’s \( \nu \) distribution. When the degree of freedom goes to infinity and one (\( v_i \rightarrow \infty \) and \( v_i = 1 \)), the EST distribution reduces to the skewed normal distribution of Azzalini and Dalla Valle (1996) and the skewed Cauchy distribution of Arnold and Beaver (2000), respectively. When the extension parameter equals zero (\( \tau_i = 0 \)), the EST distribution reduces to the ST distribution of Azzalini and Capitanio (2003). For detailed discussions of the EST, see Arellano-Valle and Genton (2010).

### 4.4. Application of the MEST model

To consider whether to apply the MEST model developed in this paper for research questions in finance, we can evaluate the two main improvements of our MEST model compared to other more restricted distributions, such as

†As DCC models have been criticized in a number of papers, e.g., Füss et al. (2013), this study focuses on the static copula development. For the sake of completeness, we report estimation results with the DCC in the Appendix.
Gaussian and Student’s $t$ distribution. At the same time, we will provide some potential areas where we should use our MEST model.

First, the MEST model allows the marginal distributions to be with non-zero skewness and excess kurtosis, which will be especially useful in the content of risk management because we care more about extreme situations. The property of the distribution on these distributions under the extreme value region will be especially important to help us set up the hedging strategies. In fact, one of the reasons why subprime mortgage crisis hurts the banking system severely is that the VaR calculation used in the banks provides a presumably safe hurdle which in fact, provides much less safety than we thought. Thus, a distribution with flexible fat or thin tail definition will be much useful in the content of risk management. Other possible areas which we could apply the MEST models may involve data with high frequencies or non-linear payoff securities, such as some derivatives. In both cases, the normality assumptions will be clearly violated and the MEST model will provide a more accurate estimations.

Second, the MEST model developed in this paper allows for multiple variables to be estimated at the same time. In practice, it is seldom that investors choose to hold only small numbers of securities. Thus, when the research questions we are interested in are with non-normal and unidentified distributions, we will need our multivariate joint distribution model to determine the expected return or risk of the portfolio held by investors, and apply the results to the research questions.

In sum, our MEST model can be used for the scholars or practitioners under the following situations (not mutually exclusive): high-frequency data, non-linear payoff securities, risk management and investing portfolios with multiple assets. Although it will be better to demonstrate the power of this model by satisfying the four characteristics at the same time, we believe it will be better to use this model with a straightforward and interesting research question with economic meaning.

In this paper, we choose to apply the MEST model on the research question whether the USD and gold can be used as hedge or safe haven assets for domestic equity market investors. The empirical answer of this question requires a joint distribution with USD, gold and domestic equity market index as three variables to be estimated at the same time. Although the hedging ability of an asset, as defined previously, is usually not affected by the excess kurtosis of the marginal distribution, the safe haven ability should be affected by the excess kurtosis because investors would like to know what will be the risk deduction by including the USD or gold in their portfolio under extreme situations in the equity market. Hence, our research question is suitable to apply the MEST model.

Our research question can be further extended under the same econometric framework. For example, we can further diversify the domestic investors’ portfolio into a globalized equity portfolio, which contains equity index from different countries. Our result can be easily extended into this setting given the multivariate assumption in our model. There are two main reasons why we choose not to apply this model under the new setting. First, investors empirically show strong home-bias when investing in equity market. A long list of literatures starting from early 90’s, such as French and Poterba (1991), shows that although it is possible for investors to diversify their portfolios worldwide, they may not choose to do so, or at least to a lower extent. Second, if investors decide to diversify their portfolios worldwide, they will need to invest in these indexes by the local currencies. Thus, the exchange rate factors will be embedded into the equity index and, from our opinions, bring possibly more confusions to the big picture.

The other possible extension of this question is to consider whether the crisis is regional or global. To achieve that, we can include the world equity index as the fourth assets into our joint distribution estimations. In this setting, the local crisis can be identified as the extreme bad scenario for the domestic equity market, and the global crisis can be identified as the intersection of the extreme bad scenarios for both the domestic and world equity market. This will be also an interesting application of the MEST model. Given that we are not even sure whether gold and USD can be used as safe haven assets, we choose not to investigate this possibility in the current paper because we want to focus on our main research question. We believe distinguishing local and global market crisis is an interesting and separated topic to investigate in the future extension on this stream of research.

4.5. Comparison of the MEST model with ST model

We conclude this section with a brief discussion on the difference between the MEST and the ST distribution. The EST copula derived from Arellano-Valle and Genton (2010) is more flexible than the ST copula from Demarta and McNeil (2005).

First, for asymmetric distributions the concept of fat-tailness is not uniquely defined. It is entirely likely we have one ‘fat’ tail and one ‘thin’ tail in the conventional sense. The merit of the EST distribution is that it introduces an extension parameter to the ST distribution to provide further flexibility.

Second, the EST copula has a more flexible third and fourth moments than the ST copula. Please refer to Proposition 7 in Arellano-Valle and Genton (2010).

Third, it requires less restriction to retain a finite second moment in the EST copula than the ST copula. For example, the degree of freedom should be larger than four to have a finite second moment in the ST copula, but it should be larger than two in the EST copula. Please refer to Section 5.1 in Demarta and McNeil (2005) and Proposition 7 in Arellano-Valle and Genton (2010).

5. Lower tail dependence coefficient for the MEST model

Since the global financial crisis of 2007, the extreme lower tail dependence has played an increasingly important role in
measuring downside risk in finance. The lower tail dependence coefficient for the regular distribution has been well developed; see Joe (1997) and Nelsen (1999). Several studies provide approximations of lower tail dependence for some irregular distributions, such as skewed elliptical distributions. For example, Fung and Seneta (2010) study the lower tail dependence of skewed normal and ST distributions. Bortot (2010) finds that the ST is asymptotically dependent in lower tails. In this study, we derive the asymptotic lower tail dependence of the bivariate EST distribution.

The lower tail dependence coefficient for \( Y = [Y_1, Y_2]^T \) is defined as follows:

\[
\lambda_L = \lim_{u \to -\infty} P(Y_1 \leq F_1^{-1}(u)|Y_2 \leq F_2^{-1}(u))
\]

\[
= \lim_{u \to -\infty} \frac{C(u,v)}{v} = \lim_{u \to -\infty} \frac{dc(u,v)}{dv}
\]

\[
= \lim_{y \to -\infty} P(Y_2 \leq F_2^{-1}(F_1(y))|Y_1 = y)
+ \lim_{y \to -\infty} P(Y_1 \leq F_1^{-1}(F_2(y))|Y_2 = y).
\]

Suppose that \( Y \) follows a bivariate EST (\( \Phi, \theta, \tau, \nu \)) with mean zero. Define \( \Phi = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \), \( \theta = [\theta_1, \theta_2]^T \), \( \lambda_1 = \frac{\theta_1 + \rho \theta_2}{\sqrt{1 + \rho^2(1 - \rho^2)}} \), \( \lambda_2 = \frac{\theta_2 + \rho \theta_1}{\sqrt{1 + \rho^2(1 - \rho^2)}} \), \( \tau_1 = \frac{\tau}{\sqrt{1 + \rho^2(1 - \rho^2)}} \), and \( \tau_2 = \frac{\tau}{\sqrt{1 + \rho^2(1 - \rho^2)}} \). The first term of (11) takes the form:

\[
\frac{T_1 \left( \frac{\tau_1}{\sqrt{1 + \lambda_1^2}}; v \right)}{T_1 \left( \frac{\tau}{\sqrt{1 + \theta^2 \Phi \theta^T}}; v \right)} \int_{-\infty}^{t_1(z;v+1)} \frac{dz}{T_1 \left( -\lambda_1 \sqrt{v+1}; v+1 \right)}
\]

\[
\times \frac{T_1 \left\{ \left( \frac{\theta_1 \sqrt{v+1}}{\sqrt{1 + \ell_2^2}} - (\rho \theta_2 + \theta_1) \right) \sqrt{\frac{v+2}{v+1}}; v+2 \right\}}{T_1 \left( -\lambda_2 \sqrt{v+1}; v+1 \right)}
\]

where

\[
a_{21} = \left( \frac{T_1 \left( -\lambda_2 \sqrt{v+1}; v+1 \right) T_1 \left( \frac{\tau_1}{\sqrt{1 + \lambda_1^2}}; v \right)}{T_1 \left( -\lambda_1 \sqrt{v+1}; v+1 \right) T_1 \left( \frac{\tau_2}{\sqrt{1 + \lambda_2^2}}; v \right)} \right)^{1/\nu} - \rho \sqrt{\frac{v+1}{1 - \rho^2}}.
\]

Similarly, the second term of (11) is given by:

\[
\frac{T_1 \left( \frac{\tau_2}{\sqrt{1 + \lambda_2^2}}; v \right)}{T_1 \left( \frac{\tau}{\sqrt{1 + \theta^2 \Phi \theta^T}}; v \right)} \int_{-\infty}^{t_1(z;v+1)} \frac{dz}{T_1 \left( -\lambda_2 \sqrt{v+1}; v+1 \right)}
\]

\[
\times \frac{T_1 \left\{ \left( \frac{\theta_1 \sqrt{v+1}}{\sqrt{1 + \ell_2^2}} - (\rho \theta_2 + \theta_1) \right) \sqrt{\frac{v+2}{v+1}}; v+2 \right\}}{T_1 \left( -\lambda_2 \sqrt{v+1}; v+1 \right)}
\]

6. Empirical analysis

6.1. Data

Weekly data of stock indices, nominal exchange rates and the gold price are collected over the period 5 January 2000–13 November 2013. The countries under consideration are the UK, Germany, Switzerland, the USA, Canada, Japan and Australia. Our sample period begins in 2000, as the euro was launched as a currency in the financial market in 1999. All data used in this paper are obtained from Datastream. Although we carefully select our sample countries to include only one country, Germany, that switched local currency to Euro, we believe the existence of Euro will affect the use of currencies, such foreign currency reserves, and the exchange rate of Euro/USD which is affected by the economy of countries other than Germany. Because exchange rates are one of the chosen hedge assets in this study, we prefer to avoid the significant structure change associated with the introduction of Euro and thus decide to use the observations only after 2000. Moreover, we use weekly data in our investigation and feel that more than 700 observations per series suffice to ensure proper power of our econometric tests.

The stock indices collected are the UK’s FTSE All-Share Index, Germany’s DAX composite index, Switzerland’s
SMI, the US S&P500 composite, Canada’s S&P/TSX 300 composite, Japan’s Nikkei 225 Stock Average and Australia’s S&P/ASX 300 composite. The price of gold is measured in domestic currency per ounce (gold bullion in domestic currency $ per troy ounce). The nominal exchange rates are measured as the domestic currency per unit of USD (an exchange rate increase means appreciation of USD). Exchange rate data are collected for currencies as follows: the British pound (GBP), the Euro, the Swiss franc (CHF), the Canadian dollar (CAD), the Japanese yen (JPY) and the Australian dollar (AUD). The set of countries used for this study includes the vast majority of market traders in international exchange. For the US data, we considered the Broad Trade-Weighted Exchange Index (TWEXB) of the US Federal Reserve in order to examine the dependence among the US aggregate exchange rate, stock and gold price. Thus, the interpretation of the results in the US is different from that of other countries. The return series are constructed from the log-difference of the corresponding stock index, USD and gold price.

Note that all three assets used in our analysis are quoted in domestic currencies. This specification ensures that our test can be applied in real risk management practices because domestic investors can decide to invest in three alternative targets: the domestic stock index, USD and gold. Thus, the returns on these three investments need to be measured in the domestic currency, which is usually ignored in previous studies regarding the hedge and safe haven roles of gold and/or USD.

Table 1 provides the summary statistics of the weekly returns of the three assets. All average returns (in absolute values) are smaller than their standard deviations, suggesting relatively high volatilities. The annualized average returns of gold measured in different currencies are between 7.55% in Switzerland and 11.53% in the UK, while the annualized average returns of the stock market index in different countries are between −1.72% in Japan and 4.11% in Canada. Given that stock and gold show relatively closed standard deviations, this result suggests that during our sample period, gold seems to be a better investment target than stock according to their Sharpe ratios. One of the possible explanations of this finding is the choice of our sample period, which contains 2 years (2001 and 2008) or 24 months of contraction periods. This observation seems to suggest that gold is a relatively better investment target during economic downturns.

When adding the exchange rates into the analyses, stock and gold returns show higher standard deviations than the exchange rate returns. Stock and gold returns are skewed to the left, and exchange rates show no clear skewness. The negative skewness of stock returns suggests the possibility of extremely bad outcomes, which suggests the importance of safe haven assets for pure stock market portfolios. All three return series show excessive kurtosis ranging from 5.16 to 14.73. Of the three investments, stock returns usually have the highest kurtosis, indicating considerable fat tails relative to the normal distribution. Both the skewness and kurtosis suggest that none of the return series are normally distributed. The large Jarque-Bera test statistics strongly reject the normality of all the return distributions. This finding provides an important clue to support adopting a distribution more flexible than the Gaussian distribution.

To take a first look at the hedging abilities of gold and USD, we report the Pearson’s pairwise correlations and the partial correlations between the stock–USD and stock–gold return pairs in the last two columns of Table 1.‡ From the results, we observe the following patterns. First, except for the UK, the Pearson’s pairwise correlations of stock–USD pairs are considerably larger than those of stock–gold pairs in absolute value, suggesting the stronger dependence of stock–USD pairs than that of stock–gold pairs. However, the correlations range from −46.85% in Australia to 33.40% in Japan, suggesting that the hedging effects of USD exist only for some countries and that the differences in country characteristics may affect the co-movements of stock market and USD. Second, except for Australia, the stock–gold pairs are positively related, indicating that the increase (decrease) in returns of the local stock markets is associated with the increase (decrease) in the gold returns. Although the observed close to zero correlation is consistent with the hypothesis that gold can serve as a weak hedging asset for stock market performances, the generally positive correlations between the two assets indeed place a question mark on whether gold is a strong hedging asset against the domestic stock index.

As argued in previous sections, there are reasons that both gold and USD can serve as hedge or safe haven assets, but which investment is preferred for risk management purposes is still an empirical question. To control for the effect of a third asset on the correlation, we also report the partial correlations in the last column of Table 1. The general pattern of the partial correlations is consistent with that of the Pearson’s correlation for the stock–USD pairs, but the partial correlation of the stock–gold pairs seems to weaken when controlling for the USD fluctuations. When controlling the USD changes, the largest positive correlation for the stock–gold pairs drops from 21.71 to 11.96% (in Japan), and the largest negative correlation changes from −21.03 to −6.34% (in Australia.)

In general, the changes of USD, rather than the gold prices, are more correlated with the stock index changes, with or without controlling for the other alternative investments. The low correlation between stock and gold returns suggests that gold may be a weak but not a strong hedge asset for stock market movements. The large magnitude with undetermined signs of the correlations between the stock market and USD implies the possibility that USD can serve as a potential strong hedge asset for some countries, but so far, we do not have enough evidence to make a more confident conclusion about why and whether USD can be a hedge asset for some countries but not others. To provide further evidence of the roles of USD and gold in risk

‡The term ‘domestic’ means countries other than the US, such as the UK and Germany. We will use this definition throughout the paper.

‡The partial correlation between \( x \) and \( y \) after controlling for the effect of \( z \) is defined as \( r_{xy \cdot z} = \frac{r_{xy} - r_{xz}r_{yz}}{\sqrt{1 - r_{xz}^2}} \sqrt{1 - r_{yz}^2} \), where \( r_{xy} \) is the Pearson’s correlation between \( x \) and \( y \).
management, we will estimate the joint distributions of stock returns, USD returns and gold returns under our newly developed econometric framework.

### 6.2. Estimation procedure

To test the hedging and safe haven hypotheses as mentioned previously, one of the most important procedures is to measure the dependence structure among the three assets: stocks, gold and USD. This study applies the proposed MEST copula model to capture the possible skewness and fat-tailed properties and to allow for more flexibility in the joint distribution of stocks, gold and USD returns. To estimate the parameters in the proposed MEST copula model, this paper applies the canonical maximum likelihood proposed by Genest et al. (1995). We present the procedures in the following order.

1. Model selection and estimation of parameters of marginal models for each return.
2. Non-parametric transformation of filtered residuals.
3. Estimation of parameters of the MEST copula.

First, we fit each return series an AR($p$)-GJR(1,1) model (Glosten et al. 1993), using the quasi-maximum likelihood estimation. For the mean equation, the lag order ($p$) ranges from one to ten. The selection for the optimal order of lag of GJR(1,1) depends on the Bayesian information criterion. After selecting the lag length and model, the estimated parameters \( \{ \hat{a}_i, \hat{b}_i, \hat{\phi}_i, \hat{\gamma}_i, \hat{\delta}_i, \hat{\omega}_i \} \) are given by:

\[
\text{arg} \max_{a_i; \phi_i; \gamma_i; \delta_i; \omega_i} \sum_{t=1}^{T} \ln f_i(r_{it})
\]

where $f_i(r_{it})$ follows a univariate EST distribution.

After the parameters are estimated by the aforementioned maximum likelihood process, we obtain the residuals \( \hat{e}_t \) and the associated standardized residuals \( \hat{\nu}_t = \left( \frac{\hat{e}_{1,t}}{\sqrt{h_{1,t}}}, \ldots, \frac{\hat{e}_{T,t}}{\sqrt{h_{T,t}}} \right) \) for each return.

As noted in Section 4.1, this study uses a non-parametric estimation in the transformation of the standardized residuals. The transformed residuals can be obtained by:

\[
\hat{u}_{it} = \Phi(\hat{\nu}_{it}) \quad \text{with} \quad i = 1, 2, 3, \quad \text{where}
\]

---

### Table 1. Descriptive statistics of stock, gold and exchange rate return series.

<table>
<thead>
<tr>
<th>Country</th>
<th>Mean (in %)</th>
<th>Median (in %)</th>
<th>SD</th>
<th>Min</th>
<th>Max</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>J-B test</th>
<th>Pearson</th>
<th>Partial</th>
</tr>
</thead>
<tbody>
<tr>
<td>UK</td>
<td>0.0020</td>
<td>0.2400</td>
<td>0.0251</td>
<td>-0.1273</td>
<td>0.1359</td>
<td>-0.3902</td>
<td>6.8983</td>
<td>476.1520***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stock</td>
<td>0.0034</td>
<td>-0.0511</td>
<td>0.0133</td>
<td>-0.0530</td>
<td>0.0717</td>
<td>0.5149</td>
<td>5.8910</td>
<td>283.7203***</td>
<td>-0.0823</td>
<td>-0.0893</td>
</tr>
<tr>
<td>Exchange rate</td>
<td>0.2100</td>
<td>0.1600</td>
<td>0.0235</td>
<td>-0.1162</td>
<td>0.1029</td>
<td>-0.3359</td>
<td>5.7018</td>
<td>233.4951***</td>
<td>0.0473</td>
<td>0.0587</td>
</tr>
<tr>
<td>Gold</td>
<td>0.2400</td>
<td>0.1800</td>
<td>0.0232</td>
<td>-0.1225</td>
<td>0.0889</td>
<td>-0.4057</td>
<td>5.7452</td>
<td>246.8637***</td>
<td>0.1461</td>
<td>0.1106</td>
</tr>
<tr>
<td>Germany</td>
<td>Stock</td>
<td>0.0458</td>
<td>0.4600</td>
<td>0.0342</td>
<td>-0.1680</td>
<td>0.1715</td>
<td>-0.7008</td>
<td>6.9215</td>
<td>522.4521***</td>
<td></td>
</tr>
<tr>
<td>Exchange rate</td>
<td>-0.0359</td>
<td>-0.0608</td>
<td>0.0145</td>
<td>-0.0981</td>
<td>0.0509</td>
<td>-0.2029</td>
<td>5.6925</td>
<td>223.3469***</td>
<td>-0.0814</td>
<td>-0.0839</td>
</tr>
<tr>
<td>Gold</td>
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Notes: ‘SD’ and ‘J-B test’ denote the standard deviation and the Jarque-Beta normality test, respectively. ‘Pearson’ and ‘Partial’ denote the Pearson’s correlation and the partial correlation, respectively. The number of observation is 723.

***Denotes significance at 1%.
\( \bar{F}(\mu) = \frac{1}{T} \sum_{i=1}^{T} I(\mu \leq \mu_i) \) is the rescaled empirical cumulative distribution function.

Following Li (2005), the marginal densities and the copula density in our model can be individually estimated. Given the estimated marginal parameters and the rescaled empirical cumulative distribution function, \( u_1, u_2, u_3 \) as inputs of the MEST copula function, we have the trivariate EST copula function:

\[
c(u_1, u_2, u_3; \Phi, \theta(\lambda), \lambda, \tau, v) = \frac{f(F_1^{-1}(u_1), F_2^{-1}(u_2), F_3^{-1}(u_3); \Phi, \theta(\lambda), \lambda, \tau, (v + 3))}{\prod_i f(F_i^{-1}(u_i); \gamma_i, \tau, v + 1)}
\]

(15)

We can estimate the parameters \( \Psi_2 = (\Phi, \theta(\lambda), \lambda, \tau, v) \) by maximizing the log-likelihood function of the MEST copula function of (15) based on the quasi-maximum likelihood theory:

\[
\Psi_2 = (\Phi, \theta(\lambda), \lambda, \tau, v)
\]

(16)

### 6.3. Estimation results of marginal models

Table 2 reports the estimates of the marginal models for returns of the stock price, USD and the gold price. The slope parameters in the conditional variance function are generally significant at conventional levels.

To examine whether the selected models adequately remove the intertemporal dependence in the returns, we examine the serial correlation and autoregressive conditional heteroscedasticity in the residuals with the Q and ARCH-LM tests. The Q statistics fail to reject the hypothesis of no serial correlation at the 5% level. Meanwhile, the ARCH-LM statistics fail to reject the hypothesis of no autoregressive conditional heteroscedasticity in the residuals at the 5% level.

### 6.4. Estimation results of the EST copula model

After estimating the filtered residuals in the first stage, we estimate the MEST copula model for each country. The copula parameter estimates (\( \hat{v}, \hat{\lambda}_1, \hat{\lambda}_2, \hat{\lambda}_3 \) and \( \hat{\tau} \)) are reported in Table 3. The joint distribution of the markets in the UK has heavier tails than those in other countries, as its estimated degrees of freedom \( \hat{v} \) is relatively small, suggesting a higher joint probability of extreme events in stock, exchange rates and gold markets. The estimated shape parameters \( \hat{\lambda}_1, \hat{\lambda}_2 \) and \( \hat{\lambda}_3 \) which control the skewness of the marginal and joint distributions, are significant in all examined countries, indicating that the joint distributions of the three markets for all countries are asymmetric. The estimated extension parameter \( \hat{\tau} \), which captures the heavy/thin-tailed property of joint distribution, is significant in all countries except Canada, indicating that the EST copula model is more appropriate than the ST copula model based on Azzalini and Capitanio (2003). Overall, the significance of the shape and extension parameters indicates that skewness and heavy/thin-tailed features exist in most of the markets, demonstrating the suitability of our proposed model.

### 6.5. Unconditional analysis of hedging and the safe haven effect of gold and USD

Recall that the first aim of our investigation is to examine hypotheses H1a (gold is a hedge against stocks) and H1b (USD is a hedge against stocks). First, we analyse the overall dependence of the stock–gold pair and stock–USD returns separately without considering the interaction between gold and USD. The conventional way to examine the dependence structure in the copula model is through the single copula parameter. In our study, the MEST copula model has multiple parameters, and the significance of the parameters in MEST copula is not appropriate for the analysis of the dependence of variables. We use the Kendal’s \( \tau \), which measures the association between two distributions using a non-parametric method of ordering as an index of the overall dependence measure, and we use the lower tail dependence coefficient (\( \hat{\lambda}_L \)), which measures the probability of extreme outcomes of one distribution conditional on the observation of extreme outcomes on the other distribution, to measure tail dependence. These quantities are defined as follows:

\[
\tau = 4 \int_{[0,1]^2} C(u_1, u_2) dC(u_1, u_2) - 1
\]

(17)

\[
\hat{\lambda}_L = \lim_{u \to 0^+} \Pr[F_1(y_1) < u | F_2(y_2) < u].
\]

(18)

#### 6.5.1. Hedging analysis of gold and USD

Table 4 reports the unconditional analysis for the Kendal’s \( \tau \) and \( \hat{\lambda}_L \) for the pairs of stock and USD returns and of stock and gold returns for different countries. Panel A.1 presents the Kendal’s \( \tau \) for the stock–gold returns. The estimated Kendal’s \( \tau \) varies from 0.0038 to 0.0148. The order of Kendal’s \( \tau \) is qualitatively different from that of the Pearson’s correlation in Table 1. Canada shows the strongest positive dependence, followed by Japan and the UK. On the other hand, Australia shows the weakest dependence among the countries in the sample. Due to the positive dependence between stock and gold returns, i.e. stock and gold prices move in the same direction, the hypothesis of gold acting as a hedge against stocks in all the countries considered in the sample should be rejected.

Panel B.1 of Table 4 presents the Kendal’s \( \tau \) for the stock–USD pair. Note that the measure of the exchange rate series (USD) in the US is, by nature, different from other currencies and should be treated differently. The estimated Kendal’s \( \tau \) varies from −0.0142 to 0.0213. Approximately half of the sample countries—those other than the USA,
### Table 2. Estimation results of marginal functions.

<table>
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<th>UK</th>
<th>Germany</th>
<th>Switzerland</th>
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<td>Stock AR(3)-GJR</td>
<td>Exchange rate AR(3)-GJR</td>
<td>Gold AR(7)-GJR</td>
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<td>0.0119***</td>
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<th>Japan</th>
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|                | Stock AR(1)-GJR             | Exchange rate AR(4)-GJR     | Gold AR(1)-GJR              |
| \(\alpha\)     | 0.0082***                   | -0.0007                     | -4.31e-05                   |
| (0.0018)       | (0.0006)                    | (0.0051)                    |
| \(\beta_i\)    | -0.1039**                   | -0.0837**                   | 0.0407                      |
| (0.0399)       | (0.0361)                    | (0.0619)                    |
| \(\beta_p\)    | -0.0526*                    | 0.0091                      | 0.0003                      |
| (0.0316)       | (0.0331)                    | (0.0344)                    |
| \(\phi\)       | 2.12e-05**                  | 2.58e-06***                 | 3.11e-05***                 |
| (1.08e-05)     | (3.66e-07)                  | (4.25e-06)                  |
| \(\gamma\)     | 0.1951**                    | 0.1310**                    | 0.0863**                    |
| (0.0960)       | (0.0319)                    | (0.051)                     |
| \(\delta\)     | 0.8253**                    | 0.8439**                    | 0.8543**                    |
| (0.0623)       | (0.0430)                    | (0.0430)                    |
| \(w\)          | 1.39e-05                    | 3.18e-05                    | 1.07e-06                    |
| (0.0896)       | (0.0804)                    | (0.921e-05)                 |
| \(Q\)          | 19.8519                     | 13.2917                     | 25.5283                     |
| (0.4672)       | [0.8645]                    | [0.1820]                    |
| LM             | 0.3430                      | 0.2120                      | 0.1343                      |
| (0.5100)       | [0.6383]                    | [0.7140]                    |

<p>|                | Stock AR(5)-GJR             | Exchange rate AR(4)-GJR     | Gold AR(1)-GJR              |
| (\alpha)     | 0.0143**                    | 0.0005                      | 0.0023**                    |
| (0.0058)       | (0.0011)                    | (0.0008)                    |
| (\beta_i)    | -0.0122                     | 0.0255                      | -0.0702                     |
| (1.0058)       | (0.0493)                    | (0.0638)                    |
| (\beta_p)    | -0.0555***                  | 0.0465                      | -0.0555***                  |
| (0.1008)       | (0.0434)                    | (0.0171)                    |
| (\phi)       | 0.0001**                    | 1.26e-05***                 | 3.26e-05***                 |
| (5.09e-05)     | (1.16e-06)                  | (3.39e-06)                  |
| (\gamma)     | 0.1089*                     | 0.0712*                     | 0.111***                    |
| (0.6211)       | (0.0304)                    | (0.0430)                    |
| (\delta)     | 0.8023**                    | 0.8575**                    | 0.8422**                    |
| (0.0982)       | (0.0751)                    | (0.0819)                    |
| (w)          | 0.0692                      | 0.0146                      | 0.0819                      |
| (0.1462)       | (0.0141)                    |
| (Q)          | 19.8519                     | 13.2917                     | 25.5283                     |
| (0.4672)       | [0.8645]                    | [0.1820]                    |
| LM             | 0.3430                      | 0.2120                      | 0.1343                      |
| (0.5100)       | [0.6383]                    | [0.7140]                    |</p>
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</tr>
</tbody>
</table>

Notes: \( r_i \) denotes log returns of stock price, exchange rate and gold price. Numbers in brackets are \( p \)-values and numbers in parentheses are standard deviations. \( Q \) stands for \( Q \)-statistics for testing the hypothesis of no serial correlation. \( LM \) stands for ARCH-LM statistics for the hypothesis of no autoregressive conditional heteroscedasticity.

***Denote significance at 1%.
**Denote significance at 5%.
*Denote significance at 10%.
Canada and Australia—show a positive dependence, with the strongest dependence for Japan. For the countries with negative dependences, Canada shows the strongest negative dependence, followed by Australia and the USA. From the estimation results in this table, we conclude that USD could act as a strong hedge against stocks in Canada and Australia, which is also consistent with our findings in Table 1. This partially supports our hypothesis H1b.

For the evaluation of the hedging role of gold and USD, the analysis given the MEST distribution assumption does not differ much from our previous results. Although the overall dependences observed for stock–USD pairs here are all positive, they are relatively small in magnitude, which is consistent with the corresponding Pearson’s correlations reported in Table 1. On the other hand, USD could serve as a hedge asset for investors in Canada and Australia, which is also consistent with the previous findings. Although it seems that our new econometric method does not provide too many qualitatively different results, the goal here is to provide a benchmark when we evaluate the safe haven roles of gold and USD. Therefore, we can later judge whether these assets can be correlated (uncorrelated) with the domestic stock performance in general but uncorrelated (correlated) when domestic stock markets crash.

### 6.5.2. Weak safe haven analysis of gold and USD

Panel A2 of Table 4 shows the \( \lambda_L \) for the stock–gold returns. For the observed tail dependences, we choose 0.01 as the threshold for being a weak safe haven in our study.† Recall that the tail dependence measures the probability of extreme outcomes of one distribution conditional on the observation of extreme outcomes on the other distribution, and the choice of the 0.01† threshold means that when the stock is doing extremely poorly and passes the threshold, there is a 1% or higher chance that the alternative investment, such as gold or USD, will also perform extremely poorly. From our estimation, Switzerland has the largest \( \lambda_L \) (0.0563), followed by Germany (0.0045) and the UK (0.0003). Except for Switzerland, the \( \lambda_L \) for other countries are less than 0.01 and are insignificant at 5% statistical level. Our results show that gold can act as a weak safe

---

**Table 3. Estimated coefficient of EST copula functions.**

<table>
<thead>
<tr>
<th></th>
<th>UK</th>
<th>Germany</th>
<th>Switzerland</th>
<th>USA</th>
<th>Canada</th>
<th>Japan</th>
<th>Australia</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(3.7855)</td>
<td>(2.7930)</td>
<td>(2.4363)</td>
<td>(0.0850)</td>
<td>(0.1199)</td>
<td>(1.9648)</td>
<td>(0.8971)</td>
</tr>
<tr>
<td>( \lambda_1 )</td>
<td>0.1117</td>
<td>0.0379***</td>
<td>0.0811***</td>
<td>-1.5834</td>
<td>0.2283***</td>
<td>0.3050***</td>
<td>-0.3813***</td>
</tr>
<tr>
<td></td>
<td>(0.0789)</td>
<td>(0.0065)</td>
<td>(0.0193)</td>
<td>(1.0949)</td>
<td>(0.0573)</td>
<td>(0.0146)</td>
<td>(0.0689)</td>
</tr>
<tr>
<td>( \lambda_2 )</td>
<td>7.6448***</td>
<td>9.1266***</td>
<td>9.2239***</td>
<td>0.1906***</td>
<td>0.3038***</td>
<td>0.3828***</td>
<td>10.1732***</td>
</tr>
<tr>
<td></td>
<td>(3.1728)</td>
<td>(2.8844)</td>
<td>(2.3009)</td>
<td>(0.0783)</td>
<td>(0.1029)</td>
<td>(0.0296)</td>
<td>(3.1518)</td>
</tr>
<tr>
<td>( \lambda_3 )</td>
<td>0.0983</td>
<td>0.1042***</td>
<td>0.2708***</td>
<td>0.0124</td>
<td>3.9434***</td>
<td>10.8447***</td>
<td>0.3789***</td>
</tr>
<tr>
<td></td>
<td>(0.0745)</td>
<td>(0.0056)</td>
<td>(0.0580)</td>
<td>(0.0534)</td>
<td>(0.8328)</td>
<td>(1.3227)</td>
<td>(0.0285)</td>
</tr>
<tr>
<td>( \tau )</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>(10.0031)</td>
<td>(2.4800)</td>
<td>(8.2504)</td>
<td>(0.0004)</td>
<td>(13.1143)</td>
<td>(3.6512)</td>
<td>(1.0684)</td>
</tr>
</tbody>
</table>

Notes: Numbers in parentheses are standard deviations. \( \lambda \) are the shape parameters for marginal distribution, \( \tau \) is an extension parameter and \( \nu \) is the degree of freedom.

***Denote significance at 1%.
**Denote significance at 5%.
*Denote significance at 10%.

---

**Table 4. Unconditional dependence for hedge and weak safe haven investigations.**

<table>
<thead>
<tr>
<th></th>
<th>UK</th>
<th>Germany</th>
<th>Switzerland</th>
<th>USA</th>
<th>Canada</th>
<th>Japan</th>
<th>Australia</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A1: Kendall’s ( \tau ) between stocks and gold</td>
<td>0.0131</td>
<td>0.0103</td>
<td>0.0124</td>
<td>0.0057</td>
<td>0.0148</td>
<td>0.0140</td>
<td>0.0038</td>
</tr>
<tr>
<td>Panel A2: ( \lambda_L ) between stocks and gold</td>
<td>0.0003</td>
<td>0.0045*</td>
<td>0.0563***</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Panel B1: Kendall’s ( \tau ) between stocks and USD</td>
<td>0.0059</td>
<td>0.0033</td>
<td>0.0157</td>
<td>-0.0133</td>
<td>-0.0142</td>
<td>0.0213</td>
<td>-0.0138</td>
</tr>
<tr>
<td>Panel B2: ( \lambda_L ) between stocks and USD</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0165***</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0148**</td>
</tr>
</tbody>
</table>

Notes: We report the estimates of Kendall’s \( \tau \) and lower tail dependence coefficient for stock–gold (Panel A) and stock–USD (Panel B) pairs using MEST copula model.

***Denote significance at 1%.
**Denote significance at 5%.
*Denote significance at 10%.

†Although there is no closed-form solution for the distribution of the tail dependence, it is possible to use bootstrapping to obtain the approximate significances.

‡Under Basel Accord, financial institutions need to calculate 10-day VaR with 99% confidence level for the financial assets on the trading book. Because we use weekly data, which is somewhat close to the 10-day interval as regulated, we believe a 1% threshold is reasonable also for the practice matters.
Hedges or safe havens role of gold and USD against stock

6.5.3. Strong safe haven effect of gold and USD. The lower tail dependence coefficient with a range between zero and one is used to detect whether two factors have positive or zero correlations. Because a strong safe haven is defined as an asset that is negatively correlated with another asset under extreme market conditions, the lower tail dependence coefficient estimated in the previous section is not appropriate to verify whether gold/USD is a strong safe haven against a domestic stock market crash. According to the definition, a strong safe haven can nevertheless be deduced in another way: an asset that is positively correlated with another asset with negative values. This implies that hypothesis H3a (H3b) can be identified by determining whether stock returns and negative gold returns (negative USD returns) are positively correlated. Table 5 reports the estimated \( \lambda_L \) with negative gold returns and negative USD returns.

Panel A of Table 5 shows the \( \lambda_L \) for the stock and negative gold returns. The estimated \( \lambda_L \) varies from 0.0002 in Switzerland to 0.1535 in Germany. In three of the seven countries in our sample, the \( \lambda_L \) are above 0.1 and are significant at 5% statistical level, which suggests there is a greater than 10% chance that the gold price is going to surge when the stock market crashes. The results are broadly consistent with H3a, supporting that gold is a strong safe haven against stocks in the USA, the UK, Germany and, probably, Australia. Panel B of Table 5 shows the \( \lambda_L \) for the stock and negative USD returns. The estimated \( \lambda_L \) are all greater than 0.01 and are significant at 5% statistical level except for the USA and Switzerland, which have a near-zero value. Four of the seven countries in our sample have \( \lambda_L > 0.1 \). The results support hypothesis H3b that USD is a strong safe haven against stocks in the UK, Germany, Japan, Australia and, probably, Canada.

6.5.4. Discussion. To conclude, we find the following observations from our estimated model. Gold serves as at the best weak hedge asset against the stock market performances for all seven countries, but USD can possibly serve as strong hedge asset for Canada and Australia. During a stock market crisis, gold can serve as a weak safe haven asset for all countries except Switzerland and as a strong safe haven asset for the UK, and Germany and the USA. USD can serve as a weak safe haven asset for almost all countries, with the exception of Switzerland and Australia, and as a strong safe haven asset for the UK, Germany, Japan and Australia.

The strong hedging effects of USD for Canadian and Australian investors are consistent with the empirical results in Campbell et al. (2010). Without considering our generalization on the return distributions, our empirical model is qualitatively similar to the first model presented in their paper, which examines the case of an investor who is fully invested in a single-country equity portfolio and is considering whether exposure to one of the other currencies (USD only, in our paper) would help reduce her portfolio volatilities. They find that only Canadian and Australian investors would optimally choose to hold significant long position on the USD, implying that the USD can help hedge their exposure to the domestic stock market risk. They attribute these findings to the observation that Australian and Canadian dollars tend to depreciate against all currencies when their stock markets fall, which may be caused by the unusually...
commodity-dependent economies in Canada and Australia, and thus foreign currency (USD) can serve as a hedge against fluctuations in these stock markets.†

Our results regarding the (insignificant) safe haven property of gold and USD in Switzerland are consistent with the findings in Ranaldo and Soderlind (2010). They find that for the US investors, Swiss franc shows the strongest safe haven patterns with the most significant coefficients and the largest R-squares. They argue that their findings are consistent with the traditional view that Swiss franc provide on average safe haven or hedging benefits to the US investors. Their arguments imply that neither USD nor gold may be necessary for investors in Switzerland to hedge their risk in normal or extreme market time.‡ In contrast, we find the investors in the UK can enjoy the most benefits from holding USD and gold as safe haven assets, consistent with Ranaldo and Soderlind’s findings that British pound shows the weakest safe haven pattern for the US investors.

From the summary of these results, we note the following patterns. First, although USD can serve as a hedge asset for investors in some countries, it is positively correlated with the domestic stock market in others. This finding implies that USD cannot serve as a hedge asset for each country and thus the hedging ability of USD should be correlated with some country-specific characteristics, such as imports and exports. Second, if an alternative investment is a strong safe haven asset against stock market crashes, it is also a weak safe haven asset.§ Note that although this pattern seems to be guaranteed, it is not necessarily true in theory. For example, if the gold (or USD) returns become extremely volatile when stock markets crash, we might observe a very high tail dependence on both sides, as we did in detecting the weak and strong safe haven assets. Third, if an alternative investment is a hedge asset against normal stock market movements, it is not necessarily a good safe haven asset against stock market crashes. We can illustrate this idea by comparing the results of the stock–USD pair in Canada and Japan. The general co-movement as measured by Kendall’s τ is negative and large in magnitude in Canada. However, USD can serve as only a weak safe haven for Canadian investors. On the other hand, the Pearson’s coefficient and Kendall’s τ between USD and stock market returns are all positive in Japan, indicating that USD is not a good hedge asset for Japanese investors. However, USD serves not only as a weak safe haven, but also a strong safe haven during Japanese stock market crashes.

These findings, combined with the discussions above, suggest that the fundamental factors driving the general co-movement and the co-movement under extreme market conditions might be different. The above findings are consistent with our hypotheses that the hedging ability of these alternative assets may be related to the inflation hedging ability, as argued by Jaffe (1989) and McCown and Zimmernan (2007), while the safe haven ability of these assets may be caused by the flight-to-quality hypothesis suggested by Caballero and Krishnamurthy (2008). A more detailed analysis is beyond the scope of our current study and should prove an interesting topic for further studies. However, our methodologies provide a general framework for future researchers to analyse and distinguish the hedging and safe haven roles of alternative investments.

Our finding of significant tail dependence in the stock–gold and stock–USD pairs has important implications in risk management and asset pricing. A left tail dependence indicates the potential of a simultaneous large loss in both the stock and gold (or USD) markets. This joint downside risk has been well documented in the literature of stock market. Stock foreign exchange rate joint downside risk is important to global investors when investing in foreign stock markets. For example, an extreme market event involving a 10% loss in the UK stock market would, under stock foreign exchange rate tail dependence, imply the potential of a large loss, for instance 5%, in the exchange rate. Consequently, a US investor who invested in the UK stock market would suffer from a loss which is larger than 10% in USD. Thus, the existence of lower tail dependence implies a much higher downside risk in foreign stock market investment than the case of no tail dependence.

Although in this paper we do not suggest a portfolio choice which can optimize the downward risk under the extreme market condition, our result does imply that the traditional mean-variance efficient portfolio may not protect investors under stock market crashes. The main reasons of this result come from two realistic assumptions: First, the asset returns are not distributed normally, and second, the dependence between two return series are not uniformly identical in different regions of their distributions. Hence, a portfolio which is efficient given normal market conditions may not be efficient under extreme market conditions, and investors may need to make some trade-off when deciding their asset allocations if they would want to protect their investment from market crashes.

### 6.6. Conditional safe haven analysis based on the joint estimation of gold, USD and stock

In the existing literature, gold and USD are considered candidates for hedge or safe haven assets against stocks. From our analysis above, gold and USD can serve as safe haven assets for most of the countries studied. An interesting question is whether investors in these countries should
choose gold or USD for hedging against domestic stock market crashes. Moreover, we also want to know whether the safe haven effects exist when two alternative investments are included in the portfolio. To answer these questions, we need to consider stock market performance, gold and USD simultaneously. However, to our knowledge, there is no literature discussing the joint behaviour of gold, USD and stocks. The econometric framework developed in our paper can help us easily analyse the joint behaviour of these three assets and choose the strongest one. Particularly, we try to examine hypotheses H4a and H4b by studying the extreme dependence of two pairs: stock–gold conditional on the USD market and stock–USD conditional on the gold market.

6.6.1. Conditional analysis of joint lower tail probability. As derived earlier, the conditional joint lower tail probability can be used to test whether stock and one alternative investment are correlated when the other alternative investment is not performing well. In other words, we can use the joint lower tail probability to test whether gold (USD) can serve as a strong safe haven asset when USD (gold) performs extremely poorly.

To estimate the joint lower tail probability, we first calculate the joint density of the \( y_1 \) and \( y_2 \) markets conditional on the \( y_3 \) markets:

\[
\hat{f}(y_1, y_2|y_3 \in \Delta),
\]

where \( \Delta \) refers to a given region of either gold or USD returns. The conditional joint tail probability is then obtained as follows:

\[
P_L = \Pr[y_1 < F^{-1}_1(z) \text{ and } y_2 < F^{-1}_2(z)|y_3 \in \Delta] = C(x, z|y_3 \in \Delta), \text{ } z \in (0, 1)
\]

where \( C(\cdot) \) refers to the MEST copula distribution function. In particular, we consider two extreme market scenarios:

\[
\Delta_L = \{y_3 : F^{-1}_3(0) \leq y_3 \leq F^{-1}_3(5\%\})
\]

\[
\Delta_H = \{y_3 : F^{-1}_3(95\%) \leq y_3 \leq F^{-1}_3(100\%\})
\]

where \( F_3 \) is the marginal distribution of either gold or USD. We report the joint lower tail probability using the above specifications in Table 6.

In Panel A of Table 6, the estimated joint probabilities of stock and gold returns conditional on the USD market under the 5, 3 and 1% quantiles are reported. As shown in the results, the estimates of the lower joint tail probability vary from 0.0268% (0.5020%) to 0.0301% (0.5630%) at the 1% (5%) quantile, as USD returns fall between the 0 and 5% quantiles. Note that countries with a higher joint lower probability with one specific \( \alpha \) always have a higher joint lower probability with an alternative choice of \( \alpha \) as well. Thus, we will interpret the results only when \( \alpha = 1\% \), and the interpretations should be extended to different choices of \( \alpha \).

If the gold and stock returns are perfectly correlated given the conditional information set of the choices of USD changes, the reported joint lower probability should equal \( \alpha = 0.01, \) or 1%. On the other hand, if the two returns are uncorrelated, the reported joint lower probability should equal to 0.0001, or 0.01%. With greater than 0.01% joint lower probabilities across all countries or different conditional information sets, the results imply that when the USD returns fall into the extreme cases on both tails, the gold and stock returns tend to fall into the lower tails more often than when these two asset returns are uncorrelated.

Although the above pattern exists across all countries, when compared with other countries, the estimated joint lower tail probabilities in Canada are smaller at various levels of quantiles. This indicates that in Canada, gold is relatively uncorrelated with stocks in terms of extreme market movements when USD returns are between the 0 and 5% quantiles. In contrast, the estimates of joint lower tail probability vary from 0.0206% (0.3915%) to 0.0383% (0.7047%) at the 1% (5%) quantile, as USD returns are between the 95 and 100% quantiles, and the estimated joint lower tail probabilities in Japan are smaller than those in other countries. Thus, gold may act as a relatively weak safe haven for Canadian investors when the USD strongly appreciates and for Japanese investors when the USD strongly appreciates. However, gold cannot be a strong safe haven for all the countries conditional on strong deprecations of domestic currencies because the joint lower tail probabilities for all countries are above 0.01%, which is the threshold for conditional zero correlation.

Panel B of Table 6 presents the conditional joint probability of stock–USD pairs under the 5, 3 and 1% quantiles. Because the gold returns fall between the 0 and 5% quantiles, the estimated joint lower tail probabilities vary from 0.0275% (0.5154%) to 0.0363% (0.6737%) at the 1% (5%) quantiles. Of all the countries considered, the estimated joint lower tail probability in Japan is smaller than those in other countries at every quantile. This observation indicates that USD is relatively uncorrelated with Japanese stocks in terms of extreme market movement when gold returns fall between the 0 and 5% quantiles. When gold is in the bull market, the estimated joint lower tail probabilities vary from 0.0215% (0.4071%) to 0.0392% (0.5723%) at the 1% (5%) quantile. This indicates that there are 0.03% (0.50%) chances on average for the stock and USD to decrease together at the lowest 1% (5%) quantile. Among the markets in the sample, the estimated lower joint tail probability of USD and Japanese stocks is the smallest given that the gold returns fall into either the top or bottom quantiles, indicating the probability of simultaneous large losses in USD and Japanese stocks is smaller than the pairing of USD and other stocks. This indicates that USD is most likely a weak safe haven for Japanese investors. Again, similar to gold, USD cannot be a strong safe haven for all the countries conditional on large drops in gold prices. Thus, neither USD nor gold can serve as strong safe haven assets when the other alternative asset performs poorly.
6.6.2. Conditional independence analysis of lower tail probability. The above empirical results indicate that the safe haven effects of both gold and USD disappear when the other alternative investment is not doing well. This finding implies that the inclusions of both USD and gold in domestic stock portfolios will not provide stronger safe haven effects. However, it is still possible that the dependencies between stock and USD (gold) can be close to zero in certain circumstances, thus leading to weak safe haven effects of both gold and USD disappear when controlling for extremely poor performance of USD and gold prices, and gold can be a weak safe haven for the domestic stock markets. For example, we can calculate the conditional density of the stock market conditional on gold returns in the USA and Australia. However, from the results in Panel B of Table 7, the differences of the two conditional probabilities in absolute value vary from 0.3938% (the UK) to 6.9862% (the USA). This finding reveals that stock returns and USD returns are independent conditional on gold returns in the UK and Germany, as the difference between the two probabilities is less than 1%. Thus, we argue that USD can be a weak safe haven for the UK and Germany conditional on large drops in gold prices, and gold can be a weak safe haven for the USA and Australia conditional on the poor performance of USD.

\[ f(y_1|y_3 < F_3^{-1}(x), y_2 < F_2^{-1}(x), z) \]  

(23)

where \( y_1, y_2 \) and \( y_3 \) refer to stock, USD and gold returns, respectively. \( z \) refers to a given quantile. The corresponding conditional tail probability is then obtained by \( \Pr[y_1 < F_1^{-1}(x)|y_2 < F_2^{-1}(x), y_3 < F_3^{-1}(x)] \). In addition, we calculate the conditional density of the \( y_1 \) conditional on the \( y_3 \):

\[ \hat{f}(y_1|y_3 < F_3^{-1}(x)) \]  

(24)

Similarly, the conditional tail probability on \( y_3 \) is \( \Pr[y_1 < F_1^{-1}(x)|y_3 < F_3^{-1}(x)] \). If two conditional probabilities are equivalent, then stock returns and USD returns are independent conditional on a specific status of gold returns.

\begin{table}[h]
\centering
\caption{Joint safe haven effects: conditional joint lower tail probability.}
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline
\textbf{Exchange rate} & \textbf{UK} & \textbf{Germany} & \textbf{Switzerland} & \textbf{USA} & \textbf{Canada} & \textbf{Japan} & \textbf{Australia} \\
\hline
\textbf{Panel A: joint lower tail probability between stocks and gold conditional on USD (in %)} & & & & & & & \\
\hline
\textbf{0–5\%} & \( \alpha = 1\% \) & 0.0286 & 0.0286 & 0.0286 & 0.0301 & 0.0268 & 0.0280 & 0.0277 \\
\textbf{0–5\%} & \( \alpha = 3\% \) & 0.2032 & 0.2032 & 0.2033 & 0.2138 & 0.1904 & 0.1991 & 0.1971 \\
\textbf{95–100\%} & \( \alpha = 1\% \) & 0.5358 & 0.5359 & 0.5362 & 0.5630 & 0.5020 & 0.5251 & 0.5197 \\
\textbf{95–100\%} & \( \alpha = 3\% \) & 0.0261 & 0.0264 & 0.0235 & 0.0383 & 0.0284 & 0.0206 & 0.0284 \\
\textbf{95–100\%} & \( \alpha = 5\% \) & 0.1858 & 0.1882 & 0.1677 & 0.2695 & 0.2017 & 0.1474 & 0.2017 \\
\hline
\textbf{Panel B: joint lower tail probability between stocks and USD conditional on gold (in %)} & & & & & & & \\
\hline
\textbf{Gold} & \( \alpha = 1\% \) & 0.0293 & 0.0293 & 0.0283 & 0.0363 & 0.0300 & 0.0275 & 0.0299 \\
\textbf{Gold} & \( \alpha = 3\% \) & 0.2081 & 0.2083 & 0.2014 & 0.2566 & 0.2134 & 0.1954 & 0.2127 \\
\textbf{Gold} & \( \alpha = 5\% \) & 0.5488 & 0.5493 & 0.5312 & 0.6737 & 0.5628 & 0.5154 & 0.5610 \\
\hline
\textbf{Notes:} & & & & & & & \\
We report the estimates of the conditional joint tail probability for stock–gold and stock–USD pairs. The upper panel reports the estimated joint probabilities of stock and gold returns, conditioning on USD market, under the 5, 3 and 1% quantiles. The lower panel reports the estimated joint probabilities of stock and USD returns, conditioning on gold market, under the 5, 3 and 1% quantiles.
\end{tabular}
\end{table}

6.6.3. Discussion. In general, when stocks and two alternative investments are considered within one framework, the results provide some evidence consistent with our hypotheses regarding safe haven properties and some new insights requiring our investigation. When one alternative investment (gold or USD) is performing extremely well, the chances that stock and the other alternative investment will drop together are often high. This finding can be consistent with the flight-to-quality hypothesis. Suppose that domestic investors can invest in only these three assets.
When very good news comes with one asset, investors may choose to withdraw their investments in the other two assets and cause a sudden drop in the values of these assets.

However, when we check the cases when the conditional asset returns fall into the bottom quintile, the lower tail dependencies are close to, or even larger than, their counterpart when the conditional asset returns fall into the top quintile. In other words, when USD (gold) prices drop significantly, the chances that stock and gold (USD) also drop significantly are higher than assuming that the latter two are uncorrelated. Thus, the strong safe haven effects as detected in the two-variable model no longer exist when the third asset’s returns also fall into the bottom quintile. Although the results under the conditional independence analysis still show that gold (USD) can still conditionally serve as a weak safe haven asset for a subset of the countries identified in the unconditional analysis, the zero or positive conditional correlation implied in our model still brings into question the joint safe haven effects of gold and USD.

The above findings may be a surprising result at first glance, but they are in fact consistent with economic intuition. Sandoval and Franca (2012) show that the global stock market tends to have a high correlation during great crashes, and this is also the time when global stock market volatilities are high (see e.g. Schwert 2011). Thus, the conditional distributions of all of the assets, including stocks, tend to have fatter tails than their unconditional distributions, and the chances that the remaining assets also perform extremely poor will be higher than those for unconditional cases. However, this observation does raise the question of whether the safe haven properties of these alternative investments may exist only under some circumstances. The trivariable cases used here demonstrate the situation when the prices of the assets used as safe haven assets also drop significantly, and we should be cautious in making quick conclusions about the safe haven roles of these assets before considering the other alternative investment performances and other macroeconomic issues.

In short, our conditional analysis provides an important answer for a usually ignored question: Can investors use two or more alternative investments to prevent large losses during stock market crashes? Our results indicate that the safe haven effects of USD and gold will decline when both assets are used as safe haven assets. Neither asset will dominate the other during stock market crashes, and the joint safe haven strategy does not outperform the single safe haven strategy. Previous studies with no consideration of the joint properties of USD and gold may lead to a naive conclusion for risk management strategies.

7. Robustness test

In this section, we test whether the returns on the exchange rate influence the overall and tail correlations of stocks and gold returns using the gold returns evaluated in terms of USD. Although we believe the approach used in our paper, evaluating gold returns in terms of domestic currencies, is more consistent with the portfolio choice problems faced by domestic investors, it is common to see scholars use gold returns in only one currency. The advantage of the latter approach is that we can isolate the exchange rate fluctuations from the gold returns. To check whether our estimations are affected by the specifications of the data in the paper, we reevaluate Kendal’s $\tau$ and $\lambda_L$ of stock–gold in Table 4 and present the revised results in Table 8.

The estimated Kendal’s $\tau$ and $\lambda_L$ between stocks and gold in Table 8 are broadly consistent with the results in Table 4, except probably for Switzerland. The correlation between stock and gold during the normal and extreme times is slightly different for Switzerland. The correlation provides qualitatively similar conclusions.

8. Comparison of lower tail dependence coefficients with the ST copula model

Another question to ask is whether our more generalized MEST copula model can provide more accurate or different results from the more restricted approaches in previous studies. We compare our proposed model with the ST copula model discussed by Demarta and McNeil (2005) and Christoffersen et al. (2012) in the estimation of $\lambda_L$. The results are presented in Table 9. The estimated lower tail dependence coefficients by the MEST copula model are systematically lower than those by the ST copula model. This property strengthens the importance of controlling heavy/thin tails in the estimation of joint distributions.

The results reported in Table 9 are the estimates of the lower tail dependence coefficients for stock–gold and

<table>
<thead>
<tr>
<th>Panel</th>
<th>Joint lower tail probability between stocks and gold conditional on USD (in %)</th>
<th>UK</th>
<th>Germany</th>
<th>Switzerland</th>
<th>USA</th>
<th>Canada</th>
<th>Japan</th>
<th>Australia</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: joint lower tail probability between stocks and gold conditional on USD (in %)</td>
<td>$P_x(y_1 &lt; F_{y_1}^{-1}(y_2)</td>
<td>y_2 &lt; F_{y_2}^{-1}(z))$</td>
<td>1.6715</td>
<td>1.6753</td>
<td>1.6738</td>
<td>1.8050</td>
<td>1.6169</td>
<td>1.6483</td>
</tr>
<tr>
<td>$P_x(y_1 &lt; F_{y_1}^{-1}(z)</td>
<td>y_2 &lt; F_{y_2}^{-1}(z), y_3 &lt; F_{y_3}^{-1}(x))$</td>
<td>1.7132</td>
<td>1.7042</td>
<td>1.7148</td>
<td>1.7934</td>
<td>1.6784</td>
<td>1.7011</td>
<td>1.6696</td>
</tr>
<tr>
<td>% difference between two probability</td>
<td>2.4948</td>
<td>1.7251</td>
<td>2.4495</td>
<td>-0.6427</td>
<td>3.8036</td>
<td>3.2033</td>
<td>-0.3938</td>
<td></td>
</tr>
<tr>
<td>Panel B: conditional lower tail probability between stocks and USD conditional on gold (in %)</td>
<td>$P_x(y_1 &lt; F_{y_1}^{-1}(z)</td>
<td>y_2 &lt; F_{y_2}^{-1}(z))$</td>
<td>1.7122</td>
<td>1.7195</td>
<td>1.6515</td>
<td>1.9281</td>
<td>1.7967</td>
<td>1.6042</td>
</tr>
<tr>
<td>$P_x(y_1 &lt; F_{y_1}^{-1}(z)</td>
<td>y_2 &lt; F_{y_2}^{-1}(z), y_3 &lt; F_{y_3}^{-1}(x))$</td>
<td>1.7132</td>
<td>1.7142</td>
<td>1.7148</td>
<td>1.7934</td>
<td>1.6784</td>
<td>1.7011</td>
<td>1.6696</td>
</tr>
<tr>
<td>% difference between two probability</td>
<td>0.0584</td>
<td>-0.3082</td>
<td>3.8329</td>
<td>-6.9862</td>
<td>-6.4126</td>
<td>-6.0404</td>
<td>-6.4126</td>
<td></td>
</tr>
</tbody>
</table>

Notes: We report the conditional lower tail probabilities of $y_1$ on $y_2$ and $y_3$ and the conditional tail probabilities of $y_1$ on $y_3$, respectively. $\alpha = 0.01$. 

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21
stock–USD pairs in Panels A and B using the ST copula model. Compared with our model, the ST model assumes the extension coefficient to be zero so that the fat-tailed property always comes with the distribution. If the real distributions are not compatible with this specific type of fat tail, the lower tail dependence coefficients may be overestimated. Compared with Panels A.2 and B.2 in Table 4, the results generally confirm the overestimated lower tail dependence by the ST model. The estimated lower tail dependences for six of the seven countries are quite significant. Thus, the inappropriate restrictions placed on the ST model can yield incorrect inferences about whether these alternative investments can serve as safe haven assets during stock crises.

9. Concluding remarks

Since the recent financial crisis, risk management has been an important issue in practice and in theory, and the development of alternative investment markets has accelerated the investigation of using these alternative investments to hedge the traditional stock portfolio returns under extreme market conditions in this decade. In this paper, we try to answer the questions of whether gold and USD can serve as hedge or safe haven assets for investors in seven different sample countries. We develop a MEST copula model and derive the associated lower tail dependence coefficient to examine the overall dependence and tail dependence for stock, the exchange rate and gold markets. We find that when gold and USD are used separately to hedge stock movements, USD is a better hedge asset under normal market conditions. When the domestic stock market crashes, these two alternative investments can serve as weak safe haven assets most of time and strong safe haven assets for a few countries in our sample. However, being a hedge asset for investors from a certain country does not guarantee that this asset, whether USD or gold, can also be a safe haven asset in that country. This finding provides partial evidence of the fundamentally different drivers of hedge and safe haven roles. When stock, USD, and gold are jointly estimated and analysed, we find that none of the assets will dominate the other during stock market crashes, which implies that the joint safe haven strategy does not outperform the single safe haven strategy. While our general conclusion complements recent studies on the hedge and safe haven roles of alternative investments with non-normal distributions, the joint estimation of stock, gold and USD, which can be obtained only in our model, addresses the importance of using a multivariate distribution model to determine the hedge and safe haven effects of these investments.

In addition to the economic interpretations in our empirical analysis, the empirical applications of gold and USD using the proposed model illustrate how to apply our econometric framework to portfolio allocation problems given that the returns on assets in interest clearly violate the normality assumptions. Although low-frequency stock returns are generally considered close to the normal distribution, the distribution of high-frequency stock returns or the returns of many newly developed alternative investment markets, such as the exchange rate, commodities and interest rates, can severely violate normality assumptions. In this

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</tr>
</thead>
<tbody>
<tr>
<td>Panel A.1: Kendal’s $\tau$ between stocks and gold</td>
<td>0.0104</td>
<td>0.0106</td>
<td>0.0553</td>
<td>0.0085</td>
<td>0.0162</td>
<td>0.0138</td>
</tr>
<tr>
<td>Panel A.2: $\lambda_\tau$ between stocks and gold</td>
<td>0.0007</td>
<td>0.0000</td>
<td>0.0071*</td>
<td>0.0001</td>
<td>0.0002</td>
<td>0.0017</td>
</tr>
</tbody>
</table>

Notes: We report the estimates of Kendal’s $\tau$ and lower tail dependence coefficient for stock–gold in which the gold prices are calculated in USD.

**Denote significance at 5%.

*Denote significance at 10%.

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<tr>
<th>UK</th>
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<th>Japan</th>
<th>Australia</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: $\lambda_\tau$ between stocks and gold by ST</td>
<td>0.2464</td>
<td>0.2318</td>
<td>0.2608</td>
<td>0.1965</td>
<td>0.0041</td>
<td>0.2900</td>
</tr>
<tr>
<td>Panel B: $\lambda_\tau$ between stocks and USD by ST</td>
<td>0.2141</td>
<td>0.2039</td>
<td>0.2788</td>
<td>0.1146</td>
<td>0.0001</td>
<td>0.3089</td>
</tr>
</tbody>
</table>

Note: We report the estimates of lower tail dependence coefficient for stock–gold and stock–USD pairs in Panel A.1 and Panel B.1 using ST copula model.
case, our more generalized econometric model can provide relatively precise statistical inference regarding the economic questions we are interested in. Furthermore, the development of the lower tail dependence coefficient in our study can help analyse the probability that two assets crash together when one of them is performing extremely poorly. However, this lower tail dependence coefficient is built on the bivariate case, so we can only evaluate the extreme risk of a pairwise portfolio. Because a market portfolio usually contains more than two assets, extending the bivariate lower tail dependence coefficient to the multivariate one would be an important issue in future research.

Although our paper provides a comprehensive analysis of the joint behaviours of stock, USD and gold returns under normal and extreme market conditions, we can only infer that the economic forces driving the safe haven and hedge effects are different and that the alternative investments considered in this paper, i.e. gold and USD, may not fulfil their independent safe haven roles on domestic investors’ portfolios from the current empirical evidence. It would be interesting to further examine the forces behind the risk management roles of these alternative investments. For example, one hypothesis is that the balance of trade can affect the imports of foreign products and the exports of local products and materials and, thus, may also adjust the consumer or producer price level in a country. In that sense, USD may serve as a hedge asset for some countries because it is correlated with domestic inflation. On the other hand, the econometric framework with multivariate non-normal distributions introduced in this paper can help us further investigate whether other alternative investments, such as commodities, real estate and fixed-income products, can also serve as better hedge and safe haven assets. In reality, it is unreasonable to assume that investors include only stocks and a few types of other financial assets in their portfolios. The joint effects on the risk of investors’ portfolios raise another interesting question and should attract further research.

Disclosure statement

No potential conflict of interest was reported by the authors.

References


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### Appendix A: Derivation of lower tail dependence coefficient

As presented in Demarta and McNeil (2005), the lower tail dependence coefficient

$$
\lambda_L = \lim_{u \to 0^-} P(Y_1 \leq F_1^{-1}(u_1)|Y_2 \leq F_2^{-1}(u_2))
$$

$$
= \lim_{u \to 0^-} \frac{C(u,u)}{u} = \lim_{u \to 0^-} \frac{dC(u,u)}{du}
$$

$$
= \lim_{u \to 0^-} P(U_2 \leq u|U_1 = u) + \lim_{u \to 0^-} P(U_1 \leq u|U_2 = u).
$$

We also have

$$
\lim_{u \to 0^-} P(U_2 \leq u|U_1 = u) = \lim_{u \to 0^-} \frac{P(U_2 \leq u, U_1 = u)}{P(U_1 = u)}
$$

$$
= \lim_{u \to 0^-} \frac{P(F_2(Y_2) \leq u, F_1(Y_1) = u)}{P(F_1(Y_1) = u)}
$$

$$
= \lim_{y \to \infty} \frac{P(F_2(Y_2) \leq F_1(y), Y_1 = y)}{P(Y_1 = y)}
$$

$$
= \lim_{y \to \infty} \frac{P(F_2(Y_2) \leq F_1(y)|Y_1 = y)}{P(Y_1 = y)}
$$

$$
= \lim_{y \to \infty} P(Y_2 \leq F_2^{-1}(F_1(y))|Y_1 = y)
$$

$$
= \lim_{y \to \infty} P(Y_2 \leq F_2^{-1}(F_1(y))|Y_1 = y)
$$
Similarly,
\[
\lim_{u \to 0} P(U_1 \leq u | U_2 = u) = \lim_{y \to -\infty} P(Y_1 \leq F_2^{-1}(F_2(y)) | Y_2 = y).
\]

Therefore,
\[
\lambda_L = \lim_{y \to -\infty} P(Y_2 \leq F_2^{-1}(F_2(y)) | Y_1 = y) + \lim_{y \to -\infty} P(Y_1 \leq F_1^{-1}(F_2(y)) | Y_2 = y).
\]

Let us first derive the first term \( \lim_{y \to -\infty} P(Y_2 \leq F_2^{-1}(F_2(y)) | Y_1 = y) \).

Following Proposition 3 of Arellano-Valle and Genton (2010) and (5) and (8), the bivariate EST and the associated marginal EST are:
\[
f(Y = x) = \frac{1}{T_1(\tau/1 + \theta^T \Phi \theta; \nu)} \Gamma\left(\frac{\nu + 1}{2}\right) \left(1 + \frac{x^T \Phi^{-1} x}{\nu}\right)^{-\nu(v+1)/2} \times \frac{1}{T_1\left(\frac{\nu + 1}{2}\right)} \left(1 + \frac{x^T \Phi^{-1} x}{\nu}\right)^{-\nu(v+1)/2}.
\]

and for \( Y_1 = x \)
\[
f(Y_1 = x) = \frac{1}{T_1(\tau/1 + \theta^T \Phi \theta; \nu)} \Gamma\left(\frac{\nu + 1}{2}\right) \left(1 + \frac{x^T \Phi^{-1} x}{\nu}\right)^{-\nu(v+1)/2} \times \frac{1}{T_1\left(\frac{\nu + 1}{2}\right)} \left(1 + \frac{x^T \Phi^{-1} x}{\nu}\right)^{-\nu(v+1)/2}.
\]

where
\[
\lambda = \begin{bmatrix} \lambda_1, \lambda_2, \lambda_3 \end{bmatrix}^T, \quad \theta = \frac{\Delta(\Phi \Phi^T - \rho^2 I_3)}{\sqrt{1 + \lambda_1^2} \sqrt{1 + \lambda_2^2} \sqrt{1 + \lambda_3^2}}.
\]

Suppose that \( \{\tau, \tau_1, \tau_2\} > -\infty \) such that the bivariate and the marginal EST do not degenerate to zero. As \( x \to -\infty \) the lower tail behaviour of \( Y_1 \):
\[
P(Y_1 \leq x) \sim \frac{1}{T_1(\tau/1 + \theta^T \Phi \theta; \nu)} \Gamma\left(\frac{\nu + 1}{2}\right) \left(1 + \frac{x^T \Phi^{-1} x}{\nu}\right)^{-\nu(v+1)/2} \times \frac{1}{T_1\left(\frac{\nu + 1}{2}\right)} \left(1 + \frac{x^T \Phi^{-1} x}{\nu}\right)^{-\nu(v+1)/2}.
\]

Using (25), (26), the conditional density of \( Y_{2|1} = (Y_2 | Y_1 = y) \) is
\[
f_{Y_{2|1}}(x) = \Xi \cdot t_1(x, \rho y, s(y); v + 1) + \frac{T_1\left(\frac{\nu + 1}{2}\right)}{T_1\left(\frac{\nu + 1}{2}\right)} \left(1 + \frac{x^T \Phi^{-1} x}{\nu}\right)^{-\nu(v+1)/2}.
\]

where \( \Xi \equiv t_1\left(\frac{\nu + 1}{2}\right) \) and \( t_1(x, \rho y, s(y); v + 1) \) is a Student’s t density with location \( \rho y \), scale \( s(y) = \sqrt{\frac{(1 - \rho^2)v^2 + 1}{v + 1}} \) and the degree of freedom \( v + 1 \).

Next, we derive the quantile function \( F_2^{-1}(F_2(y)) \) as \( y \to -\infty \).

Suppose \( F_2^{-1}(F_2(y)) = c(y) \). From (27), we observe that
\[
c(y) \sim \frac{T_1\left(-\lambda_1 \sqrt{v + 1}; v + 1\right) T_1\left(\frac{\nu + 1}{2}; v\right)}{T_1\left(-\lambda_1 \sqrt{v + 1}; v + 1\right) T_1\left(\frac{\nu + 1}{2}; v\right)} y \text{ as } y \to \infty.
\]

(29)

Let \( z = \frac{\nu + 1}{\nu} y \), we have
\[
P(Y_2 \leq F_2^{-1}(F_2(y)) | Y_1 = y) = \Xi \cdot \frac{\nu}{\nu + 1}, \quad t_1(z; 0, 1; v + 1)
\]

As \( y \to -\infty \) and \( \tau_1 > -\infty \),
\[
T_1\left(\frac{\nu + 1}{2}; v + 1\right) > 0
\]

implies
\[
\lim_{y \to -\infty} \left(T_1\left(\frac{\nu + 1}{2}; v + 1\right)\right)^{-1} < \infty
\]

By the dominated convergence theorem,
\[
\lim_{y \to -\infty} P(Y_2 \leq F_2^{-1}(F_2(y)) | Y_1 = y) = \frac{T_1\left(\frac{\nu + 1}{2}; v + 1\right)}{T_1\left(\frac{\nu + 1}{2}; v + 1\right)} - \frac{\nu}{\nu + 1}, \quad t_1(z; 0, 1; v + 1)
\]

\[
\times \frac{T_1\left(\frac{\nu + 1}{2}; v + 1\right)}{T_1\left(\frac{\nu + 1}{2}; v + 1\right)} \left(1 + \frac{x^T \Phi^{-1} x}{\nu}\right)^{-\nu(v+1)/2}.
\]

where
Table 10. Estimated coefficient of DCC model.

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<th>Japan</th>
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<tbody>
<tr>
<td>$c_1$</td>
<td>0.0189</td>
<td>0.0432**</td>
<td>0.0531</td>
<td>0.0659</td>
<td>0.0232</td>
<td>0.0301</td>
<td>0.0342</td>
</tr>
<tr>
<td></td>
<td>(0.0742)</td>
<td>(0.0198)</td>
<td>(0.2018)</td>
<td>(0.1266)</td>
<td>(0.1069)</td>
<td>(0.0520)</td>
<td>(0.0726)</td>
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<tr>
<td>$c_2$</td>
<td>0.9726***</td>
<td>0.9371***</td>
<td>0.8529</td>
<td>0.7601</td>
<td>0.6649*</td>
<td>0.9614***</td>
<td>0.5560</td>
</tr>
<tr>
<td></td>
<td>(0.0116)</td>
<td>(0.0353)</td>
<td>(0.8402)</td>
<td>(0.9302)</td>
<td>(0.5177)</td>
<td>(0.0177)</td>
<td>(0.4114)</td>
</tr>
</tbody>
</table>

Notes: Numbers in parentheses are standard deviations.
***Denote significance at 1%.
**Denote significance at 5%.
*Denote significance at 10%.

Table 11. Upper tail dependence coefficients.

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<tr>
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</thead>
<tbody>
<tr>
<td>Panel A: $\lambda_{Uj}$ between stocks and gold</td>
<td>0.0001</td>
<td>0.0013</td>
<td>0.0118**</td>
<td>0.0011</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0001</td>
</tr>
<tr>
<td>Panel B: $\lambda_{Uj}$ between stocks and USD</td>
<td>0.0053*</td>
<td>0.0096*</td>
<td>0.0253***</td>
<td>0.0046*</td>
<td>0.0002</td>
<td>0.0345**</td>
<td>0.0026</td>
</tr>
</tbody>
</table>

***Denote significance at 1%.
**Denote significance at 5%.
*Denote significance at 10%.

Appendix B: The EST copula with dynamic conditional correlation

B.1: DCC-EST copula model

We follow the dynamic conditional Correlation (DCC) model of Engel (2002). Let $Q_t$ be the auxiliary matrix driving the rank correlation dynamics and $Q_t = E(y_jy_j^T)$ with sample analogue $\frac{1}{n-1}\sum_{i=1}^{n}(y_iy_i^T)$. The dynamics of the process is modelled as follows:

$$Q_t = (1 - c_1 - c_2)\hat{Q} + c_1y_{t-1}y_{t-1}^T + c_2\hat{Q}_{t-1}$$

where $c_1, c_2 > 0$, $c_1 + c_2 < 1$. The correlation matrix $\Phi_t$ is a matrix of rank correlation given by

$$\Phi_t = \text{diag}(Q_t)^{-0.5} \cdot Q_t \cdot \text{diag}(Q_t)^{-0.5}$$

It follows that the dynamic EST copula density is given by:

$$c(u_1, \ldots, u_p; \Phi_t, \theta(\lambda), \lambda, \tau, v)$$

$$= f(F^{-1}(u_1), \ldots, F^{-1}(u_p); \Phi_t, \theta(\lambda), \lambda, \tau, v + p)$$

$$\prod_{i=1}^{p} f_i^{-1}(F^{-1}(u_i); \lambda_i, \tau, v + 1)$$

B.2: Estimation results

The estimated coefficients $c_1$ and $c_2$ in the dynamic conditional copula correlation $Q_t$ still can illustrate the dynamic nature of the copula correlations. The estimated values of $c_1$ and $c_2$ are mainly insignificant in Table 10, with the estimated coefficients of $c_2$ in the ranges of 0.97 for the UK and 0.76 for the USA. This indicates that the time-varying feature in the copula correlation cannot well capture the persistence of the dependence in our variables comparing with the static copula correlation.
Appendix C: The upper tail dependence coefficient

As the difference between the upper and lower tail dependence functions is that the former function uses the survival function of the copula model. Therefore, this study only reports the estimated results of upper tail dependence coefficients for stock–gold and stock–USD pairs. Pane A in Table 11 shows the $\lambda_U$ for stock–gold pair. From our estimation, Switzerland has the largest $\lambda_U$ (0.0118), followed by Germany (0.0013) and the UK (0.0011). Except for Switzerland, the $\lambda_U$ for other countries are less than 0.01 and statistically insignificant at 5% level.

Panel B of Table 11 reports the for the stock–USD pair. Japan has the largest $\lambda_U$ (0.0345), followed by Switzerland (0.0253). Except for Switzerland and Japan, the $\lambda_U$ for other countries are less than 0.01 and statistically insignificant at 5% level.