Name: KEY

UIN: ____________________________________________

Class Time (Please Circle): 11:10am to 12:25pm or 12:45pm to 2:00pm

Instructions:

1. Please provide your name and UIN.

2. Please circle the correct class time.

3. To get full credit on answer for this exam, be clear, rigorous, and thorough in your responses.

4. You cannot get credit (full or partial) unless something is written.

5. Sign the Aggie Pledge:

   “On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work.”

_____________________________________  ______________________________________
Signature                          Date
(7pts) 1. Potpourri

(1pt) (a) Name a pioneer of LP applications:

\textit{George Dantzig}

\textit{Naresh Karwan}

(1pt) (b) What is the name of the technique used to arrive at a solution to any LP problem?

\textit{Simplex Method}

(2pts) (c) \textcolor{red}{\text{True}} or False.

If \( \frac{\partial^2 s}{\partial x^2} = 6, \frac{\partial^2 s}{\partial y^2} = 9, \) and \( \frac{\partial^2 s}{\partial x \partial y} = \frac{\partial^2 s}{\partial y \partial x} = 8, \) then

the \textit{SOC} of this multivariate optimization problem is consistent with a
minimum solution.

\[ \frac{\partial^2 s}{\partial x^2} > 0, \quad \frac{\partial^2 s}{\partial y^2} > 0 \quad (\frac{\partial^2 s}{\partial x^2})(\frac{\partial^2 s}{\partial y^2}) - (\frac{\partial^2 s}{\partial x \partial y})(\frac{\partial^2 s}{\partial y \partial x}) < 0 \]

\[ 54 - 64 = -10 \]

(1pt) (d) The shadow price or shadow value of a non-binding constraint in any LP problem is equal to \( \boxed{0} \).

(1pt) (e) Trend, seasonal, and cyclical (smoothing) models yield \textit{conditional} or \textit{unconditional} forecasts? Circle the correct answer.

(1pt) (f) The only data needed to implement the models mentioned in (e) are for the forecast object. This approach is called \textit{univariate} or \textit{multivariate} approach. Circle the correct answer.
(10pts) 2. Using 30 observations, we estimate the demand function for Kellogg’s Raisin Bran as follows:

\[
Q_{KRB} = 0.75 - 1.65P_{KRB} + 0.85P_{GMC} + 0.53P_{KFF} + 0.22I + 0.03A,
\]

\[
(0.21) \quad (0.01) \quad (0.03) \quad (0.02) \quad (0.04) \quad (0.01)
\]

where \( R^2 = 0.94 \). The standard error of the regression is equal to 0.54.

\( Q_{KRB} \) denotes the quantity of Kellogg’s Raisin Bran sold (in thousands), \( P_{KRB} \) denotes the price of Kellogg’s Raisin Bran, \( P_{GMC} \) denotes the price of General Mills Cheerios, \( P_{KFF} \) represents the price of Kellogg’s Frosted Flakes, \( I \) represents U.S. disposable income (in thousands), and \( A \) represents the level of advertising expenditures associated with Kellogg’s Raisin Bran (in thousands).

p-values are given in parentheses. The level of significance chosen for this analysis is 0.05.

(1pts) (a) What is the name of the estimation technique used to obtain the estimated coefficients of the explanatory variables?

**OLS (Ordinary Least Squares)**

(4pts) (b) Use the regression model to forecast the amount of Kellogg’s Raisin Bran to be sold in a new market conditional on \( P_{KRB} = 4 \); \( P_{GMC} = 4 \); \( P_{KFF} = 4 \); \( I = 40 \), and \( A = 20 \). Recall that the units of measurement for the quantity sold are in thousands.

\[
\hat{Q}_{KRB} = 9.07 \Rightarrow 9,070 \text{ units sold of KRB}
\]

\[
\hat{Q}_{KRB} = 0.75 - 1.65(4) + 0.85(4) + 0.53(4) + 0.22(40) + 0.03(20)
\]

(2pts) (c) A critical value of the t-distribution is needed to construct a 95% confidence interval for your forecasts. What is the appropriate degrees of freedom associated with the t-statistic?

\[
\frac{2}{n - 1} \quad n = 30; p = 6
\]

(3pts) (d) Construct the confidence interval for your forecast in (b) assuming the critical value of the t-distribution is 2.

\[
9.07 \pm (0.54)(2) \Rightarrow 9.07 \pm (1.08)
\]

\[
[7.99, 10.15]
\]

7,990 to 10,150 units sold of KRB corresponds to the 95% confidence interval in this analysis.
3. The average total cost function for McClelland Corporation is given by 
\[ ATC = 5,000 - 100Q + Q^2, \]
where \( Q \) corresponds to output level.

(a) Determine the output level that minimizes average total cost.

\[ \frac{\partial ATC}{\partial Q} = -100 + 2Q = 0 \]

\[ 2Q = 100 \]

\[ Q = 50. \]

(b) Show that your answer in (a) minimizes rather than maximizes average total costs.

\[ \frac{\partial^2 ATC}{\partial Q^2} = 2 > 0 \]

\[ Q = 50 \text{ MINIMIZES ATC}. \]

(c) What is the minimum ATC?

\[ ATC = 5,000 - 100(50) + (50)^2 \]

\[ ATC = 4,500 \]

(d) Why is this information important to McClelland Corporation?

\[ \text{Min } ATC \text{ CORRESPONDS TO THE BREAK-EVEN POINT} \]

\[ \text{FOR McCLELLAND CORPORATION, ANY OUTPUT} \]

\[ \text{ABOVE } 50,500 \text{ => McCLELLAND CORPORATION WILL MAKE PROFITS} \]

\[ \text{THE BREAK-EVEN QUANTITY = 50 UNITS}. \]
The TimBurr Company manufactures two types of furniture products. Product A is made from wood harvested from pecan trees. Product B is made from wood harvested from oak trees. Information concerning the resource requirements and availability is exhibited in the table presented below.

<table>
<thead>
<tr>
<th>Resource Constraint</th>
<th>Quantity of Resources Required per Unit of Output</th>
<th>Resources Available During Production Period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Product A</td>
<td>Product B</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

The profit contribution of Product A is $100/unit, and the profit contribution of Product B is $125/unit.

(2pts) (a) Provide the objective function for the TimBurr Company.

\[
\text{max } \pi = 100A + 125B
\]

(4pts) (b) Provide the constraints for this problem. Label all constraints accordingly.

1. \(A + 2B \leq 2,000\)
2. \(A + B \leq 1,400\)
3. \(B \leq 800\)

\(A \geq 0, \quad B > 0\).

(6pts) (c) Graphically provide the feasible solution space.

![Graphical representation of feasible solution space]
(4pts) (d) Find the optimal number of Product A units and the optimal number of Product B units for the TimBurr Company to manufacture.

\[ \begin{align*}
1. \quad A &= 1400, \quad B = 0 \\
2. \quad A &= 800, \quad B = 600 \\
3. \quad A &= 400, \quad B = 800 \\
4. \quad A &= 0, \quad B = 800 \\
\end{align*} \]

(2pts) (e) Find the maximum profit for the TimBurr Company.

\[ \begin{align*}
\text{Profit at Point 1} &= \$140,000 \\
\text{Profit at Point 2} &= \$155,000 \\
\text{Profit at Point 3} &= \$140,000 \\
\text{Profit at Point 4} &= \$100,000 \\
\end{align*} \]

(3pts) (f) Which of the three resource constraints are binding?

When \( A = 800 \) and \( B = 600 \),

\( \text{Resource Constraints 1 and 2 are binding.} \)

\( \text{Resource Constraint 3 is not binding.} \)

(1pt) (g) True or False: The shadow price associated with resource constraint 2 is greater than zero. Circle the correct answer.

\( \text{True} \)

(2pts) (h) List two reasons why LP is applicable in this instance.

\begin{itemize}
  \item Linear combinations of decision variables in the objective function and constraints
  \item Inequality constraints
  \item Multiple decision variables
\end{itemize}
(11 pts) 5. Suppose the production of two goods, X and Y, yields the profit function specified below, where \( Q_x \) and \( Q_y \) correspond to the amount of X and Y produced.

\[
\pi = -100 + 50Q_x + 65Q_y - 10Q_x^2 - 5Q_y^2 - 5Q_xQ_y
\]

(3pts) (a) Determine the output levels that maximize profit.

\[
\frac{\partial \pi}{\partial Q_x} = 50 - 20Q_x - 5Q_y = 0 \quad Q_x = 1
\]

\[
\frac{\partial \pi}{\partial Q_y} = 65 - 10Q_y - 5Q_x = 0 \quad Q_y = 6
\]

(2pts) (b) Verify that the output levels in (a) maximize rather than minimize profit.

\[
\frac{\partial^2 \pi}{\partial Q_x^2} = -20; \quad \frac{\partial^2 \pi}{\partial Q_y^2} = -10; \quad \frac{\partial^2 \pi}{\partial Q_x \partial Q_y} = -5 = \frac{\partial^2 \pi}{\partial Q_y \partial Q_x} < 0 \quad \left( \frac{\partial^2 \pi}{\partial Q_x^2} \right) \left( \frac{\partial^2 \pi}{\partial Q_y^2} \right) - \left( \frac{\partial^2 \pi}{\partial Q_x \partial Q_y} \right)^2 > 0
\]

(4pts) (c) Suppose that the production capacity is exactly 5 units of output altogether. Use the Lagrangian Multiplier Method to determine the optimal output level of X and Y that maximize profit.

\[
\pi = -100 + 50Q_x + 65Q_y - 10Q_x^2 - 5Q_y^2 - 5Q_xQ_y - \lambda (Q_x + Q_y - 5)
\]

\[
\frac{\partial \pi}{\partial Q_x} = 50 - 20Q_x - 5Q_y - \lambda = 0 \quad 50 + 20Q_x - 5Q_y = 65 - 10Q_y - 5Q_x
\]

\[
\frac{\partial \pi}{\partial Q_y} = 65 - 10Q_y - 5Q_x - \lambda = 0 \quad 15 + 15Q_x - 5Q_y = 0
\]

or \( 3 + 3Q_x - Q_y = 0 \)

\[
\frac{\partial \pi}{\partial \lambda} = Q_x + Q_y - 5 = 0 \quad Q_x + Q_y = 5
\]

or \( Q_x = \frac{3 + 3Q_x}{2} \)

(2pts) (d) Calculate and interpret \( \lambda \), the Lagrangian Multiplier from (c).

\[
\lambda = 50 - 20Q_x - 5Q_y = 50 - 10 - 22.5 = 17.5 \quad Q_x = \frac{1}{2} \text{ or } 1.5
\]

\[
\lambda = 65 - 10Q_y - 5Q_x = 65 - 45 - 2.5 = 17.5 \quad Q_y = \frac{5}{6} \text{ or } 4.5
\]

\[
\lambda = 17.5
\]

If the constraint on production capacity changes by 1 unit,

then profits will change by 17.5 units in the same direction.
(8pts) 6. The trend analysis corresponding to the stock price $p_t$ (in dollars) of a certain company over time $t$ yields the following results:

- Model 1: $\hat{p}_t = 50 + 10t$
- Model 2: $\hat{p}_t = 2 - 5t + t^2$
- Model 3: $\ln(\hat{p}_t) = 2.4 + .15t$

Suppose that this analysis uses annual observations from 1990 to 2010, a total of 21 observations.

(1pt) (a) What is the technical name of Model 1? 

**LINEAR TREND**

(1pt) (b) What is the technical name of Model 2? 

**QUADRATIC TREND**

(1pt) (c) What is the technical name of Model 3? 

**EXPONENTIAL TREND**

(1pt) (d) According to Model 1, the stock price increases by $10$ dollars each year.

(1pt) (e) According to Model 3, the stock price increases by $15$ percent each year.

(3pts) (f) Which model provides the most optimistic forecast of the stock price for 2011? To answer this question, you must provide forecasts of all three models.

For 2011, $t = 22$.

- **Linear Trend** $\hat{p}_t = 50 + 10(22) = 270$
- **Quadratic Trend** $\hat{p}_t = 2 - 5(22) + (22)^2 = 376$
- **Exponential Trend** $\hat{p}_t = 2.4 + .15(22) = 5.7$

$\hat{p}_t = \exp(5.7) = 298.87$

**Quadratic Trend Model provides the most optimistic forecast.**
(10 pts) 7. Suppose you have been hired by the Walt Disney World Company to analyze attendance by season at the Universal Studios theme park in Orlando, FL. You collect data from the company and you estimate the following model:

\[
\text{Attendance} = 15 \text{ million} - 3 \text{ million} \times Q1 + 6 \text{ million} \times Q2 + 7 \text{ million} \times Q3
\]

Q1 corresponds to the months of January, February, and March (Quarter 1), Q2 corresponds to the months of April, May, and June (Quarter 2), and Q3 corresponds to the months of July, August, and September (Quarter 3). Q1, Q2, and Q3 are dummy variables or indicator variables.

(2 pts) (a) What values do the variables Q1, Q2, and Q3 take on in this analysis?

\[Q1, Q2, Q3\]

There are dummy variables.

(2 pts) (b) By how much does attendance differ between the third quarter and the fourth quarter?

Attendance in the third quarter is higher by 7 million relative to the fourth quarter.

(2 pts) (c) Forecast the attendance for the second quarter.

21 million

(4 pts) (d) If the base or reference quarter were Q3 in lieu of Q4, compute each of the estimated coefficients associated with this regression analysis. That is, what is the new coefficient associated with the

i) the intercept 22 million?

ii) Q1 -10 million?

iii) Q2 -1 million?

iv) Q4 -7 million?
(10pts) 8. Suppose the monthly sales for Fish Daddy's, a local restaurant in College Station, TX, from June 2011 to October 2011 are given as follows:

<table>
<thead>
<tr>
<th>Month</th>
<th>Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>June 2011</td>
<td>$120,000</td>
</tr>
<tr>
<td>July 2011</td>
<td>$165,000</td>
</tr>
<tr>
<td>August 2011</td>
<td>$90,000</td>
</tr>
<tr>
<td>September 2011</td>
<td>$225,000</td>
</tr>
<tr>
<td>October 2011</td>
<td>$180,000</td>
</tr>
</tbody>
</table>

(2pts) (a) Give the general model representation of a MA(2) process.

\[
\text{SALES}_t = \frac{1}{2} (\text{SALES}_{t-1} + \text{SALES}_{t-2}) + \nu_t
\]

(3pts) (b) Calculate the two-period moving average prediction of sales for August 2011, September 2011, and October 2011.

- August 2011: \( \$142,500 \)
- September 2011: \( \$127,500 \)
- October 2011: \( \$157,500 \)

(3pts) (c) Calculate the mean absolute error (MAE) for your predictions.

\[
\text{Forecast Errors} \quad \begin{align*}
\text{August 2011} & : (\$90,000 - \$142,500) = -\$52,500 \\
\text{September 2011} & : (\$225,000 - \$127,500) = \$97,500 \\
\text{October 2011} & : (\$180,000 - \$157,500) = \$22,500
\end{align*}
\]

\[
\text{MAD or MAE} = \frac{|-\$52,500| + |\$97,500| + |\$22,500|}{3} = \frac{\$173,500}{3} = \$57,833.33
\]

(1pt) (d) Forecast sales for November 2011.

\[
\text{November 2011} = \frac{1}{2} \left[ \$180,000 + \$225,000 \right] = \$202,500
\]

(1pt) (e) Forecast sales for December 2011.

\[
\text{December 2011} = \frac{1}{2} \left[ \$202,500 + \$180,000 \right] = \$191,250
\]
9. Suppose we wish to analyze the Euro to U.S. dollar exchange rate. The exchange rate deals with the number of Euros per U.S. dollar. In conducting this analysis, suppose the following regression results are obtained:

<table>
<thead>
<tr>
<th>Model</th>
<th>Equation</th>
<th>AIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>Euro $US_t = .15 + .8 \text{ Euro } US_{t-1}$</td>
<td>20.28</td>
</tr>
<tr>
<td>Model 2</td>
<td>Euro $US_t = -.59 + 1.3 \text{ Euro } US_{t-1} + .6 \text{ Euro } US_{t-2}$</td>
<td>19.65</td>
</tr>
</tbody>
</table>

The variable Euro $US_t$ represents the Euro to U.S. dollar exchange rate at time period $t$.

(a) What is the technical name of Model 1?

\[ AIC \]

Autoregressive model of order 1

(b) What is the technical name of Model 2?

\[ AIC \]

Autoregressive model of order 2

(c) What does the acronym AIC stand for?

AIC = \text{Akaike Information Criterion}

(d) Which model is preferred? Why?

\[ AIC(2) \text{ lower AIC} \]

\text{Model (2) has more data to calculate AIC, which is probably why it is lower.}

(e) Suppose that the Euro to U.S. dollar exchange rates for September 2011 and October 2011 were 0.6834 and 0.6778 respectively. For the model you selected in (d), provide the forecast for the exchange rate for November 2011. Show all work.

\[ \text{November 2011 forecast from } \text{ Model (2)} = -.59 + 1.3(0.6778) + .6(0.6834) = 0.70118 \]

\[ \text{November 2011 forecast from } \text{ Model (1)} = .15 + .8(0.6778) = 0.69224 \]

(f) The explanatory factors in this analysis are typically called \text{lagged independent variables.}
(7 pts) 10. We wish to center attention on the out-of-sample forecasting ability of a regression model for units sold of Pepsi. Actual and forecasted values of units sold of Pepsi are given for two periods in the table below.

<table>
<thead>
<tr>
<th>Period</th>
<th>Actual Units Sold</th>
<th>Forecasted Units Sold</th>
<th>Forecast Error</th>
<th>Squared Error</th>
<th>Absolute Percent Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4169</td>
<td>4000</td>
<td>169</td>
<td>28,561</td>
<td>4.05</td>
</tr>
<tr>
<td>2</td>
<td>4572</td>
<td>4863</td>
<td>-291</td>
<td>84,681</td>
<td>6.36</td>
</tr>
</tbody>
</table>

(3 pts) (a) Fill in the blank entries of this table. That is, calculate forecast error, squared error, and absolute percent error for period 1 and 2.

See entries in the table

(3 pts) (b) Compute the MSE, RMSE, and MAPE.

\[
MSE = 56,621
\]

\[
RMSE = \sqrt{MSE} = 237.95
\]

\[
MAPE = 5.21
\]

(1 pt) (c) Interpret your calculation of the MAPE metric.

On average, the forecast error relative to actual units sold is 5.21 percent.