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HOMOGENEITY AND ENDOGENEITY IN SYSTEMS OF DEMAND EQUATIONS

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In estimating systems of demand equations one of the right-hand-side explanatory variables, expenditure, may be endogenous in the sense that it is correlated with the equation error. If the assumption of homogeneity of degree zero in prices and nominal income is imposed on the system, it turns out it is still possible to estimate the parameters of the system even when expenditure is endogenous. The estimation procedure is simple requiring just one additional ordinary least squares regression.

The paper also demonstrates that a model in which homogeneity is tested with expenditure assumed exogenous is exactly equivalent to a model in which the exogeneity of expenditure is tested with homogeneity imposed. Previous tests of demand systems which have rejected the homogeneity postulate might therefore be reinterpreted instead as rejecting the hypothesis of exogeneity of expenditure with homogeneity of degree zero in prices and nominal income taken as given.

1. Introduction

A problem that arises in the estimation of systems of demand equations such as the Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980) or the Rotterdam Model of Theil (1965, 1976) and Barten (1969) is that one of the right-hand-side explanatory variables, expenditure, may be endogenous in the sense that it is correlated with the equation errors. One source of the endogeneity it is argued [cf. Summers (1959), Lluch and Williams (1974), Deaton (1980, pp. 12–13)], arises from the simultaneous equation nature of the problem. That is, since total expenditure is *defined* as the sum of expenditures on individual commodities and as these expenditures are assumed to be endogenous, we might expect total expenditure to be jointly endogenous. Even if a two-stage budgeting process is assumed [cf. Prais (1959)], whereby a household first decides on its total expenditure and then allocates it to various

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commodities so that *true* expenditure is exogenous, if true expenditure is measured with error, then *observed* expenditure is likely to be correlated with the error term – the classical errors in variables result. If, for whatever reason, expenditure is correlated with the equation errors resulting estimators will be both biased and inconsistent.

The purpose of this paper is to show that if the assumption of homogeneity of degree zero is *imposed* on the price and nominal income coefficients of each equation in a demand system, it is possible to estimate the covariances between the expenditure variable and the equation errors, thereby obtaining consistent estimators of the remaining structural parameters in the model. That is, the homogeneity restriction is sufficient to identify all the structural parameters in the model including the covariances between expenditure and the equation errors. Identification does not depend upon the introduction of further exogenous variables.[1] The estimation procedure turns out to be very simple requiring just one ordinary least squares regression in addition to those already carried out in the standard estimation of demand equations.

An important conclusion of the analysis is that a test for homogeneity, with expenditure assumed exogenous, is equivalent to a test for endogenous expenditure assuming homogeneity. Hence previous tests on demand systems which have rejected the hypothesis of homogeneity might be reinterpreted instead as rejecting the hypothesis of exogeneity of the expenditure variable where homogeneity is assumed as part of the maintained hypothesis.

The next section, section 2, derives the conditions for identification of the structural parameters in the model. Section 3 develops the estimation procedure, shows how the analysis can be put into a single-equation OLS framework and derives a method for estimating asymptotic standard errors. Section 4 applies the method to the AIDS model, and section 5 concludes the paper.

2. Identification of structural parameters

A well known example of a differential demand system is the Rotterdam model,

$$w_{it}\Delta \ln q_{it} = \sum_{j}^{k} \gamma_{ij}\Delta \ln p_{jt} + \alpha_i \Delta \ln m_t + u_{it}, \qquad i=1,\ k-1, \quad t=1,n, \tag{1}$$

where w_{it} is the budget share of the ith commodity, $w_{it} = p_{it}q_{it}/\sum_j^k p_{jt}q_{jt}$, q_{it} is

[1] If a further set of exogenous variables is available which can be assumed to be the determinants of expenditure, in a model where expenditure is assumed endogenous, then it should be possible to test the homogeneity postulate directly.

demand for commodity i, p_{it} the price of commodity i, and m_t is total real expenditure on all commodities at time t. We assume that one equation has been dropped to overcome the problem of the singularity of the covariance matrix of the errors u_{it}.

To simplify notation, rewrite the system in (1) as

$$y_t = \Gamma x_t + \alpha z_t + u_t, \qquad t = 1, \ldots, n, \tag{2}$$

where

$$y_t' = (w_{1t}\Delta \ln q_{1t}, \ldots, w_{k-1t}\Delta \ln q_{k-1t}),$$

$$x_t' = (\Delta \ln p_{1t}, \ldots, \Delta \ln p_{k-1t}),$$

$$z_t = \Delta \ln m_t,$$

$$\Gamma = \begin{bmatrix} \gamma_{11}, \ldots, \gamma_{1k} \\ \gamma_{k-1,1}, \ldots, \gamma_{k-1,k} \end{bmatrix},$$

$$\alpha' = (\alpha_1, \ldots, \alpha_{k-1}).$$

If now z_t is endogenous then $\mathrm{E}(u_t\, z_t) = \sigma_{uz} \neq \mathbf{0}$ and estimators of Γ and α will be biased and inconsistent.[2]

The idea in this paper is in essence to estimate the elements of σ_{uz} as free parameters in addition to the unknown elements in Γ and α. That it is possible to identify the elements of σ_{uz} can be seen by noting that with homogeneity imposed one element in each row of Γ is restricted and, given a non-zero correlation between x_t and z_t, this restriction can be exploited to estimate an element of σ_{uz}.

More formally, the model in (2) can be rewritten as

$$y_t = \pi_1 x_t + \pi_2 z_t + v_t,$$

where

$$v_t = \Gamma x_t + \alpha z_t + u_t - \pi_1 x_t - \pi_2 z_t.$$

For consistent estimators of π_1 and π_2, we then require $\mathrm{E}(v_t x_t') = \mathbf{0}$ and $\mathrm{E}(v_t z_t) = \mathbf{0}$ which implies that

$$\pi_1 = \Gamma - \sigma_{uz}\sigma_{zx}'\Sigma_{xx}^{-1} w^2 \quad \text{and} \quad \pi_2 = \alpha + \sigma_{uz} \cdot w^2, \tag{3}$$

where

$$\sigma_{zx}' = \mathrm{E}(z_t x_t'), \quad \Sigma_{xx} = \mathrm{E}(x_t x_t'), \quad w^2 = (\sigma_{zz} - \sigma_{zx}'\Sigma_{xx}^{-1}\sigma_{xz})^{-1},$$

[2] The standard assumptions, $\mathrm{E}(u) = \mathbf{0}$, $\mathrm{E}(xu') = 0$ are applicable.

with

$$\sigma_{zz} = \mathrm{E}(z_t)^2,$$

and where

$$(\pi_1, \boldsymbol{\pi}_2) = \left(\Sigma'_{yx}\boldsymbol{\sigma}_{yz}\right)\begin{bmatrix} \Sigma_{xx} & \boldsymbol{\sigma}_{xz} \\ \boldsymbol{\sigma}'_{zx} & \sigma_{zz} \end{bmatrix}^{-1}.$$

Substituting for π_1 and $\boldsymbol{\pi}_2$ we obtain the model

$$y_t = \left(\Gamma - \boldsymbol{\sigma}_{uz}\boldsymbol{\sigma}'_{zx}\Sigma_{xx}^{-1}w^2\right)\boldsymbol{x}_t + \left(\boldsymbol{\alpha} + \boldsymbol{\sigma}_{uz}\cdot w^2\right)z_t + \boldsymbol{v}_t. \tag{4}$$

Since sample moments are consistent estimators of population moments, sample moments replace $\boldsymbol{\sigma}'_{zx}$, Σ_{xx} and w^2 in (3), and we then require estimators for the unknown elements in Γ, $\boldsymbol{\sigma}_{uz}$ and $\boldsymbol{\alpha}$. That these elements are all identifiable can be deduced from an analysis similar to Rothenberg (1973, p. 39).

If the left-hand sides of the expressions in (3), π_1 and $\boldsymbol{\pi}_2$, are considered as the unrestricted reduced form coefficients, then the identifiability of the structural parameters reduces to the question of whether we can solve the equations in (3) for the structural parameters. Stacking the elements in π_1 by rows, denoted by the column vector $VEC(\pi_1)$, let $\boldsymbol{\pi}' = [VEC(\pi_1)', \boldsymbol{\pi}'_2]$, $\boldsymbol{\gamma}' = VEC(\Gamma)$, and $\boldsymbol{\theta}' = (\boldsymbol{\gamma}', \boldsymbol{\alpha}', \boldsymbol{\sigma}'_{zu})$, then we wish to evaluate the rank of the matrix

$$\Phi = \begin{bmatrix} \partial\boldsymbol{\pi}/\partial\boldsymbol{\theta}' \\ \Psi \end{bmatrix},$$

where Ψ denotes the matrix of first derivatives of the constraint functions on the structural parameters in $\boldsymbol{\theta}$. If Φ is of full column rank $(k-1)(k+2)$, the elements in $\boldsymbol{\theta}'$ are identified. For the model in (4) we have

$$\Phi = \begin{array}{ccc|c} \boldsymbol{\gamma}' & \boldsymbol{\alpha}' & \boldsymbol{\sigma}'_{zu} & \\ \hline I_{k(k-1)} & 0 & \underset{((k-1)k\times(k-1))}{\left(I_{k-1}\otimes\Sigma_{xx}^{-1}\sigma_{xz}w^2\right)} & VEC(\pi_1) \\ \underset{((k-1)\times k(k-1))}{0} & I_{k-1} & w^2 I_{k-1} & \boldsymbol{\pi}_2 \\ \Psi_\gamma & 0 & 0 & \text{constraints} \end{array}$$

where Ψ_γ denotes the derivatives of the restrictions on the Γ matrix, and it is assumed that the elements in $\boldsymbol{\alpha}$ and in $\boldsymbol{\sigma}_{uz}$ are unconstrained. Since we are using homogeneity restrictions only, we have that $\Psi_\gamma = (I_{k-1}\otimes\boldsymbol{\iota}')$, where $\boldsymbol{\iota}'$ is a $(1 \times k)$ vector of units. Therefore,

$$\underset{(k-1)(k+2)\times(k-1)(k+2)}{\boldsymbol{\Phi}} = \begin{bmatrix} I_{k(k-1)} & 0 & \left(I_{k-1} \otimes \Sigma_{xx}^{-1}\boldsymbol{\sigma}_{xz}w^2\right) \\ 0 & I_{k-1} & w^2 I_{k-1} \\ I_{k-1} \otimes \boldsymbol{\iota}' & 0 & 0 \end{bmatrix},$$

and so

$$\rho(\boldsymbol{\Phi}) = k(k-1) + (k-1) + \rho\left(I_{k-1} \otimes \boldsymbol{\iota}'\Sigma_{xx}^{-1}\boldsymbol{\sigma}_{xz}w^2\right),$$

where ρ denotes rank. But since

$$I_{k-1} \otimes \boldsymbol{\iota}'\Sigma_{xx}^{-1}\boldsymbol{\sigma}_{xz}w^2 = dI_{k-1},$$

where

$$d = \boldsymbol{\iota}'\Sigma_{xx}^{-1}\boldsymbol{\sigma}_{xz}w^2,$$

provided $d \neq 0$, we have $\rho(\boldsymbol{\Phi}) = (k+2)(k-1)$ and the structural parameters are identified.

Notice that $\boldsymbol{\Phi}$ has $(k-1)(k+2)$ rows and so the model is 'just identified', i.e., each homogeneity restriction allows the identification of one element of $\boldsymbol{\sigma}_{uz}$. Note also that symmetry constraints on Γ may be imposed and tested since they are overidentifying constraints.

The condition for identification, that $d = \boldsymbol{\iota}'\Sigma_{xx}^{-1}\boldsymbol{\sigma}_{xz}w^2 \neq 0$, has a simple interpretation. The term $\Sigma_{xx}^{-1}\boldsymbol{\sigma}_{xz}$ is the vector of coefficients of a linear regression of z on $\boldsymbol{x}$, and the term w^2 is the reciprocal of the variance of the error in this same regression. d is therefore the *sum* of the coefficients from a regression of z on x divided by the error variance from the same regression. For identification we require at least one of the regression coefficients to be non-zero.

3. Estimation of the structural parameters

Given that the structural parameters are identified we can return to eq. (3) to obtain an analytic solution to the problem. Let $\boldsymbol{\beta}' = \Sigma_{xx}^{-1}\boldsymbol{\sigma}_{xz}$, the regression of z on $\boldsymbol{x}$, then the equations in (3) become

$$\pi_1 = \Gamma - \boldsymbol{\sigma}_{uz}\boldsymbol{\beta}'w^2, \qquad \pi_2 = \boldsymbol{\alpha} + \boldsymbol{\sigma}_{uz}w^2. \tag{5}$$

Since we are imposing homogeneity restrictions on the rows of Γ, we may write

$$\Gamma = \left[\begin{array}{c|c} \boldsymbol{\gamma}_1' & -\boldsymbol{\gamma}_1'\boldsymbol{\iota} \\ \vdots & \\ \boldsymbol{\gamma}_{k-1}' & -\boldsymbol{\gamma}_{k-1}'\boldsymbol{\iota} \end{array}\right] = \left[\Gamma_1 \mid -\Gamma_1\boldsymbol{\iota}\right],$$

where the last element in each row of Γ is the negative of the sum of the first $(k-1)$ elements.

Partition $\boldsymbol{\beta}'$ into $(\boldsymbol{\beta}_1', \beta_2)$ where $\boldsymbol{\beta}_1'$ is $(1 \times (k-1))$, and π_1 into (π_{11}, π_{12}), and then

$$\pi_{11} = \Gamma_1 - \boldsymbol{\sigma}_{uz}\boldsymbol{\beta}_1' w^2, \qquad \boldsymbol{\pi}_{12} = -\Gamma_1 \boldsymbol{\iota} - \boldsymbol{\sigma}_{uz}\beta_2 w^2. \tag{6}$$

Since Γ_1 in the first equation in (6) is now unconstrained we can solve for Γ_1 and substitute the solution into the second equation in (6) to produce

$$\boldsymbol{\sigma}_{uz} = -\pi_1 \boldsymbol{\iota} / (\boldsymbol{\iota}'\boldsymbol{\beta}) w^2. \tag{7}$$

So for the jth equation in the system the jth element of $\boldsymbol{\sigma}_{uz}$, the covariance between the jth equation error and z, is obtained by summing over the reduced form coefficients, the π's, on all the elements of $\boldsymbol{x}$, and dividing by the sum of the coefficients of the regression of z on $\boldsymbol{x}$, the β's, and multiplying by the variance of the equation error in this regression, i.e., $1/w^2$. The term $1/w^2$ acts as a scaling factor for the problem in that if all the data is weighted by some constant, while π_1 and $\boldsymbol{\beta}$ will remain unchanged, each element of $\boldsymbol{\sigma}_{uz}$ will be multiplied by the square of the constant – as we would expect.

A consistent estimator of $\boldsymbol{\sigma}_{uz}$ in eq. (7) can therefore be obtained by k ordinary least squares regressions, i.e., $(k-1)$ demand equations and one regression of expenditure, z, on prices, $\boldsymbol{x}$. Having secured $\hat{\boldsymbol{\sigma}}_{uz}$, an estimator of $\boldsymbol{\sigma}_{uz}$, the remaining parameters of the system are consistently estimated from

$$\hat{\Gamma}_1 = \hat{\pi}_{11} + \hat{\boldsymbol{\sigma}}_{uz}\hat{\boldsymbol{\beta}}_1'\hat{w}^2 \quad \text{and} \quad \hat{\boldsymbol{\alpha}} = \hat{\boldsymbol{\pi}}_2 - \hat{\boldsymbol{\sigma}}_{uz}\hat{w}^2. \tag{8}$$

What is of interest is that, from eq. (7), $\boldsymbol{\sigma}_{uz} = \mathbf{0}$ only if $\pi_1\boldsymbol{\iota} = \mathbf{0}$. But the hypothesis $\pi_1\boldsymbol{\iota} = \mathbf{0}$ is exactly the null hypothesis tested by researchers testing for linear homogeneity. A rejection of the hypothesis that $\pi_1\boldsymbol{\iota} = \mathbf{0}$ can therefore be interpreted as either a rejection of homogeneity *given* that expenditure is exogenous, *or* equivalently, as a rejection of the hypothesis that z is exogenous *given* homogeneity.

There is another way of viewing the model which is illuminating and leads to considerable simplicity in obtaining estimates of asymptotic standard errors. If we assume that the vector $(\boldsymbol{x}_t', z_t)$ is multivariate normal, then, using the theorem in Anderson (1958, p. 28) and expressing variables as deviations from sample means, we may express z_t as a linear function of $\boldsymbol{x}_t$, i.e., $z_t = \boldsymbol{\beta}'\boldsymbol{x}_t + \xi_t$, where ξ_t is normal with zero mean and constant variance. Then, consider the structure

$$\begin{aligned} \boldsymbol{y}_t &= \Gamma \boldsymbol{x}_t + \boldsymbol{\alpha} z_t + \boldsymbol{u}_t, \\ z_t &= \boldsymbol{\beta}'\boldsymbol{x}_t + \xi_t. \end{aligned} \tag{9}$$

The equations in (9) make explicit our assumptions that the elements of $\boldsymbol{x}$ are the only truly exogenous variables in the system and that z is correlated with the elements of $\boldsymbol{x}$. Moreover, the structure in (9) is formally equivalent, with suitable restrictions, to Hausman's 'augmented structural model' (1977, eqs. 4-7b, 4-7c, p. 395). He adopts the method to overcome the 'errors in variables' problem, i.e., to take into account the correlation between an explanatory variable and an equation error. We wish to allow for the same correlation – between z_t and u_t in eq. (9).

Substituting out z we obtain

$$y_t = (\Gamma + \boldsymbol{\alpha}\boldsymbol{\beta}')\boldsymbol{x}_t + \boldsymbol{u}_t + \boldsymbol{\alpha}\xi_t = (\Gamma + \boldsymbol{\alpha}\boldsymbol{\beta}')\boldsymbol{x}_t + \varepsilon_{1t},$$
$$z_t = \boldsymbol{\beta}'\boldsymbol{x}_t + \xi_t = \boldsymbol{\beta}'\boldsymbol{x}_t + \varepsilon_{2t}. \tag{10}$$

In (9) and (10) we have retained the same notation for $\boldsymbol{\beta}$ as used throughout this section since it follows from Zellner's (1962) 'seemingly unrelated regression model' that $\boldsymbol{\beta} = \Sigma^{-1}\boldsymbol{\sigma}_{xz}$ and that

$$\mathrm{E}(\varepsilon_{2t})^2 = \omega^{-2} = (\sigma_{zz} - \boldsymbol{\sigma}'_{zx}\Sigma_{xx}^{-1}\boldsymbol{\sigma}_{xz}).$$

Also, since the parameters in Γ and α are just identified, from the first equation in (10) we have

$$\Gamma + \boldsymbol{\alpha}\boldsymbol{\beta}' = \Sigma_{xx}^{-1}\Sigma_{xy}. \tag{11}$$

Now from the eqs. (5) at the beginning of this section we have

$$\pi_1 + \boldsymbol{\pi}_2\boldsymbol{\beta}' = \Gamma + \boldsymbol{\alpha}\boldsymbol{\beta}', \tag{12}$$

and since

$$\pi_1 + \boldsymbol{\pi}_2\boldsymbol{\beta}' = (\Sigma_{yx}, \boldsymbol{\sigma}_{yz})\begin{bmatrix} \Sigma_{xx} & \boldsymbol{\sigma}_{xz} \\ \boldsymbol{\sigma}'_{zx} & \sigma_{zz} \end{bmatrix}^{-1}\begin{bmatrix} I \\ \boldsymbol{\beta}' \end{bmatrix},$$

we have

$$\Gamma + \boldsymbol{\alpha}\boldsymbol{\beta}' = \pi_1 + \boldsymbol{\pi}_2\boldsymbol{\beta}' = \Sigma_{xx}^{-1}\Sigma_{xy},$$

so the estimators of Γ and α obtained from estimating the system in (10) are identical with the estimators obtained by the procedure developed at the beginning of this section.

The advantage of the structure in (10) is that the likelihood function for the sample is easily derived and structural parameters can be estimated by standard non-linear maximum likelihood routines and estimates of the asymptotic standard errors constructed from the negative of the inverse of the hessian matrix in the usual manner. The form of the likelihood function is given in Hausman (1977, p. 397) with the determinant of the Jacobian equal to unity.

4. An application

The AIDS model of Deaton and Muellbauer (1980) in levels is

$$y_{it} = \alpha_i + \sum_j^k \gamma_{ij} x_{jt} + \phi_i z_t + u_{it}, \qquad i = 1, \ldots, k-1, \quad t = 1, \ldots, n, \tag{13}$$

where

$$y_{it} = w_{it} = p_{it} q_{it} \Big/ \sum_j^k p_{jt} q_{jt},$$

$$x_{jt} = \ln p_{jt},$$

$$z_t = \ln \sum_j^k p_{jt} q_{jt} - \ln p_t,$$

$$\ln p_t = \sum_j^k w_{jt} \ln p_{jt}.$$

Table 1a gives the set of unconstrained estimates from (13) – the π matrix of this paper. Unconstrained estimates are of course the same as those in table 1 in Deaton and Muellbauer (DM) (1980, p. 319).[3] Table 1b gives the set of estimates derived on the assumption that demand equations are homogeneous of degree zero in prices and nominal income and allowing for an endogenous expenditure variable, i.e., the set of estimates obtained from eqs. (7) and (8). The final row in table 1b gives the coefficients of the regression of total expenditure on prices. With the exception of the price of fuel no price coefficient appears significantly different from zero.[4] However, a joint test on all coefficients using the F distribution strongly rejects the hypothesis that all price coefficients are zero. Collinearity among prices is probably responsible for the high standard errors on individual coefficients. There appears to be enough evidence that at least one of the price coefficients is non-zero and so the structural model is identified.[5]

[3] Table 1a gives the results of doing things in the 'normal way', i.e., a test of homogeneity assuming expenditure is exogenous and measured without error.

[4] The model was estimated with an intercept, but since there are no restrictions on these parameters they do not help with the identification of the structural coefficients.

[5] Of course, if there are no significant relationships between prices and total expenditure then there will be no bias in the price coefficients when expenditure is correlated with the error term. The only bias will be in the coefficients on expenditure.

Comparing the estimates in table 1b with the DM estimates in table 1a, we see that the intercept terms are larger in absolute value for all categories except housing; some price coefficients have different signs but are of the same order of magnitude as the DM estimates, while some of the expenditure coefficients change sign. Very few of the price coefficients in table 1a are significantly differrent from zero. However, when endogeneity of expenditure is allowed for, the resulting matrix of coefficients on prices, $\Gamma + \boldsymbol{\alpha}\boldsymbol{\beta}'$ as in eq. (10), is much more strongly determined. These estimates are given in table 2.

Own price elasticities computed on the same basis as in DM have the same sign as those given in table 2 of DM, with clothing and housing being rather more price elastic.

Much more implausible are the expenditure elasticities computed from the estimates in table 1b. Elasticities for food, clothing and fuel are negative implying that these commodities are inferior goods. These results suggest that the problem with the expenditure variable is more complex than an errors-in-variable type problem in which expenditure is correlated with the equation error because of measurement error. If this were the case we would expect these consistent estimators of the expenditure elasticities to be more 'sensible'. Another explanation for the result is the one given in the introduction; i.e., if total expenditure is endogenous and depends upon prices, budget shares and some omitted variables, then the concept of elasticity of demand with respect to expenditure has no meaning. In this case the correct procedure is to model the expenditure variable appropriately and compute demand elasticities with respect to prices and the omitted exogenous variables.

The final column of table 1b gives estimates of the covariances between expenditure and the equation errors. Using a likelihood ratio test the covariance is strongly significant for the categories of food, clothing, housing and transport and communications with 'other services' on the borderline. This is exactly what we would have anticipated from the results in table 1a since food, clothing, housing and transport are the categories where homogeneity is strongly rejected by DM and where the 'other services' category is borderline.

Notice that the adding up restrictions are still satisfied for the estimates in table 1b in that the intercepts sum to unity, the expenditure coefficients sum to zero and the sum of each column of price coefficients is zero. Note also that the sum of the covariances between expenditure and the equation errors is also zero. This is as it should be since the equation errors themselves sum to zero.[6]

[6]Since the analysis was carried out one equation at a time, it was unnecessary to drop one equation to avoid the problem of singularity of the error covariance matrix. The estimates of asymptotic standard errors in table 1b were obtained by estimating the equations in the system in (10) by a non-linear maximum likelihood programme using analytic first derivatives to obtain an estimate of the information matrix [Attfield and Dunn (1982)].

Table 1a

The unconstrained parameter estimates and tests of homogeneity for the AIDS model (t-values in parentheses).

Commodity i	Inter-cept	Expenditure coefficient	Price coefficients p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	Sum of price coeffs.	$S.E.E$ (10^{-2})	R^2	$D.W.$
Food	1.221	−0.160	0.186	−0.077	−0.013	−0.020	−0.058	0.032	0.015	−0.098	−0.033	.113	0.999	2.33
	(7.4)	(−6.1)	(9.8)	(−4.3)	(−0.8)	(−1.1)	(−6.2)	(1.3)	(0.7)	(−4.2)	(−4.4)			
Clothing	−0.482	0.091	0.033	0.016	−0.024	−0.026	−0.029	0.014	0.033	−0.049	−0.032	.106	0.984	2.29
	(−3.1)	(3.7)	(1.8)	(1.0)	(−1.6)	(−1.5)	(−3.3)	(0.6)	(1.6)	(−2.2)	(−4.5)			
Housing	0.793	−0.104	−0.082	−0.009	0.088	0.009	0.033	−0.055	−0.030	0.098	0.061	.086	0.999	1.89
	(6.3)	(−5.1)	(−5.6)	(−0.7)	(7.2)	(0.7)	(4.7)	(−2.9)	(−1.8)	(5.5)	(9.1)			
Fuel	−0.159	0.033	−0.042	0.010	−0.011	0.037	−0.004	0.022	0.007	−0.031	−0.010	.140	0.883	2.25
	(−0.8)	(1.0)	(−1.8)	(0.4)	(−0.5)	(1.6)	(−0.3)	(0.7)	(0.3)	(−1.1)	(−1.1)			
Drink and tobacco	−0.043	0.028	−0.043	0.034	−0.027	−0.020	0.056	0.005	−0.018	0.014	0.001	.099	0.969	2.96
	(−0.3)	(1.2)	(−2.6)	(2.2)	(−1.9)	(−1.2)	(6.9)	(0.2)	(−0.9)	(0.7)	(0.0)			
Transport and communication	−0.061	0.029	−0.022	−0.012	0.002	−0.011	−0.060	−0.023	0.024	0.053	0.040	.047	1.000	2.24
	(−0.9)	(2.6)	(−2.7)	(−1.6)	(−0.3)	(1.4)	(15.2)	(−2.2)	(−2.6)	(5.3)	(13.1)			
Other goods	−0.38	0.022	0.001	−0.003	−0.001	−006	−0.030	0.007	0.032	−0.006	−0.005	.108	0.885	1.92
	(−0.2)	(0.9)	(0.0)	(0.2)	(−0.0)	−0.3	(−3.4)	(0.3)	(1.5)	(−0.2)	(0.7)			
Other services	−0.231	0.060	−0.032	0.041	−0.011	0.014	−0.028	−0.003	−0.015	0.019	−0.014	.107	0.843	2.27
	(−1.5)	(2.4)	(−1.8)	(2.4)	(−0.7)	(0.8)	(−3.1)	(−0.1)	(−0.7)	(0.9)	(−2.0)			

Table 1b

Parameter estimates of the AIDS model assuming linear homogeneity and endogeneity of the expenditure variable (estimates of asymptotic standard errors in parentheses).

Commodity i	Inter-cept	Expenditure coefficient	Price coefficients p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	$(\times 10^{-4})$ $\hat{\sigma}_{u_i z}$		
Food	3.055	−0.453	0.202	−0.142	0.094	−0.020	−0.074	0.093	−0.058	−0.097	0.254		
	(0.957)	(0.152)	(0.051)	(0.071)	(0.082)	(0.047)	(0.027)	(0.081)	(0.052)	(0.059)			
Clothing	1.271	−0.188	0.047	−0.046	0.080	−0.026	−0.044	0.072	−0.037	−0.047	0.242		
	(0.913)	(0.145)	(0.048)	(0.068)	(0.078)	(0.044)	(0.025)	(0.077)	(0.049)	(0.056)			
Housing	−2.089	0.356	−0.106	0.093	−0.084	0.093	0.058	−0.151	0.086	0.095	−0.398		
	(1.455)	(0.232)	(0.077)	(0.108)	(0.125)	(0.071)	(0.041)	(0.123)	(0.079)	(0.089)			
Fuel	0.419	−0.059	−0.037	−0.011	0.024	0.037	−0.009	0.042	−0.016	−0.030	0.080		
	(0.446)	(0.071)	(0.024)	(0.033)	(0.038)	(0.022)	(0.013)	(0.038)	(0.024)	(0.027)			
Drink and tobacco	−0.114	0.040	−0.044	0.037	−0.031	−0.020	0.057	0.003	−0.016	0.014	−0.010		
	(0.242)	(0.039)	(0.013)	(0.018)	(0.021)	(0.012)	(0.007)	(0.020)	(0.013)	(0.015)			
Transport and communications	−2.333	0.391	−0.041	0.069	−0.137	0.011	0.079	−0.099	0.067	0.050	−0.314		
	(1.141)	(0.182)	(0.060)	(0.084)	(0.098)	(0.055)	(0.032)	(0.097)	(0.062)	(0.070)			
Other goods	0.252	−0.024	0.003	−0.014	0.017	−0.006	−0.033	0.017	0.020	−0.005	0.040		
	(0.299)	(0.048)	(0.016)	(0.022)	(0.026)	(0.015)	(0.009)	(0.025)	(0.016)	(0.018)			
Other services	0.539	−0.063	−0.026)	0.014	0.035	0.014	−0.034	0.023	−0.046	0.020	0.106		
	(0.464)	(0.074)	(0.025)	(0.034)	(0.040)	(0.023)	(0.013)	(0.039)	(0.025)	(0.028)			
Regression of expenditure on prices	6.276		0.052	−0.223	0.374	−0.0001	−0.054	0.209	−0.253	0.006	*RMSE*	R^2	*D.W*
	(0.006)		(0.157)	(0.141)	(0.105)	(0.153)	(0.076)	(0.201)	(0.176)	(0.194)	0.0093	0.994	1.16

Table 2

Parameter estimates of the regression of budget shares on prices, i.e., $y_t = (\Gamma + \boldsymbol{\alpha\beta}')\boldsymbol{x}_t + \boldsymbol{\varepsilon}_{1t}$ (estimates of asymptotic standard errors in parentheses).

Commodity	Inter-cept	Price coefficients p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	$RMSE$ (10^{-2})	R^2	$D.W.$
Food	0.214	0.178	−0.042	−0.073	−0.020	−0.049	−0.002	0.056	−0.099	0.170	0.997	1.34
	(0.001)	(0.028)	(0.025)	(0.019)	(0.028)	(0.014)	(0.037)	(0.032)	(0.035)			
Clothing	0.091	0.038	−0.004	0.010	−0.026	−0.034	0.033	0.010	−0.048	0.115	0.965	2.02
	(0.001)	(0.019)	(0.017)	(0.013)	(0.019)	(0.009)	(0.025)	(0.022)	(0.024)			
Housing	0.142	−0.087	0.014	0.049	0.009	0.039	−0.077	−0.004	0.097	0.115	0.997	1.38
	(0.007)	(0.019)	(0.017)	(0.013)	(0.019)	(0.009)	(0.025)	(0.022)	(0.024)			
Fuel	0.051	−0.040	0.003	0.002	0.037	−0.006	0.030	−0.001	−0.031	0.106	0.872	1.86
	(0.001)	(0.018)	(0.016)	(0.012)	(0.017)	(0.009)	(0.023)	(0.020)	(0.022)			
Drink and tobacco	0.135	−0.042	0.028	−0.016	−0.020	0.055	0.011	−0.026	0.014	0.076	0.965	2.91
	(0.001)	(0.013)	(0.012)	(0.009)	(0.012)	(0.006)	(0.017)	(0.014)	(0.016)			
Transport and communications	0.120	−0.020	−0.018	0.009	0.011	0.058	−0.017	−0.031	0.053	0.044	0.999	2.11
	(0.001)	(0.007)	(0.007)	(0.005)	(0.007)	(0.004)	(0.009)	(0.008)	(0.009)			
Other goods	0.102	0.002	−0.008	−0.008	−0.006	−0.031	0.012	0.026	−0.005	0.081	0.877	2.04
	(0.001)	(0.014)	(0.012)	(0.009)	(0.013)	(0.007)	(0.017)	(0.015)	(0.017)			
Other services	0.145	−0.029	0.028	0.011	0.014	−0.031	0.010	−0.030	0.020	0.095	0.762	1.44
	(0.001)	(0.016)	(0.014)	(0.011)	(0.016)	(0.008)	(0.021)	(0.018)	(0.020)			

5. Conclusion

The conclusion of the paper is that the failure of linear homogeneity in demand systems reported by a number of authors [cf. the papers cited in Deaton and Muellbauer (1980, p. 320)] may be attributed instead to the presence of an endogenous expenditure variable. That is, the hypothesis of non-homogeneity with expenditure exogenous is observationally equivalent to the hypothesis of endogeneity of expenditure with homogeneity taken as given. More information is required to break this deadlock.

One solution may come from the observation that most demand theorists more readily believe the postulate of homogeneity rather than that expenditure is exogenous. Failure of homogeneity may then be ascribed to the presence of a non-zero correlation between equation errors and expenditure, in which case the type of analysis undertaken in section 4 would be valid and the covariances would then be estimated. Another approach might be to obtain further information in the form of an appropriate instrumental variable for total expenditure. Given such an instrument the expenditure variable can be tested directly for correlation with the equation error in the manner suggested by Hausman (1978). Whatever the approach, the estimation procedure in section 3 is so simple that it should provide a useful research tool since it can be routinely applied by investigators analysing systems of demand equations.

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