Introduction to Computable General Equilibrium Model (CGE)

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Course Outline

- Overview of CGE
- An Introduction to the Structure of CGE
- An Introduction to GAMS
- Casting CGE models into GAMS
- Data for CGE Models & Calibration
- Incorporating a trade & a basic CGE application
- Evaluating impacts of policy changes and casting nested functions & a trade in GAMS
- Mixed Complementary Problems (MCP)
This Week’s Road Map

- What is GAMS?
- Using GAMS
- Dissecting GAMS Formulation
- A user interface - GAMS IDE
  - steps to create and run programs
- Features of the GAMS IDE
- GAMS Documentation
What is GAMS?

- **Generalized Algebraic Modeling System**
  - a language for setting up and solving mathematical programming optimization models
  - all-in-one package that allows one to
    - specify the structure of an optimization model
    - specify and calculate data that go into that model
    - solve that model
    - conduct report writing on a model
    - perform a comparative static analysis
Two approaches to using GAMS

1) Traditional method:
   use a text editor set up the model then use DOS (or UNIX) command line instructions to find errors in and run the model

2) GAMS IDE alternative:
   A graphical interface to create, debug, edit and run GAMS files. We will use GAMS through the IDE for this class.
Formulation of a Simple Market Clearing

- **Demand:** \( P \geq P_d = 6 - 0.3 \cdot Q_d \)
- **Supply:** \( P \leq P_s = 1 + 0.2 \cdot Q_s \)
- **Equilibrium:** \( Q_s \geq Q_d \) and \( P, Q_s, Q_d \geq 0 \)

**POSITIVE VARIABLE**
- \( P \): Equilibrium price
- \( Q_d \): Quantity demanded
- \( Q_s \): Quantity supplied

**EQUATION**
- **DemandPrice** Demand equation
- **SupplyPrice** Supply equation
- **Qbalance** Equilibrium equation

<table>
<thead>
<tr>
<th>Equation</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>DemandPrice..</td>
<td>( P = 6 - 0.3 \cdot Q_d )</td>
</tr>
<tr>
<td>SupplyPrice..</td>
<td>( 1 + 0.2 \cdot Q_s = P )</td>
</tr>
<tr>
<td>Qbalance..</td>
<td>( Q_s = Q_d )</td>
</tr>
</tbody>
</table>

**MODEL EQUIL**

/ DemandPrice.Qd
SupplyPrice.Qs
Qbalance.P / ;

**OPTION** MCP = PATH ;
**SOLVE** EQUIL using MCP ;
Formulation of a Simple Market Clearing

Demand:
\[ P \geq 6 - 0.3*Q_d \]
\[ [ P - (6 - 0.3*Q_d) ]Q_d = 0 \]
\[ Q_d = 0 \text{ then } P > 6 - 0.3*Q_d \]

Supply:
\[ P \leq 1 + 0.2Q_s \]
\[ [ P - (1 + 0.2) ]Q_s = 0 \]
\[ Q_s = 0 \text{ then } P < 1 + 0.2Q_s \]

Equilibrium:
\[ Q_s \geq Q_d \]
\[ (Q_s - Q_d)P = 0 \]
\[ P = 0 \text{ then } Q_s > Q_d \]
Formulation of a Simple Market Clearing

IF

\[ Q_d > 0 \text{ then } P = 6 - 0.3Q_d \]
\[ Q_s > 0 \text{ then } P = 1 + 0.2Q_s \]
\[ P > 0 \text{ then } Q_s = Q_d \]

\[ \text{Implies that } P_d = P_s = P \]

Price (\$)

\[ Q_d = Q_s \]

Quantity
GAMS Solution

Solution

At Equilibrium:

\[ \text{Pd} = \text{Ps} = \text{P} \Rightarrow \text{Pd} = 6 - 0.3 \times 10 = 3 \]
\[ \text{Ps} = 1 + 0.2 \times 10 = 3 \]
\[ \text{Qd} = \text{Qs} = 10 \]
Dissecting GAMS

**POSITIVE VARIABLE**

- \( P \)  
  Equilibrium price
- \( Q_d \)  
  Quantity demanded
- \( Q_s \)  
  Quantity supply

**EQUATION**

- **DemandPrice**  
  Demand equation
- **SupplyPrice**  
  Supply equation
- **Qbalance**  
  Equilibrium equation

DemandPrice..  
\[ P =G= 6-0.3*Q_d \];

SupplyPrice..  
\[ 1+0.2*Q_s =G= P \];

Qbalance..  
\[ Q_s =G= Q_d \];

**MODEL** EQUIL  
/DemandPrice.Qd
SupplyPrice.Qs
Qbalance.P /

**OPTION**  
MCP = PATH ;

**SOLVE**  
EQUIL using MCP ;
Variable Specification

GAMS requires variables in each problem to be identified. In the example, we have variables P, Qd, Qs

**POSITIVE VARIABLE**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>Equilibrium price</td>
</tr>
<tr>
<td>Qd</td>
<td>Quantity demanded</td>
</tr>
<tr>
<td>Qs</td>
<td>Quantity supply</td>
</tr>
</tbody>
</table>

2 types of variables

- **VARIABLE**: unrestricted variables
- **POSITIVE VARIABLE**: restricted variables to be nonnegative

\[ \begin{align*} 
P & \geq 0 \\
Qd & \geq 0 \\
Qs & \geq 0
\end{align*} \]
Equation Specification consists of two parts.

(1) Naming equations:
GAMS requires the modeler name each equation, which is active in the model. In the example, the equations are named in the EQUATION line.

The name for each equation can be anything up to 31 characters.
(2) Specifying algebraic structure:

After naming equations, the exact algebraic structure of equations must be specified by using .. notation.

DemandPrice..  P =G= 6 - 0.3*Qd ;  P \geq 6 - 0.3*Qd
SupplyPrice..  1 + 0.2*Qs =G= P ;  P \leq 1 + 0.2*Qs
Qbalance..     Qs =G= Qd ;  Qs \geq Qd

This algebraic form involves use of a special syntax to tell the exact form of the equation that may actually be an inequality.

=E= indicates an equality constraint
=<= indicates a less than or equal to constraint
=>= indicates a greater than or equal to constraint
Model Specification

**MODEL** statement is used to identify models that will be solved. It involves 2 steps:
- give name of the model (e.g. EQUIL)
- specify equations that will be included in the model in slashes / /

**MCP = Mixed Complementary Problem**
**MCP** uses ‘.’ as complementary

```
MODEL EQUIL /DemandPrice.Qd
   SupplyPrice.Qs
   Qbalance.P / ;
```

```
MODEL EQUIL /DemandPrice.Qd
   SupplyPrice.Qs /;
```

Omitting Qbalance equation
Dissecting GAMS

- **Solve Specification**

  **SOLVE** causes GAMS to use a solver to the model named (EQUIL) immediately after the SOLVE statement.

  ```gams
  SOLVE EQUIL using MCP ;
  ``

  **MCP = Mixed Complementary Problem**

  That model must already have been defined in a **MODEL** statement.

  ```gams
  MODEL EQUIL / DemandPrice.Qd
  SupplyPrice.Qs
  Qbalance .P / ;
  ```
Dissecting GAMS

■ **Specification**

GAMS requires to terminate each statement with a `;`.

**POSITIVE VARIABLE**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>Equilibrium price</td>
</tr>
<tr>
<td>Qd</td>
<td>Quantity demanded</td>
</tr>
<tr>
<td>Qs</td>
<td>Quantity supply</td>
</tr>
</tbody>
</table>

**EQUATION**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>DemandPrice..</td>
<td>Demand equation</td>
</tr>
<tr>
<td>SupplyPrice..</td>
<td>Supply equation</td>
</tr>
<tr>
<td>Qbalance..</td>
<td>Equilibrium equation</td>
</tr>
</tbody>
</table>

\[
\text{DemandPrice.. } P = G = 6 - 0.3 \times Qd ; \\
\text{SupplyPrice.. } 1 + 0.2 \times Qs = G = P ; \\
\text{Qbalance.. } Qs = G = Qd ;
\]

**MODEL** EQUIL /DemandPrice.Qd \
SupplyPrice.Qs  \
Qbalance.P / ;

**OPTION** MCP = PATH ;
**SOLVE** EQUIL using MCP ;

`;` is a very important part of the syntax. The omission often causes many syntax errors.
Dissecting GAMS – Finding errors

\[
\text{DemandPrice..} \quad P \quad =G= \quad 6-0.3*Qd \\
\text{SupplyPrice..} \quad 1+0.2*Qs \quad =G= \quad P \\
\text{Qbalance..} \quad Qs \quad =G= \quad Qd
\]

**Error Messages**

--- Starting compilation
--- SMALLMCP.GMS(14) 1 Mb 1 Error
*** Error 409 in C:\TASANA\685CGEPROJECT\SMALLMCP.GMS
Unrecognizable item - skip to find a new statement
   looking for a ';' or a key word to get started again
--- SMALLMCP.GMS(22) 1 Mb 2 Errors
*** Error 257 in C:\TASANA\685CGEPROJECT\SMALLMCP.GMS
Solve statement not checked because of previous errors
--- SMALLMCP.GMS(24) 1 Mb 5 Errors
*** Error 141 in C:\TASANA\685CGEPROJECT\SMALLMCP.GMS
Symbol neither initialized nor assigned
   A wild shot: You may have spurious commas in the explanatory
text of a declaration. Check symbol reference list.
Steps to using GAMS IDE

1. Install GAMS and IDE
2. Start the IDE
3. Create and open files
4. House keeping
5. Run GAMS
6. Navigate around outputs
1. **Install GAMS and the IDE on your computer**

   The IDE is automatically installed when GAMS is installed. To install do the following steps:
   
   a. load the GAMS CD into your machine
   
   b. start the installation using the Windows Explorer and go into the systems subdirectory called **win**
      then double click on **setup.exe**
   
   c. supply the location for a license (on your floppy)
d. right click on gamside.exe in the GAMS system directory, choose “create shortcut” and drag the shortcut to your desktop
2. Start the IDE using it’s icon

a. double click the icon

b. create a directory for your work by opening the File menu and select Project and New project

The IDE uses a “Project” file for two purposes.

: to determine where all saved files are placed and where GAMS looks for files when executing

: to save file names and program options associated with the effort.
c. define project name and location. All files associated with this project will be saved here

In the “File name” area type in a name for the project file you wish to use. This defines the directory where your files are located.
GAMS IDE - How to open library files

3. Open existing files
   a. from the model library

GAMS Model Library Version 11.0

<table>
<thead>
<tr>
<th>Name</th>
<th>Application Area</th>
<th>Type</th>
<th>Contributor</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>TSP42</td>
<td>Recreational Models</td>
<td>MIP</td>
<td>Dantzig, G B</td>
<td>TSP solution for 4 cities</td>
</tr>
<tr>
<td>TURKEY</td>
<td>Agricultural Economics</td>
<td>NLP</td>
<td>Le-Si, V</td>
<td>Turkey Agriculture</td>
</tr>
<tr>
<td>TURKPOW</td>
<td>Energy Economics</td>
<td>LP</td>
<td>Turvey, R</td>
<td>Turkey Power Plant</td>
</tr>
<tr>
<td>TWO3MCP</td>
<td>Applied General Equilibrium</td>
<td>MCP</td>
<td>Shoven, J</td>
<td>Simple 2 x 2 x 2</td>
</tr>
<tr>
<td>UIMP</td>
<td>Management Science and OR</td>
<td>LP</td>
<td>Ellison, E F</td>
<td>LIMP - Products</td>
</tr>
<tr>
<td>UNSTMGE</td>
<td>Applied General Equilibrium</td>
<td>MPSGE</td>
<td>Scarf, H</td>
<td>Globally Unstable</td>
</tr>
</tbody>
</table>

sets
   f factors /labor, capital/;
s sectors /mfrs, nonmfrs/;
h households /rich, poor/;
b. from your directory
4. Create new files

a. open existing files and with save as dialogue from file menu change it’s name.
b. open the file menu and use the new option.

You will then get a file called *untitled* with an empty screen that you can create (type) your own program.

```plaintext
VARIABLE
DemandPrice       Demand equations
SupplyPrice         Supply equations       ;
```
5. Do a little housekeeping

: use the options dialog under File to set the output page length to 9999 and under the execute dialog check the box update process window
GAMS IDE - Fixes display
GAMS IDE - Choose solvers

5. Do a little housekeeping (con’t)

: make IDE the *.gms file processor

: set up solvers
  e.g. if NLP using MINOS
  if LP using CPLEX
  if MCP using PATH
6. Run GAMSIDE

: clicking the run button or pressing F9

As GAMS is running, the process window giving a log of steps will appear.
6. Run GAMSIDE (con’t)

--- Starting compilation
--- SMALLMCP.GMS(26) 1 Mb

=> check if your file is ok
=> (26) indicate line it is on
=> execute your file

--- Starting execution
--- SMALLMCP.GMS(20) 2 Mb

=> set up the problem
=> size of the problem

--- Generating model EQUIL
--- SMALLMCP.GMS(22) 2 Mb

=> start solver and gives a name for which solver is used

--- 3 rows, 3 columns, and 6 non-zeroes.
--- SMALLMCP.GMS(22) 2 Mb

--- Executing PATH

--- Restarting execution
--- SMALLMCP.GMS(22) 0 Mb

=> GAMS restarts

--- Reading solution for model EQUIL
--- SMALLMCP.GMS(25) 2 Mb

*** Status: Normal completion

=> GAMS stops without errors
6. Run GAMSIDE (con’t)

- double click on lines in the process window to access output
- positioning of your access is determined by the color of the line
  - blue lines => open *.LST file and jump to line in *.LST file
  - black lines => open *.LST file and jump to a location of previous blue line
  - red lines => jump to *.gms file (your program) where errors occur
6. Run GAMSIDE (con’t)

- red lines => jump to *.gms file (your program) where errors were made

```gams
; * #####
* #### demands equal supply for goods
* #### equation 8a

make errors
CommodMkt(Sector)..
  Production(Sector)
  =G=
  sum(HouseH,
      (HHIncome(HouseH)
        / sum(Sector1, alpha(Sector1, HouseH)
```
7. Navigate around outputs

: look for first ---- to find equation listing in *.LST file

--- Income \( \geq \) income budget constraint equation

Income(NonFarmer).. \(-\) 25*FactPrice(Capital) + HHIncome(NonFarmer) \(\geq\) 0 ; (LHS =  

Income(Farmer).. \(-\) 60*FactPrice(Labor) + HHIncome(Farmer) \(\geq\) 0 ; (LHS = 0)

: look for the word ‘solution’ or ---- to find solution

---- VAR Production  Production level

<table>
<thead>
<tr>
<th>LOWER</th>
<th>LEVEL</th>
<th>UPPER</th>
<th>MARGINAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food</td>
<td>.</td>
<td>24.235</td>
<td>+INF</td>
</tr>
<tr>
<td>NonFood</td>
<td>.</td>
<td>55.283</td>
<td>+INF</td>
</tr>
</tbody>
</table>
Useful Tools – Find text

Useful Tools

- finds the first occurrence in the current file
- finds the next occurrence in the current file
- finds all occurrences in the directory where the project is located

Click on **a red line** to open the file and on **a black line** to open the file and indexes to the particular line
Useful Tools (con’t) – Parentheses matching

parentheses match up

```
CommodMkt(Sector)
Production(Sector)
= Com
sum(HHIncome(HouseH)
   / sum(Sector1, alpha(Sector1, HouseH)
      * ComPrice(Sector1)**(1-SigmaC(HouseH)))
   ) * Alpha(Sector, HouseH) * (1 / ComPrice(Sector))**SigmaC(HouseH)
)
```

Command line saving parameters once defined in \texttt{\a1}

Command line calling/retrieving saved parameters from \texttt{\a1}

Caution: Make sure that you are working on files located in the same directory location as the project is located.
Useful Tools (con’t) – Column block

Useful Tools (con’t)

\[\text{ALT+SHIFT} \] moving column blocks of text

\begin{table}[h]
\centering
\begin{tabular}{|l|c|c|}
\hline
Factor & Food & NonFood \\
\hline
Labor & 0.6 & 0.7 \\
Capital & 0.4 & 0.3 \\
\hline
\end{tabular}
\end{table}

The copy, cut, and paste can be done with the Edit menus as in normal windows.
GAMS documentation is accessible through the help menu under the choice GAMS
GAMS Documentation – Path

GAMS/PATH User Guide
Version 4.3

Michael C. Ferris
Todd S. Munson

March 20, 2000
Wrap Up

- What is GAMS?
- GAMS IDE
- Useful Tools for GAMS IDE

Next:
- Casting CGE Modeling via GAMS
  - Set definitions
  - Data entry
  - Variables & Equations specification
  - Identifying complementarity relationship
  - Normalizing prices
  - Solution reports
  - Comparative analysis
References

McCarl, B. A. Basic GAMS class. (http://agecon.tamu.edu/faculty/mccarl/mccarl.htm).