The Measurement of Environmental and Resource Values
Theory and Methods

Third Edition

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The theory of the measurement of welfare change has been discussed by others, both at the most rigorous levels of abstraction, and in pragmatic, practical terms of application. See for examples, Johansson (1987), Just, Hueth, and Schmitz (1982, 2004), and Bockstael and McConnell (2007). The earliest work focused on the welfare effects of changes in the prices people pay for the (private) goods they consume, but the literature has expanded broadly into valuing changes in the quantity and quality of both private and public goods. The current chapter provides a systematic development of the definition and measurement of the welfare effects stemming from changes in prices and the quantities and/or qualities of nonmarket environmental and resource service flows.

Changes in environmental quality can affect individuals’ welfares through a number of channels: changes in the prices paid for goods bought in markets; changes in the quantities or qualities of nonmarketed goods (for example, public goods such as air quality); changes in the prices received for factors of production; and changes in the risks individuals face. The first two of these channels are the focus of this chapter. After a brief review of the theory of individual preferences and demand, the principles of welfare measurement for price changes are reviewed. These principles are relevant because some forms of environmental change affect people only indirectly through price effects, and because these principles provide a solid foundation for the treatment of quantity and quality changes that follow.

Chapter 8 covers the welfare effects of changes in factor prices. The extension of these principles to the valuation of changes in risk—the fourth channel—raises some interesting questions, which will be left to Chapter 5.

The principles and measures developed in this chapter apply equally to decreases and increases in individuals’ welfare. It is a basic principle of welfare economics that all costs ultimately take the form of reductions in the utility of individuals. This principle applies equally to the costs of public policies (for example, investment in resource development and the regulation of private activities), and to the costs of private uses of the environment (for example, harvesting from a common property resource and using the waste receptor services of the environment). Hence, the welfare measures developed here provide a foundation for the analysis of both the benefits and the costs of environmental change.
the prices of these goods is examined, along with the relationships among the Marshallian and Hicksian surplus measures of welfare change for continuous goods (that is, goods for which the main consumer choice problem is to choose how many units to purchase and consume). There are two reasons for choosing this order of presentation. First, it parallels the historical evolution of the theory of welfare change. Second, it makes for an easier exposition of the basic principles. This section concludes with a review of methods for obtaining exact measures of, and approximations to, the desired Hicksian surpluses. The third section examines the case of the welfare effects of changes in the quantities of continuous goods. While potentially applicable to private goods, these welfare measures will most often be relevant for public goods or nonmarket goods since the quantity available for consumption is not a matter of choice for the individual. The next section considers welfare measures for discrete goods: goods for which the key choice is not how many units to consume, but rather which goods, from two or more alternatives, are consumed. In the final section, a review is provided of some of the issues involved in aggregating measures of individual welfare change for public policy decision making and in selecting the appropriate welfare measure.

**Individual Preferences and Demand**

Before introducing the various possible welfare measures, it will be useful to review briefly the basic theory of individual preferences and the demand for goods as it relates to welfare theory. For alternative treatments of this and related topics, the reader may wish to consult other texts, such as Just, Hueb, and Schmitz (1982, 2004), Badoway and Bruce (1984), Varian (1992), and Johansson (1987). This theory starts with the premise that individuals are their own best judges of their welfare and that inferences about welfare can be drawn for each individual by observing that individual's choices among alternative bundles of goods and services. If an individual prefers bundle A to bundle B, then bundle A must convey a higher level of welfare.

What things are to be included in the bundles (such as A and B) among which individuals are assumed to have preferences? There is little controversy over the inclusion of all the goods and services that can be bought or sold in markets—consumer goods, the services of household assets such as a house or a car, and consumer durables. Since time can be used in leisure activities, or sold at some wage rate in the labor market, individuals must also have preferences among alternative uses of time, such as reading, outdoor recreation, and working at some wage rate. Since government and the environment both provide a variety of services that enhance the welfare of individuals, these services should also be included in the bundles among which people have preferences. Environmental services include those provided by cleaner air, cleaner water, and scenic amenities. Just as importantly, these environmental services are not limited to direct uses of the environment, such as breathing clean air or observing unspoiled vistas—they can also include services related to the mere presence of environmental goods, such as the knowledge that pristine wilderness areas exist, or that there is a viable breeding population of a particular endangered species.

If we assume that individuals can rank the alternative bundles according to their preferences, what properties will the resulting ordering of bundles display? For our purposes, two properties are important. The first is nontransitivity, or the “more-is-better” property. This means that a bundle with a larger quantity of an element will be preferred to a bundle with a smaller quantity of that element, other things being equal. Formally, if \( X^r \) consists of \( (x'_1, \ldots, x'_n) \) and \( X^s \) consists of \( (x''_1, \ldots, x''_n) \) and \( x'_j > x''_j \) then this individual will prefer \( X^r \) to \( X^s \).

The second property is substitutability among the components of bundles. This means that if the quantity of one element of a bundle, say \( x_j \), is decreased, it is possible to increase the quantity of another element, say \( y_j \), sufficiently to make the individual indifferent between the two bundles. More formally, suppose that \( X^r \) consists of \( (x'_1, \ldots, x'_n, x'_n, \ldots, x'_j) \); and \( X^s \) consists of \( (x''_1, \ldots, x''_n, x''_j, \ldots, x''_j) \) with \( x''_j < x'_j \). Substitutability means that there is another bundle \( X^x \) consisting of \( (x''_1, \ldots, x''_n, x''_j, \ldots, x''_j) \) with \( x''_j > x''_j \) such that the individual is indifferent to \( X^x \) and \( X^s \). In other words, \( X^r \) and \( X^s \) lie on the same indifference surface.

The property of substitutability is at the core of the economist's concept of value. This is because substitutability establishes tradeoff ratios between pairs of goods that matter to people. In this formulation, the tradeoff ratio is \( \frac{(x'_j - x''_j)}{(x''_j - x'_j)} \) or \( |\Delta x_j / \Delta x_j| \). In the limit for infinitesimally small changes, this reduces to \( |dx_j / dx_j| \), which is the definition of the marginal rate of substitution between \( x_j \) and \( x_{j'} \) or the slope of the two-dimensional indifference curve between these two elements. The money price of a market good is just a special case of a tradeoff ratio, because the money given up to purchase one unit of one element of the bundle is a proxy for the quantities of one or more of the other elements in the bundle that had to be reduced in order to make the purchase.

If the preference ordering has the properties described here, it can be represented by an ordinal preference function, or utility function, that assigns a number to each bundle as a function of the quantities of each element of the bundle. Specifically,

\[
u = u(X, Q, T), \tag{3.1}
\]

where \( X \) is a vector of the quantities of market goods, \( Q \) is a vector of public goods and environmental and resource services whose quantities or qualities are fixed for the individual, and \( T \) is a vector of the times spent in various activities that yield utility to the individual. This utility function is assumed to be increasing in

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2 Two other important properties are transitivity and quasi-concavity. If there are three bundles \( X^r, X^o, \) and \( X^s \), and the individual prefers \( X^r \) over \( X^o \) and \( X^s \) over \( X^r \), then transitivity is satisfied if the individual prefers \( X^o \) over \( X^s \). For more on the axiomatic description of these properties of preference ordering, see Badoway and Bruce (1984) or Varian (1992).
all of its arguments, and unique up to a monotonic transformation. For purposes of mathematical modeling and analysis, it is convenient also to assume that this function is continuous, convex, and twice differentiable. This preference function is not the same thing as the cardinal utility function of the classical utilitarians. Since there is no unit of measurement for this ordinal utility, it is not possible to add or otherwise compare the utilities of different individuals.

To simplify the exposition and notation, let us now consider an individual whose utility is a function only of private goods that can be bought and sold in markets. Assume that tastes and preferences (that is, the utility function) are given and do not change. The individual faces a set of given prices for these goods and is assumed to choose the quantities of the goods so as to maximize his utility, given the constraints of prices and a fixed money income $M$. The maximization problem can be expressed as

$$\text{maximize } u = u(X),$$

subject to

$$\sum_{j=1}^{J} p_j x_j = M \quad (3.2)$$

where $X = (x_1, \ldots, x_j, \ldots x_J)$ is the vector of quantities. The solution to this problem leads to a set of ordinary, or Marshallian, demand functions

$$x_j = x_j(P, M), \quad (3.3)$$

where $P = (p_1, \ldots, p_j, \ldots p_J)$ is the vector of prices.

Substituting the expressions for $x_j$ as functions of $P$ and $M$ into the direct utility function gives the indirect utility function—that is, utility as a function of prices and income, assuming optimal choices of goods:

$$v = v(P, M). \quad (3.4)$$

According to Roy’s identity, the demand functions can also be expressed in terms of derivatives of the indirect utility function,

$$x_j(P, M) = -\frac{\partial v}{\partial p_j} \frac{\partial p_j}{\partial v} \quad (3.5)$$

The expenditure function represents a useful perspective on the problem of individual choice. The expenditure function is derived by formulating the dual of the utility maximization problem. The individual is assumed to minimize total expenditure,

$$e = \sum_{j=1}^{J} p_j x_j, \quad (3.6)$$

subject to a constraint on the level of utility attained,

$$u(X) = u^0, \quad (3.7)$$

where $u^0$ is the maximum utility attained with the solution to the primal problem. Just as the solution to the utility maximization problem yields a set of ordinary demand curves conditional on prices and money income, the solution of the expenditure minimization problem yields a set of functions giving optimal quantities for given prices and utility. These are Hicks-compensated demand functions that show the quantities consumed at various prices, assuming that income is adjusted (compensated), so that utility is held constant at $u^0$. Substituting these demand functions into the expression for total expenditure yields the expenditure function. This expression gives the minimum dollar expenditure necessary to achieve a specified utility level given market prices. In functional notation:

$$e = e(P, u^0), \quad (3.8)$$

where $e$ is the dollar expenditure and $u^0$ is the specified utility level. The compensated demand functions can also be found by differentiating the expenditure function with respect to each of the prices:

$$\frac{\partial e}{\partial p_j} = h_j = h_j(P, u^0), \quad (3.9)$$

where $h_j$ is the compensated demand for $x_j$.

Now consider the set of ordinary demand functions derived from the utility maximization problem. In order to determine the functional form and parameters of these demand functions, it is necessary to know the underlying utility function, and this may not be directly observable. Suppose instead that we observe an individual’s behavior and estimate the demand functions that describe the individual’s responses to changes in prices and income. These functions are based on the same information as the underlying preferences. This is assured, provided the demand functions satisfy the so-called integrability conditions. These conditions require that the Slutsky matrix of substitution terms,

$$\frac{\partial h_j}{\partial p_i} = \frac{\partial x_j}{\partial p_i} \frac{\partial v}{\partial p_i} + \frac{\partial x_j}{\partial M} \frac{\partial v}{\partial M}, \quad (3.10)$$

is symmetric and negative semi-definite (Hurwicz and Uzawa 1971; Silberberg 1978; Varian 1992). If these conditions are satisfied, the system of demand functions can be integrated to yield the expenditure function, which in turn can be used to derive the indirect and direct utility functions. If the integrability conditions are not satisfied, the implication is that the observed demand functions are not consistent with the maximization of a well-behaved utility function. As explained below, if the integrability conditions are satisfied, it may be possible to utilize empirically derived descriptions of demand behavior to obtain a complete description of the underlying preferences, as well as exact measures of welfare change for a wide range of postulated changes in economic circumstances.
Welfare Measures for Continuous Goods: 
Price Changes

An Overview

In order to introduce the alternative welfare measures, consider first the simplest case of only two goods and the welfare gain associated with a nonmarginal decrease in the price of one of these goods. Two types of measures of this welfare change have been identified in the literature. The first is the change in ordinary consumer’s surplus, a concept with an origin that can be traced back through Alfred Marshall to Dupuit. Mishan (1960) and Currie, Murphy, and Schmitz (1971) provided useful discussions of the history and evolution of the concept of consumer’s surplus. As Marshall explained it,

[The individual] derives from a purchase a surplus of satisfaction. The excess of the price that he would be willing to pay rather than go without the thing, over that which he actually does pay, is the economic measure of this surplus of satisfaction. It may be called consumer’s surplus.

(Marshall 1920, 124)

Ordinary consumer’s surplus is measured by the area under a Marshallian ordinary demand curve, but above the horizontal price line. As we will see, the consumer surplus measure cannot be defined in terms of the underlying utility function.

The other measures of welfare change are theoretical refinements of the ordinary consumer’s surplus (Hicks 1943), and each can be defined in terms of the underlying individual preference mapping. Figure 3.1 is used to illustrate these concepts in the context of two goods ($x_1$ and $x_2$), where $x_1$ is the numeraire good (i.e., $p_1 = 1$). The figure shows two indifference curves for the individual. Assume that an environmental improvement reduces the cost of producing $x_1$, so that its price drops from $p_1'$ to $p_1''$. In response to the price reduction, the individual shifts from the consumption bundle marked $A$ at utility level $u_1$ to consumption bundle $B$ at utility level $u_2$. What is the welfare benefit of the price reduction to this individual? Two additional measures of the welfare change can be defined in terms of the numeraire good $x_2$:

1. Compensating Variation ($CV$). This measure asks what compensating payment (that is, an offsetting change in income) is necessary to make the individual indifferent between the original situation ($A$ in Figure 3.1) and the new price set. Given the new price set with consumption point $B$, the individual’s income could be reduced by the amount of $CV$ and that person would still be as well off at point $C$ as at point $A$ with the original price set and money income. The measure $CV$ is often interpreted as the maximum amount that the individual would be willing to pay for the opportunity to consume at the new price set. However, for a price increase, $CV$ measures what must be paid to the individual to make that person indifferent to the price change. For price decreases, the $CV$ cannot be greater than the individual’s income; but for a price increase, the $CV$ could exceed income.

2. Equivalent Variation ($EV$). This measure asks what change in income (given the original prices) would lead to the same utility change as the change in the price of $x_1$. As shown in Figure 3.1, given the original prices, the individual could reach utility level $u_1$ at point $D$ with an income increase equal to $EV$. $EV$ is the income change equivalent to the welfare gain due to the price change. The $EV$ measure has also been described as the minimum lump sum payment the individual would have to receive to induce that person to voluntarily forgo the opportunity to purchase at the new price set. For a price increase, $EV$ is the maximum amount the individual would be willing to pay to avoid the change in prices.

Note that both the $EV$ and $CV$ measures allow the individual to adjust the quantities consumed of both goods in response to both changes in relative prices and income levels. Hicks also described two additional measures where the levels of the goods could not be changed. He referred to them as compensating and equivalent surplus. The compensating and equivalent surplus measures for price changes do not answer very useful questions since they both arbitrarily restrict the individual to consuming a specific quantity of the good whose price has changed.

![Figure 3.1 Two measures of the welfare gain from a price decrease](image-url)
Hence, the original form suggested by Hicks will not be considered further, but we will return to measures of welfare change associated with quantity changes (with no associated price changes) later in this chapter; and note that, following Hicks, such welfare measures are often referred to as compensating and equivalent surplus. The next subsection is devoted to a comparison and evaluation of the compensating and equivalent variations and their relationship to the ordinary consumer surplus.

In the many-good case, \( x_1 \) is a composite good that can be treated as an index of the consumption levels of all other goods except \( x_1 \). The aggregation of all other goods into a composite good for graphical representation is valid so long as the prices of all the goods are assumed to move in the same proportion—that is, there are no changes in the relative prices of components of the composite good bundle. This assumption can be maintained, since we are analyzing only the consequences of the change in the price of \( x_1 \).

A Closer Look at the Welfare Measures

This section begins with a presentation of the basic welfare measure for a marginal change in one price. Then more rigorous derivations are provided for the consumer surplus, compensating variation, and equivalent variation measures of welfare change for the case of changes in price. For more detailed treatment of these topics see Silberberg (1972), Just, Hult, and Schmitz (1982, Appendix B), Varian (1992), and Johansson (1987). For a marginal change in, say, \( p_1 \), the basic welfare measure is the change in expenditure necessary to hold utility constant. Using equation (3.9) from the previous section, we have

\[
\omega_h = \frac{\partial e(P, u^0)}{\partial p_1} = h(P, u^0),
\]

(3.11)

where \( \omega_h \) is the marginal welfare measure. This result also follows from the indirect utility function and Roy's identity:

\[
\omega_h = x_1 = -\frac{\partial u/\partial p_1}{\partial u/\partial M}
\]

(3.12)

or

\[
\frac{dM}{dp_1} = x_1.
\]

(3.12')

In equation (3.12), the marginal utility of the price change is converted to monetary units by dividing by the marginal utility of income. Equation (3.12') says that the change in income required to hold utility constant is equal to the change in price multiplied by the quantity of the good being purchased.

Figure 3.2 The compensating variation and Hicks-compensated demand
Marshallian Consumer Surplus

In Figure 3.2, panel A shows one individual’s preference mapping in the simple two-good case. Suppose that the price of good $x_i$ falls from $p_i'$ to $p_i''$. The individual responds by moving from the original equilibrium at point $A$ to point $B$ on the new budget line. In panel B of Figure 3.2, these equilibrium positions are plotted in the price and quantity plane. Points $A$ and $B$ are on the ordinary demand curve, holding the price of good $x_i$ and money income constant. Since the Marshallian surplus associated with the consumption of a good at a given price is the area under the demand curve, the change in surplus for a change in the good's price is the geometric area $\Phi_1'$ in panel B of Figure 3.2. In mathematical form,

$$S = \int_{x_1}^{x_1'} (P, M) d\Phi_1,$$  \hspace{1cm} (3.13)

where $S$ is the change in surplus.

The condition under which $S$ can be interpreted as an indicator of utility change can be seen by employing Roy's identity:

$$x_i (P, M) = \frac{\partial v(P, M)}{\partial P_i} / \frac{\partial v(P, M)}{\partial M},$$  \hspace{1cm} (3.14)

and substituting this into equation (3.13) to obtain

$$S = -\int_{x_1}^{x_1'} \frac{\partial v(P, M)}{\partial M} d\Phi_1.$$  \hspace{1cm} (3.15)

If the marginal utility of income is constant over the range of the price change, this can be written as

$$S = v(p_i''p_2,M) - v(p_i'p_2,M).$$  \hspace{1cm} (3.16)

This expression shows that the Marshallian surplus can be interpreted as the utility change converted to monetary units by a weighting factor—the marginal utility of income. If the marginal utility of income is constant, then $S$ can be said to be proportional to the change in utility for any price change. However, as any one price changes, the constancy of the marginal utility of income is a restrictive condition. The marginal utility of income cannot simultaneously be invariant: with respect to income and to changes in all of the prices (Samuelson 1942; Johansson 1987, ch. 4).

Alternatively, as Eugene Silberberg explained (1978, 350–361), the integral of equation (3.15) can be viewed as the sum of a series of small steps from an initial price and income vector of $(p_1', p_2', M)$ to $(p''_1, p_2', M)$, following a path on which $p_2$ and $M$ are held constant. However, there are other paths over which (3.15) can be integrated involving changes away from the initial values for $p_2$ and/or $M$ as long as the terminal point is $(p''_1, p_2', M)$). The other paths, in general, will not lead to the same solution value for the integral. In other words, the integral in general will not be path independent.

A similar problem arises when the Marshallian surplus measure is generalized to simultaneous changes in all prices. This case, $S$ is defined as a line integral. This integral will be independent of the path of integration (that is, the order in which prices and/or incomes are assumed to change) only if the income elasticities of demand for all goods are equal. The income elasticities of all goods can be equal to each other only if they are all equal to one, in other words, if preferences are homothetic. Finally, if the prices of only a subset of all goods change, a unique $S$ exists if the marginal utility of income is constant with respect to only those prices that are changed. See Just, Hueth, and Schmitz (1982) for more details.

Compensating Variation

Suppose now that as the price of good $x_i$ is decreased, income is taken away from the individual so that he remains at the initial utility level and indifference curve $u^i$. Given the price change and the compensating income change, the individual would be in equilibrium at point $C$ in panel A of Figure 3.2. Point $C$ is also plotted in panel B of Figure 3.2. Points $A$ and $C$ are on the Hicks-compensated demand curve, a demand curve that reflects only the substitution effect of the change in relative prices. The device of compensating withdrawals of money income has eliminated the income effect of the price change. Since $x_i$ is a normal good by assumption—that is, it has an income elasticity greater than zero—the Hicks-compensated demand curve is less price-elastic than the ordinary demand curve.

The difference between the Hicks-compensated and the ordinary demand functions is one of the main considerations in the comparison of $EV$, $CV$, and consumer surplus measures of welfare change.

Panel A of Figure 3.2 shows the compensating variation measure of the welfare change associated with the price decrease—that is, the reduction in income needed to hold the individual on the original indifference curve. In terms of the indirect utility function, $CV$ is the solution to

$$v(P^*, M) = v(P'^*, M - CV) = u^i.$$  \hspace{1cm} (3.17)

The $CV$ can also be defined in terms of the expenditure function. It is the difference between the expenditures required to sustain utility level $u^i$, at the two price sets:

$$CV = e(p_i', p_2, u^i) - e(p_i'', p_2, u^i)$$
$$= M - e(p_i'', p_2, u^i) > 0.$$  \hspace{1cm} (3.18)

"
Because $CV$ is defined as the difference between two levels of expenditure, it can also be written as the integral of the marginal welfare measure (equation 3.11) over the relevant range. Specifically,

$$CV = \int_{u^0}^{u^1} \frac{\partial h(P, u)}{\partial u} du.$$  \hspace{1cm} (3.19)

Since spending $M$ at the new price set yields a higher level of utility, we can also write

$$M = e\left(\rho_0^0, \rho_1^0, u^0\right),$$  \hspace{1cm} (3.20)

and by substitution

$$CV = e\left(\rho_0^0, \rho_1^0, u^1\right) - e\left(\rho_0^0, \rho_1^0, u^0\right) > 0.$$  \hspace{1cm} (3.21)

In other words, although the $CV$ is defined in terms of $u^0$, it also measures the amount of money required to raise utility from $u^0$ to $u^1$ at the new set of prices.

The $CV$ is equal to the area to the left of the Hicks-compensated demand curve between the two prices—that is, the area $\rho\Delta q_k^n$. The partial derivative of the expenditure function with respect to $\rho_j$ gives the change in expenditure (income) necessary to keep the individual on $u^0$ for small changes in $\rho_j$. As shown above, this derivative gives the Hicks-compensated demand curve—that is, it gives the optimal quantity for $x_j$, holding utility constant. For finite changes, the integral of this derivative is the area to the left of the Hicks-compensated demand curve—that is, the $CV$. In other words,

$$CV = \int_{u^0}^{u^1} h(P, u) du.$$  \hspace{1cm} (3.22)

Unlike the Marshallian measure of surplus given by equation (3.13), this measure does not rely on any assumption about the constancy of the marginal utility of income. This is because this measure integrates along a constant utility indifference curve at $u^0$. In the many-good case, when several prices change, the $CV$ of the price changes taken together is the integral of the set of compensated demand functions evaluated by taking each price change successively. The order in which the price changes are evaluated is irrelevant. This follows from the symmetry of the cross price substitution terms—that is, $\partial x_j/\partial \rho_k = \partial x_k/\partial \rho_j$.

**Equivalent Variation**

The equivalent variation can also be derived through the expenditure function. Panel A of Figure 3.3 shows the same preference mapping and price change for an individual. With a price decrease, the $EV$ is defined as the additional expenditure (income) necessary to reach utility level $u^1$, given the initial set of prices. In terms of the indirect utility function, $EV$ is the solution to

$$v(P', M+EV) = v(P''', M) = u^1.$$  \hspace{1cm} (3.23)
In Figure 3.3, the \( EV \) is the additional expenditure necessary to sustain point \( C' \) over point \( A \) at the initial prices, or
\[
EV = \epsilon(p', p_2, u') - \epsilon(p', p_2, u^*)
\]
\[
= \epsilon(p', p_2, u') - M > 0.
\] (3.24)

Since the money expenditure levels are the same at point \( A \) and point \( B \)—that is, \( \epsilon(p', p_2, u^*) = \epsilon(p', p_2, u') \)—this can also be written as
\[
EV = \epsilon(p', p_2, u') - \epsilon(p', p_2, u').
\] (3.25)

In other words, although the \( EV \) is defined in terms of the monetary equivalent of a change from \( u^* \) to \( u' \), it can also be measured by the change in expenditure associated with price changes given utility level \( u' \).

The \( EV \) can also be written as the integral of the marginal value measure (equation 3.11):
\[
EV = \int_{p}^{p'} \frac{\partial \epsilon(p, u)}{\partial p} \, dp.
\] (3.26)

The price derivative of the expenditure function (this time holding utility constant at \( u' \)) generates another Hicks-compensated demand curve through point \( B \) in panel B of Figure 3.3. The area to the left of this Hicks-compensated demand curve between the two prices (area \( \rho(C'B) \)) is the equivalent variation welfare measure. In other words,
\[
EV = \int_{p}^{p'} h(p, u') \, dp.
\] (3.27)

As in the case of the \( CV \), this measure does not require any assumption about the constancy of the marginal utility of income; and the measure for multiple price changes is path independent.

All of this discussion has been in terms of the welfare gain due to a price decrease. The derivation of the welfare cost of a price increase can be worked out in a symmetrical fashion. In general, for any price change, the \( CV \) welfare measure is the area to the left of the Hicks-compensated demand curve that passes through the initial position. The \( EV \) measure of the welfare change is the area to the left of the Hicks-compensated demand curve that passes through the final position.

A Comparison of the Three Measures

Although the Marshallian consumer surplus has some intuitive appeal as a welfare indicator, it does not measure either of the theoretical definitions of welfare change developed here. In general, it is not a measure of gain or loss that can be employed in a potential compensation test. The Marshallian surplus does lie between the \( CV \) and the \( EV \), however, this opens the question of whether it can be a useful approximation to either of these other measures, a question that is taken up below in the subsection Consumer’s Surplus Without Apology.

In contrast, the \( CV \) and the \( EV \) do represent welfare relevant measures. The \( EV \) is the monetary equivalent of a price change. It can be interpreted as an index of utility in the sense that it imputes the same monetary value to all changes from an initial position that result in the same final utility level. This is an ordinal utility index (Morey 1984). For example, suppose a change from initial position \( A \) to position \( B \) has an \( EV \) of \$10, while a change from \( A \) to \( C \) has an \$EV \ of \$20. It cannot be inferred that the second change conveys twice as much extra utility as the first change. This is because it evaluates all changes from an initial position at the same set of prices. The \( CV \) cannot be interpreted as an index of utility—rather, it measures the offsetting income change necessary to “prevent” a utility change. As Silberberg put it, “the \( EV \) imputes a dollar evaluation to a change in utility levels for a particular path of price changes, while the \( CV \) derives dollar values necessary to hold utility constant when prices change” (1972, 948).

The two measures \( EV \) and \( CV \) will be the same if the income elasticity of demand for good \( x_i \) is zero. In this case, the ordinary and Hicks-compensated demand curves are identical. With positive income elasticity, the \( EV \) exceeds the \( CV \) for price decreases, but the \( CV \) exceeds the \( EV \) when price increases are considered. The difference between points \( C \) and \( B \) in Figure 3.2, and between points \( A \) and \( C \) in Figure 3.3, is one of income level. If the income elasticity of demand for \( x_i \) were zero, the income differences would have no effect on the purchase of \( x_i \). The \( CV \) and the \( EV \) would be exactly equal, and they both could be measured by the area under the ordinary demand curve. The higher the income elasticity of demand for \( x_i \), the larger the difference between the \( EV \) and the \( CV \), and the larger the difference between either of the measures and the ordinary consumer surplus.

There is symmetry between the \( CV \) and the \( EV \) measures that can be seen by comparing Figures 3.2 and 3.3, and by comparing equation (3.21) with equation (3.24), and by comparing equation (3.18) with equation (3.25). For simplicity, let I represent the initial price set (with \( p'_0 \)) and let II represent the second price set (with \( p''_0 \)). The \( CV \) for moving from I to II with \( u^* \) as the reference utility level is exactly equal to the \( EV \) of moving from II to I with \( u' \) as the reference utility level. The \( CV \) is a welfare measure for the move from I to II via point \( C \); the \( EV \) starts at point \( B \) and measures the reduction in income necessary to get to point \( A \), and therefore \( u' \) via point \( C \). Similarly, the \( EV \) for the move from I to II is just equal to the \( CV \) starting at \( II \) and \( u' \), and moving to \( I \).

This symmetry relates to the interpretation of \( CV \) and \( EV \) as measures of willingness to pay (WTP) and willingness to accept (WTA) compensation. The \( CV \) is sometimes described as the maximum willingness to pay for the right to purchase the good at the new price level (i.e., the lump sum payment that the individual would be willing to make that would just exhaust the potential for welfare gain from the new price). This description is accurate only for a price decrease. For a price increase, the \( CV \) defines the minimum payment to the individual sufficient to prevent a utility decrease; in other words, it defines a WTA measure. Similarly, the \( EV \) defines a WTA measure for a price decrease—that is, the sum of money the individual would require to voluntarily forgo a proposed price decrease. However,
for a proposed price increase, the EV is a WTP measure—that is, the maximum sum of money that could be taken away from the individual—yielding a loss of utility equivalent to that caused by the price change. Whatever the direction of the price change, the CV takes the initial utility as the reference point.

These two measures can also be interpreted in terms of the implied rights and obligations associated with alternative price sets. The CV carries an implicit presumption that the individual has no right to make purchases at a new set of lower prices, but does have a right to the original price set in the case of price increases. In contrast, the EV contains the presumption that the individual has a right to (an obligation to accept) the new lower (higher) price set, and must be compensated (make a payment) if the new price set is not to be attained. Based on this interpretation of the two measures, some economists have argued that the choice between them is basically an ethical one—that is, one that depends on a value judgment as to which underlying distribution of property rights is more equitable (Krutilla 1967; Mishan 1976). All of this can be summarized as in Table 3.1.

For two alternative price changes, the welfare measures should be the same if both changes place the individual on the same higher indifference curve. However, if the two price changes place the individual on different indifference curves, the welfare measure should correctly indicate the preference ranking of the two alternatives. The EV measure always provides a consistent ranking in this sense, but the CV measure does not.

Figure 3.4 illustrates why this is the case. It shows an individual in equilibrium at point A, given prices and money income. Suppose that one policy proposal would increase the price of \( x_1 \) and decrease the price of \( x_2 \) simultaneously. The individual would achieve a new equilibrium at point B. The CV measure of the welfare change is shown as \( CV_{AB} \). The second policy alternative would decrease the price of \( x_1 \) while increasing the price of \( x_2 \). This would lead to a new consumer's equilibrium at point C. Point C has been drawn on the same indifference curve as point B. Therefore, the measure of welfare change should be the same for the two policy alternatives. However, as can be seen by inspection, the CV for the second policy, \( CV_{AC} \), is larger. The CV measure would indicate a preference for the second policy while the individual is in fact indifferent between the two policies. The EV gives the same welfare measure for the two policy alternatives. This is because the EV measure bases its comparison on a point on the indifference curve passing through the new equilibrium, but with the old prices. If two policies are on the same new indifference curve, the EV measure picks the same point for measuring the welfare effects for both policies.

If the question being asked by policymakers is, “does the proposed change pass the Kaldor potential compensation test?” then EV is the measure to use. The Kaldor potential compensation test is one form of potential Pareto improvement test that asks whether it is possible for the winners to fully compensate all of the losers from the proposed policy change and still leave someone better off. For each person, the CV gives the compensating income change required to maintain that person at his or her initial utility level. If the sum of what could be collected from all gainers exceeds the sum of the required compensations for losers, the proposal passes this form of the potential Pareto improvement test. The fact that the CV cannot rank consistently two or more policy changes is no obstacle to its use in this manner. This is because the potential Pareto criterion itself provides no basis for ranking two or more proposed policy changes. If two proposed changes both pass the Kaldor potential compensation test, the potential Pareto improvement criterion provides no basis for choosing between them.

On the other hand, if the question being asked by policymakers is, “does the policy pass the Hicks version of the potential compensation test?” then EV is the appropriate measure. The Hicksian test asks whether it is possible for the losers to bribe the gainers to obtain their consent to forgo the proposed policy change. The potential gainers would accept a bribe only if it were large enough to raise their utility by the same amount as the proposed policy would have. The offered
bribe would have to be as large as each individual's EV measure of welfare gain; and the maximum bribe that would be offered by the potential losers would be their EV measure of loss. Thus if the sum of the EV of all gainers exceeded the sum of the EVs of all losers, the proposal would pass the Hicks form of the potential compensation test. Also, since the Hicks form of the compensation test is based on the EV measure, it will consistently rank two or more policy changes, provided that society is indifferent as to the distribution of gains and losses across individuals.

Measurement

Simply put, the problem posed for applied welfare economics is that the desired welfare measures, the CV or the EV, are based on the unobservable Hicks-compensated demand functions, while the one measure based on the observed Marshallian demand functions is flawed as a welfare indicator. The typical practice had been to use the Marshallian surplus anyway, and to offer such justifications as “income effects are likely to be small”; “with only one price change, path dependence is not an issue”; and “it is the only measure we have and it is better than nothing.” Then Robert Willig (1976), in a widely cited article, provided a justification for using the Marshallian surplus by examining the magnitude of the differences between S and CV or EV under different conditions. Willig argued, “In most applications the error of approximation will be very small. In fact the error will often be overshadowed by the errors involved in estimating the demand curve” (1976, 589). Following his work, several authors have developed methods for direct calculation of the CV and EV from information contained in the ordinary demand function, either through a Taylor’s series approximation (McKenzie and Pearce 1982; McKenzie 1983), or as exact measures through integration to obtain the indirect utility function and the expenditure function (see for example, Hausman 1981). The second subsection describes Hausman’s contribution.

Consumer’s Surplus without Apology

Willig (1976) has offered rigorous derivations of expressions relating CV, S, and EV. These expressions provide a way of calculating the magnitude of the differences among the three measures for given prices, quantities, and income. The differences among the three measures depend on the income elasticity of demand for the good in question and consumer surplus as a percentage of income. The differences among the measures appear to be small and almost trivial for most realistic cases. The differences are probably smaller than the errors in the estimation of the parameters of demand functions by econometric methods.

Willig’s bounds for the approximation errors are based on the fact that the differences between S and CV or EV arise from an income effect on the quantity demanded; and the size of that effect depends on the change in real income brought about by the price change and on the income elasticity of demand for the good. This can be shown in a nonrigorous way for the case of one price change with the help of Figure 3.5. Although this exposition applies to the case of only one price change, the Willig expressions can be generalized to accommodate multiple price changes (Willig 1979), provided that a specific path of integration is chosen. In Figure 3.5, the ordinary and compensated demand curves are assumed to be linear. Let S represent the area a + b + c. So:

\[ CV = a + b = S - c, \]  
(3.28)

and

\[ EV = a + b + c + d = S + d. \]  
(3.29)

The errors in using S to approximate CV and EV are equal to the areas c and d respectively. For a price change from \( p' \) to \( p'' \), the factors influencing the size of the approximation error can be seen by examining the determinants of the area c:

\[ CV - S = -c = -\frac{1}{2} \Delta p \cdot \Delta x^*, \]  
(3.30)

where \( \Delta x^* \) is the income effect on the quantity demanded of \( x \), which is associated with reducing income sufficiently to hold utility at \( u^* \). Let \( \Delta M^c \) represent this...
income change. By definition, $\Delta M'$ is $CV$. The definition of income elasticity of demand is

$$E_M = \frac{\Delta x}{\Delta M} \cdot \frac{M}{x}.$$  \hspace{1cm} (3.31)

Solving this expression for $\Delta x'$ gives

$$\Delta x' = E_M \cdot x \cdot \frac{\Delta M'}{M} = E_M \cdot x \cdot \frac{CV}{M}.$$  \hspace{1cm} (3.32)

Substituting this into equation (3.30), we obtain

$$CV - S = -\frac{\Delta x' \cdot E_M \cdot CV}{2M}.$$  \hspace{1cm} (3.33)

In general, for small changes in $p$, $\Delta x \cdot x \approx S$. This is strictly true for the linear demand curve when $x$ is evaluated at the midpoint between $x'$ and $x''$. Finally, dividing both sides by $CV$ to express the error in percentage terms gives

$$\frac{CV - S}{CV} \approx -\frac{E_M}{2} \cdot \frac{S}{M}.$$  \hspace{1cm} (3.34)

This is similar to the Willig expression for the approximation error. The principal difference is that it expresses the error as a percentage of $CV$, while Willig’s term makes the error a percentage of $S$. It says that the error is proportional to the income elasticity of demand and consumer surplus as a percentage of income. A similar line of reasoning can be used to derive the relationship between $EV$ and $S$.

Willig’s analysis is more rigorous than this in that it takes into account the possibility that for finite changes in price and quantity, the income elasticity of demand may vary over the range of the price change. Willig derived rules of thumb for calculating the maximum error in using $S$ as an approximation for $EV$ or $CV$. The rules of thumb are applicable if the following conditions are met:

$$\frac{S}{M} \cdot \frac{E_M}{2} \leq 0.05$$  \hspace{1cm} (3.35)

$$\frac{S}{M} \cdot \frac{E_M}{2} \leq 0.05$$  \hspace{1cm} (3.36)

and

$$\frac{S}{M} \leq 0.9,$$  \hspace{1cm} (3.36)

where $E_M$ and $\bar{E}_M$ are the smallest and largest values, respectively, of the income elasticity of demand for the good in the region under consideration.

Given these conditions, the rule of thumb for $CV$ is

$$\frac{S}{M} \cdot \frac{E_M}{2} \leq \left| \frac{CV - S}{S} \right| \leq \frac{S}{M} \cdot \frac{\bar{E}_M}{2}.$$  \hspace{1cm} (3.37)

and the rule of thumb for $EV$ is

$$\frac{S}{M} \cdot \frac{E_M}{2} \leq \left| \frac{S - EV}{S} \right| \leq \frac{S}{M} \cdot \frac{\bar{E}_M}{2}.$$  \hspace{1cm} (3.38)

The first thing to note is the conditions under which these rules of thumb are valid. Consider equation (3.36) first. The change in consumer surplus as a percentage of income depends on the size of the price change, the price elasticity of demand, and expenditure on this good as a percentage of total income. The smaller the price change and the smaller the proportion of income spent on the good, the smaller $S/M$ becomes. It can readily be shown that

$$\frac{S}{M} \leq \frac{\Delta p}{p} \cdot \frac{S}{M}.$$  \hspace{1cm} (3.39)

From a given initial situation, $S$ is largest when the demand curve is perfectly inelastic. Then $S = 0$ and (3.39) holds as an equality. With more elastic demand, $S < 0$ and the condition follows. For example, it shows that for a good absorbing 50 percent of total income and for a 100 percent price change, $S/M$ cannot exceed 0.5, while for a 10 percent price change for a good absorbing 10 percent of income, $S/M$ will be less than 0.1. Thus, condition (3.36) is likely to be satisfied except for very large price increases for goods with low price elasticities that also absorb a large proportion of the total budget.

As for the first condition, the smaller consumer surplus is as a percentage of income, and the smaller the income elasticity of demand is, the more likely it is that (3.35) be satisfied. For example, if consumer surplus is 5 percent of income, the income elasticity of demand can be as high as 2.0 and still satisfy (3.35). If $S/M$ just barely satisfies condition (3.36), the income elasticity cannot exceed 0.11 to satisfy (3.35).

Assuming that conditions (3.35) and (3.36) hold, then let us turn to the rules of thumb. First, according to (3.35), the maximum error involved in using $S$ as an approximation for either $CV$ or $EV$ is 5 percent. Second, the smaller the change in income elasticity over the range being considered, the more precise (3.37) and (3.38) are as statements of the error involved in using $S$ rather than $CV$ or $EV$. If the income elasticity of demand does not change over the range being considered, the left-hand and right-hand sides of (3.37) and (3.38) are equal to each other and the errors are zero, as discussed above. Finally, as the income elasticity of demand for the good decreases, the differences among ordinary consumer surplus, $CV$, and $EV$ decrease, disappearing as $E_M$ goes to zero.

Willig’s analysis has been interpreted as providing a justification for using consumer surplus as an approximation of the $CV$ or the $EV$. However, there are two reasons why one should be cautious about adopting the Willig approach to welfare measurement. The first has to do with limitations on the applicability of the Willig conditions to some kinds of problems of welfare measurement, including some of specific interest to environmental and resource economists.
this, Hanemann (1980) apparently anticipated the analysis of Hausman (1981),
discussed below.

For some questions, the variable of interest to policymakers is not \( CV \) but
some fraction of \( CV \)—for example, the dead weight loss associated with a tax
on a commodity. Suppose an excise tax raises the price of a good from \( p' \) to
\( p'' \), as shown in Figure 3.6. The consumer’s loss as measured by \( CV \) is the area
\( a + b + c \), but only \( b + c \) is an efficiency loss, since \( a \) is a revenue transfer to
the government. If the ordinary demand curve is used to approximate the consumer
loss, the area \( c \) is the error. If the Willig conditions are satisfied, \( c \) is an acceptably
small percentage of \( S \) and \( CV \), but it can be an unacceptably large percentage of
the true dead weight loss.

The second reason for being cautious about using the Willig approximation is
that better methods of welfare measurement now exist. If the demand functions
being used to calculate \( S \) reflect utility maximizing behavior on the part of
individuals, they should satisfy the integrability conditions. If this is the case, it
is possible to calculate \( CV \) and \( EV \) directly without approximation. On the other
hand, if the demand functions do not satisfy the integrability conditions, then it
is inappropriate to use the Willig approximations, since their derivation was also
based on the assumption of utility-maximizing behavior.

**Exact Welfare Measurement**

Hausman (1981) presented a procedure for exact welfare measurement based on
the recovery of the parameters of the utility function from data on consumers’
demand. His procedure, which was developed for the case of only one price
change, involves four steps. The first involves combining the ordinary demand
function and Roy’s identity to obtain a partial differential equation:

\[
x_i(P, M) = \frac{\partial v(P, M)}{\partial P_i} \frac{\partial P_i}{\partial M} \quad (3.40)
\]

If the utility function is separable so that the demand function contains only its
own price argument, and if the demand function is linear, this becomes:

\[
a - (b \cdot p_i) + (c \cdot M) = \frac{\partial v(P, M)}{\partial P_i} \frac{\partial P_i}{\partial M} \quad (3.41)
\]

where the parameters \( a, b, \) and \( c \) are estimated econometrically, and where \( p_i \) and
\( M \) are deflated by an appropriate index of the other prices. Changes in \( p_i \) and
\( M \) that involve moving along an indifference curve must satisfy

\[
\left. \frac{\partial v(\cdot)}{\partial P_i (\cdot) dt} \right| + \left. \frac{\partial v(\cdot)}{\partial M (\cdot) dt} \right| = 0 \quad (3.42)
\]
where \( t \) defines a path of price changes. Rearranging this expression, substituting into (3.41), and using the implicit function theorem gives

\[
\frac{dM(q)}{dp_t} = a - (b \cdot p_t) + (c \cdot M),
\]

(3.43)

the solution of which is

\[
M(p_t) = k \cdot \exp(-p_t) \left( \frac{a - (b \cdot p_t) - \frac{b}{c}}{c} \right),
\]

(3.44)

where \( k \) is the constant of integration, which depends on the initial level of utility. If units are arbitrarily chosen so that \( k \) is the initial utility level, the quasi-indirect utility function and quasi-expenditure function follow directly:

\[
u = k^0 \cdot \exp(-p_t) \left( \frac{M + \frac{1}{c} \left( a - (b \cdot p_t) - \frac{b}{c} \right)}{c} \right)
\]

(3.45)

and

\[
\epsilon = k^0 \cdot \exp(p_t) \left( \frac{1}{c} \left( a - (b \cdot p_t) - \frac{b}{c} \right) \right).
\]

(3.46)

These expressions are termed “quasi” functions because they do not contain information about the effects of the prices of other goods on utility or expenditure. Hausman’s method depends on the ability to solve the differential equation that is obtained from Roy’s identity. Hausman has shown a method of solution for the case when only one price changes, and has discussed in general terms the solution in the case of multiple price changes.

**Conclusions**

Selection of a welfare measure has long involved questions both of appropriateness and of practicality. The Marshallian surplus measure was frequently chosen on the grounds of practicality, even though it was recognized that the measure was inappropriate in that it did not answer any specific well-formed welfare question. Willig’s development of the bounds for the errors of approximation in using \( S \) gave encouragement to this practice. However, quickly on its heels have come new approaches to exact welfare measurement that offer the opportunity to calculate the more appropriate \( CV \) and \( EV \) measures directly.

One question related to practicality remains, however—do we know enough about the functional form of the utility function to implement the exact measurement methods? Assuming a functional form for the system of demand functions for purposes of estimation is equivalent to assuming the functional form of the underlying utility function. One approach is to assume a specific functional form for the utility function or indirect utility function, and to derive the demand functions for estimation. If this is the case taken, then plugging the estimated parameters back into the utility function to calculate welfare changes is straightforward, provided the parameter estimates of the demand function satisfy the integrability conditions. Since researchers have been reluctant to specify the functional form of the utility function, one alternative has been to specify so-called flexible forms for the indirect or direct utility function (for example, Deaton and Muellbauer 1980). Again, if the integrability conditions are satisfied, deriving “exact” welfare measures from the “approximate” flexible functional form of the utility function is straightforward. The alternative is to seek guidance from the data by selecting the functional form for the demand functions based on goodness-of-fit and consistency with the restrictions imposed by theory.

**Welfare Measures for Continuous Goods: Quantity Changes**

Many environmental policy proposals involve changes in either the quantities or the qualities of nonmarket environmental goods and services, rather than changes in the price of a marketable good. From the individual’s point of view, the most important characteristic of some environmental goods is that they are available only in fixed, unalterable quantities. These quantities act as constraints on each individual’s choice of a consumption bundle. The analysis of this class of problems is often referred to as the theory of choice welfare under quantity constraints (Johansson 1987). The imposition of quantity constraints raises some new issues in the theory of choice welfare. The analysis of these problems has evolved out of the theory of rationing as initially developed by Tobin and Houthakker (1950/1), and Neary and Roberts (1980).

This section provides a brief description of the model of individual preferences and choice under imposed quantity constraints. The corresponding measures of welfare impacts for changes in the quantities of imposed goods are then derived. These measures are essentially similar to the compensating and equivalent surplus measures for price changes presented in Hicks (1943), but the change being considered is one of a quantity or quality change, rather than price. As mentioned earlier, Hicks referred to these measures as compensating or equivalent “surplus,” and this terminology convention is continued in this chapter. The section closes with a brief discussion of the value of changes in \( q \) when \( q \) is a bad.

**The Basic Model**

Consider an individual whose utility function has the following form:

\[
u(X, Q),
\]

(3.47)

where \( X = (x_1, \ldots, x_j) \) is the vector of private goods quantities, and \( Q = (q_1, \ldots, q_k) \) is a vector of environmental and resource service flows (unpriced public goods) that is exogenous to the individual. It is possible that there is a positive price for at least some of the elements in \( Q \); but to keep the exposition simple, all prices for elements of \( Q \) are assumed to be zero. Let \( P = (p_1, \ldots, p_j) \) be the vector of prices for \( X \). The individual maximizes utility subject to a budget constraint.
where $M$ is money income. This yields a set of conditional demand functions for the marketed goods:

$$ x_i = x_i(P, M, Q). $$

(3.49)

In general, $Q$ will be an argument in these conditional demand functions, along with prices and income. The term "conditional" refers to the fact that these functions are conditioned upon the imposed $Q$.

Inserting the conditional demand functions into the utility function gives the conditional indirect utility function

$$ v = v(P, M, Q). $$

(3.50)

Inverting the conditional indirect utility function for $M$ yields a conditional expenditure function that gives the minimum expenditure on market goods required to produce utility level $u$, given $P$ and $Q$. This is

$$ e = M = e(P, Q, u). $$

(3.51)

For simplicity, in what follows $Q$ is assumed to consist of only one element, $q$. In order to make graphic presentations of some of the key points, it is assumed that $X$ is the numeraire, represented as $x$ with a price of 1. Finally, it is assumed that at the given prices and income, the individual would choose more of $q$ if given the option (i.e., $q$ is a "good").

To begin with, the marginal value of a small increase in $q$ is the reduction in income that is just sufficient to maintain utility at its original level. If $w$ is the marginal value or marginal willingness to pay for a change in $q$, it is given by the derivative of the restricted expenditure function with respect to $q$ or

$$ w_q = -\frac{\partial e}{\partial q}. $$

(3.52)

The right-hand side of this expression is also equal (in absolute value) to the slope of the indifference curve through the point at which the welfare change is being evaluated. There are several ways to present compensating surplus ($CS$) and equivalent surplus ($ES$) for changes in quantity-constrained goods.

The first way is based on the conditional indirect utility function. The $CS$ and $ES$ measures are defined implicitly as the solutions to the following expressions: $CS$ is the solution to

$$ v(P, M, q^0) = v(P, M - CS, q^1), $$

(3.53)

and $ES$ is the solution to

$$ v(P, M + ES, q^0) = v(P, M, q^1). $$

(3.54)

These two measures can also be defined in terms of the conditional expenditure function. For a change in $q$, $CS$ is

$$ CS = e(P, q^0, u^0) - e(P, q^1, u^0) = M - e(P, q^1, u^0). $$

(3.55)

The $ES$ measure given by the conditional expenditure function is

$$ ES = e(P, q^0, u^1) - e(P, q^0, u^1) = e(P, q^0, u^1) - M. $$

(3.56)

$ES$ and $CS$ are shown graphically in Figure 3.7. The increase in $q$ enables the individual to reach point $B$ with utility equal to $u^1$. The $CS$ is the distance $B-C$. Alternatively, if income increased by the $ES$ value while holding $q$ constant, the individual could achieve $u^1$ at point $D$. Thus, $ES$ is the distance $A-D$.

A second way to derive the $ES$ and $CS$ measures is also based on the conditional expenditure function. The value of a nonmarginal change in $q$ is the integral of this function taken over the relevant range, or

$$ W_q = -\int_{q^0}^{q^1} \frac{\partial e}{\partial q} dq. $$

(3.57)

This is either a $CS$ or an $ES$ measure, depending on whether $t = 0$ or $t = 1$.

Before leaving this section, note that there are two ways in which more $q$ could be a bad, rather than a good, for an individual. The first way is when $q$ has a price greater than zero and the individual would prefer to have less than the quantity being imposed given that price. The welfare measures $ES$ and $CS$ are still defined in the same way, but now they are negative for increases in $q$ and positive for decreases in $q$. 
The second way in which \( q \) can be a bad is the more fundamental one—it is when the marginal utility of \( q \) is negative. Even at a zero price, the individual would prefer to receive a smaller quantity. In both cases, the welfare measures \( ES \) and \( CS \) are defined in the same way, and again they are negative for increases in \( q \) and positive for decreases in \( q \). In addition, all of the discussion of exact welfare measurement techniques and approximations carries over with appropriate changes to the case of \( q \) as a bad.

**Welfare Measures for Discrete Goods**

In the first part of this chapter the models described for changes in price exploited the marginal equalities revealed when individuals optimize over choice variables that are continuously variable. This is not always a realistic way to model the individual choice problem. Some problems are better viewed as involving the choice of one option from a range of discrete alternatives. For example, the choice might be whether or not to take a once-in-a-lifetime cruise around the world, or whether to travel to work by private auto, bus, or on foot. The solutions of discrete choice problems of this sort are essentially corner solutions. Consequently, there are no tangencies from which a marginal rate of substitution can be inferred. Discrete choice models have been developed both to predict individuals’ behaviors in these choice contexts and to draw inferences about welfare change on the basis of observed choices.

In this section, a simple discrete choice model is presented, and measures of welfare change and value are derived from the model. Welfare measures for both price changes and quantity changes are considered. Subsequent chapters present detailed discussions of applications and estimation approaches. A wide range of environmental problems and decision making can be represented in a discrete choice setting, including; voting yes or no on a referendum question; accepting or rejecting a hypothetical offer for an environmental commodity; the choice of which of several alternative houses to live in based on part on environmental quality in their vicinity; and the choices of whether or not to undertake a specific recreation activity or to visit a specific recreation site. For expositions of the specification, estimation, and interpretation of discrete choice models generally, see Ben-Akiva and Lerman (1985) and Train (2009). Hanemann (1999) gave a more advanced exposition in the context of valuing environmental changes. See also Johansson, Kristrom, and Maier (1989) and Hanemann (1989).

Consider an individual's decision regarding which one of several alternative goods to purchase. The individual can choose one good from a set of \( J \) alternatives \( j = 1, \ldots, J \), where each good has a vector of environmental quality attributes \( \mathbf{Q}_j \) associated with it. The price for good \( j \) is \( p_j \). The individual gets utility from the discrete good chosen and the consumption of a numeraire good.

With this construction, a conditional utility function associated with each alternative can be written as:

\[
   u_j = u_j(M, p_j, \mathbf{Q}_j), \quad j = 1, \ldots, J.
\]

(3.58)

The individual will choose to consume the alternative that yields the highest utility; that is, the chosen alternative \( J^* \) will satisfy

\[
   u_j(M, p_j, \mathbf{Q}_j) > u_k(M, p_k, \mathbf{Q}_k), \quad j, k = 1, \ldots, J.
\]

(3.59)

It is straightforward at this point to implicitly define the compensating and equivalent variation associated with a price change for one or more of the alternatives. Specifically, the compensating variation associated with a decrease in all prices of the discrete alternatives can be written implicitly as:

\[
   \text{Max } u_j(M, p_j^*, \mathbf{Q}_j^*) = \text{Max } u_j(M - CV, p_j, \mathbf{Q}_j),
\]

(3.60)

where superscript \( CV \) indicates the original price, and superscript \( 1 \) indicates the new, lower set of prices. The expression makes clear that the option chosen after compensation is paid could also differ from the original alternative or the choice without compensation. Likewise, equivalent variation can be written as:

\[
   \text{Max } u_j(M + EV, p_j^*, \mathbf{Q}_j^*) = \text{Max } u_j(M, p_j, \mathbf{Q}_j),
\]

(3.61)

where the base level of utility is the utility associated with the new price vector rather than the original.

It is also straightforward to construct the compensating and equivalent surplus measures associated with a change in the vector of quality attributes associated with each alternative:

\[
   \text{Max } u_j(M, p_j, \mathbf{Q}_j) = \text{Max } u_j(M - CS, p_j, \mathbf{Q}_j),
\]

(3.62)

\[
   \text{Max } u_j(M + ES, p_j, \mathbf{Q}_j) = \text{Max } u_j(M, p_j, \mathbf{Q}_j).
\]

A common representation of the utility function is additive. By also recognizing that the budget constraint implies that the amount of the numeraire that can be consumed when alternative \( J^* \) is chosen is \( M - p_j \), the conditional utility function can be written as:

\[
   u_j = \beta(M - p_j) + u_j(\mathbf{Q}_j), \quad j = 1, \ldots, J.
\]

(3.63)

where \( \beta \) can be interpreted as the marginal utility of income, and \( u_j(\mathbf{Q}_j) \) is a function representing the utility associated with the quality aspects of the alternative. With this specification, the compensating and equivalent surpluses for

\[3\] Note that this is a conditional indirect utility function. We depart from our standard notation used throughout the rest of the book and use \( u(\cdot) \) to denote an indirect utility function in this case for consistency with the established literature in this area.
a quality change are identical (a direct result of the constant marginal utility of income, $\beta$) and can be written as:

$$CS = ES = \frac{1}{\beta} \{ \text{Max}_{j} u_{j}(M, p_{j}, Q_{j}) - \text{Max}_{j} u_{j}(M, p_{j}, Q_{j}^{0}) \}, \quad j = 1, \ldots, J.$$  \hspace{1cm} (3.64)

Expression (3.64) is intuitively appealing, as it says that the compensating and equivalent variation associated with a quality change is simply the difference in utility from the most desirable alternatives before and after the change, divided by the marginal utility of income. The marginal utility of income acts to monetize the utility difference.

Thus far, the discrete choice behavioral model and associated welfare measures have been presented in a deterministic form, just as the behavioral model underlying the continuous demand functions and their associated welfare measures were presented earlier in this chapter. Typically, however, analysts employing the discrete choice model recognize that there are individual characteristics and/or omitted variables that are not observable to the researcher, but are known to the individual making the decision. To incorporate this idea, an additive error can be added to the observable component

$$u_{j} = v_{j}(M, p_{j}, Q_{j}) + \varepsilon_{j}, \quad j = 1, \ldots, J,$$  \hspace{1cm} (3.65)

where $\varepsilon_{j}$ is a random, unobservable component of utility. As before, utility maximizers will choose the alternative that yields the highest utility, but from the perspective of the analysis, the utility is now random. This “random utility maximization” model, or RUM model (Thurstone 1927; Marschak 1960; McFadden 1974, 1978, 1981), implies the probability that the individual chooses to purchase alternative “$k$” can be expressed as the probability that the utility associated with $k$ is greater than the utilities associated with all the other alternatives:

$$\text{Pr}(k) = \text{Pr} \left[ v_{k}(M - p_{k}, Q_{k}) + \varepsilon_{k} > v_{j}(M - p_{j}, Q_{j}) + \varepsilon_{j} \right], \quad \forall j \neq k.$$  \hspace{1cm} (3.66)

McFadden (1974) demonstrated that if the error terms are independently and identically distributed (i.i.d.) with a Type I Extreme Value distribution, a logistic distribution results and this probability can be written simply, as follows:

$$\text{Pr}(k) = \frac{e^{\xi(M_{k}, Q_{k})}}{e^{\xi(M_{k}, Q_{k})} + \sum_{j \neq k} e^{\xi(M_{j}, Q_{j})}} = \left( 1 + \sum_{j \neq k} e^{-\Delta u_{jk}} \right)^{-1}$$  \hspace{1cm} (3.67)

where

$$\Delta u_{jk} = v_{j}(M - p_{j}, Q_{j}) - v_{k}(M - p_{k}, Q_{k}).$$

The logit model of choice implies certain restrictions on individuals’ choices and preferences. The most notable is that choices must have the property of the Independence of Irrelevant Alternatives (IIA). This issue, and a host of additional topics related to interpretation and estimation of RUMs, will be discussed in later chapters.

The introduction of an error term complicates welfare computation since only the probability of choosing a particular alternative under a price or quality change can be considered. A general expression for the compensating variation associated with a price change is:

$$\text{Max}_{j} \left[ v_{j}(M, p_{j}, Q_{j}) + \varepsilon_{j} \right] - \text{Max}_{j} \left[ v_{j}(M - CV, p_{j}, Q_{j}) + \varepsilon_{j} \right].$$  \hspace{1cm} (3.68)

where $CV = CV(M, P^{o}, p_{i}, Q_{i}, \varepsilon)$ and $\varepsilon = (\varepsilon_{1}, \ldots, \varepsilon_{J})$ denotes the full vector of error terms. A corresponding equivalent expression for $EV$ can be written. As the notation indicates, this welfare measure will itself be a random variable and its expected value can be computed (Small and Rosen 1981; Hanemann 1984). Using the linear functional form identified in (3.63), compensating and equivalent variations are equal to each other. If in addition, the error terms are Type I Extreme Value, then the mean $CV$ and $EV$ terms take a particularly simple form, with:

$$\text{CV}\bar{=EV} = \frac{1}{\beta} \left[ \ln \left( \sum_{j = 0}^{J} e^{\varepsilon_{j}} \right) - \ln \left( \sum_{j = 1}^{J} e^{\varepsilon_{j}} \right) \right],$$  \hspace{1cm} (3.69)

where $v_{i} = v_{i}(M, p_{i}, Q_{i})$ for $i = 0, 1$. Similar calculations can be used to obtain the value of adding or deleting a site with a specified set of characteristics from the individual’s choice set. For the addition of site $j + 1$, the expression is

$$\text{CV} = \bar{EV} = \frac{1}{\beta} \left[ \ln \left( \sum_{j = 0}^{J} e^{\varepsilon_{j}} \right) - \ln \left( \sum_{j = 0}^{J} e^{\varepsilon_{j}} \right) \right],$$  \hspace{1cm} (3.70)

and for deleting site $j$, the expression is

$$\text{CV} = \bar{EV} = \frac{1}{\beta} \left[ \ln \left( \sum_{j = 0}^{J} e^{\varepsilon_{j}} \right) - \ln \left( \sum_{j = 0}^{J} e^{\varepsilon_{j}} \right) \right].$$  \hspace{1cm} (3.71)

These measures are examples of compensating and equivalent variation approaches to defining a welfare measure using a random utility framework. Hanemann described two such approaches and examined the relationships among them (Hanemann 1999, 43–48).

**When CV and EV Diverge: Willingness to Pay versus Willingness to Accept Compensation**

The results from Willig discussed earlier imply that measures of compensating variation (or surplus) should in theory generally be very close to their associated
equivalent variation (or surplus) measures. Since these measures have willingness to pay for and willingness to accept compensation interpretations, another way to say the same thing is that WTP to acquire a good or price change should typically approximately equate WTA to do without the change. However, there is a substantial body of evidence from stated preference studies, laboratory experiments, and field experiments that suggests that differences between WTP and WTA for the same good can be quite large (Horowitz and McConnell 2002; Sayman and Onculer 2005). Efforts at explaining these differences have taken several paths.

One argument is that these divergences do not represent actual divergences in preferences but reflect experience with the good and the trading environment in which the values are elicited. List (2003, 2004) studied the divergence in an actual marketplace and found that the disparity is highly correlated with experience in the market: those who have extensive experience in buying and selling the good (sports memorabilia at trade shows) exhibit no meaningful disparity. Focusing on the experimental environment in which these values are elicited, Plott and Zeiler (2005) argued that when a full suite of experimental controls is employed, the divergence between WTP and WTA disappears. They presented findings from three experiments to support their argument and concluded that the differences between WTP and WTA reported in the literature relate to misconceptions that subjects have about the task they faced in the experiment, rather than representing a reflection of true value disparity. Fudenberg, Levine, and Maniadis (2012) undertook a similar set of experiments (though their focus was on “anchoring” effects) and found evidence for the existence of the disparity, albeit of smaller size than many previous studies. Other authors have suggested and studied explanations that relate to the value elicitation environment (Hoehn and Randall 1987; Kolstad and Guzman 1999; Guzman and Kolstad 2007).

A second path involves examining the theory of preferences and value more closely to see whether theory predicts the large disparities between true WTA and WTP. One example of this is in the work of Hanemann (1991, 1999). He showed that the price flexibility of income can be expressed as the ratio of two other terms:

$$E_s = \frac{E_M}{\sigma_q},$$

(3.72)

where $\sigma_q$ is the aggregate Allen–Uzawa elasticity of substitution between $q$ and the composite commodity $X$ and $E_M$ is the income elasticity of demand for $q$. If the elasticity of substitution (a measure of the curvature of the indifference curve between $q$ and private goods) is low, $\sigma_q$ can be close to zero. This can lead to a high value for $E_s$ and a large difference between $CS$ and $ES$. However, Hanemann’s analysis does not explain the persistent differences between the two measures in experiments with simulated markets involving commonplace goods such as lottery tickets, coffee mugs, and pens (see Knetsch and Sinden 1984; Kahneman, Knetsch, and Thaler 1990).

Figure 3.8 The value function and the endowment effect

Zhao and Kling (2001, 2004) have also offered a possible explanation that is largely consistent with the standard paradigm. They considered consumers who make decisions about whether to buy or sell goods whose value is uncertain to them when they have the opportunity to delay the decision and gather more information in the meantime. They demonstrated that there are conditions under which this will lead to lower WTP values and higher WTA values than theory would predict in the absence of this potential for learning. The dynamic welfare measures they derived will be further discussed in Chapter 5.

A final approach, and the one that seems to have gained the most traction, has been to move further from standard economic theory. Thaler (1980) proposed that the reconciliation of theory with observation can be brought about by postulating an “endowment effect” on individuals’ valuation functions and a kink in this function at the status quo point. He suggested that this is a reasonable extension and generalization of the prospect theory of Kahneman and Tversky (1979) to choices not involving uncertainty. The idea of the endowment effect and the differential valuation of gains and losses can be shown with the aid of Figure 3.8. The horizontal axis shows the quantity of an environmental good $q$. The vertical axis shows the compensating welfare measure for changes in $q$. This measure is positive (WTP) for increases and negative (WTA) for decreases from some status quo point. Suppose that the status quo is $q_0$. The associated valuation function $w_0$ shows the compensating utility (compensation) that holds utility constant for a given increase (decrease) in $q$ from $q_0$. This function is kinked at the status quo point of $q_0$, showing that the marginal valuation of increases in $q$ is substantially
lower than the marginal valuation of losses from \( q_0 \). A change in the endowment of \( q \) from \( q_0 \) to \( q_1 \) shifts the valuation function. In addition, as Figure 3.8 shows, the willingness to pay for an increase from \( q_0 \) to \( q_1 \) is substantially less than the required compensation for the decrease from \( q_1 \) to \( q_0 \).

In conclusion, although the observed large differences between WTP and WTA can be explained by replacing the standard utility model with one that incorporates an endowment effect, it is not clear that this is always necessary. These differences can also be explained by the absence of close substitutes in the case of unique and perhaps irreplaceable resources and as the rational response to uncertainty and the high cost of information about preferences.

**Aggregation and Social Welfare**

Assume now that we have obtained measures of the welfare changes, either plus or minus, for all individuals. How can we use that information to make choices about public policy alternatives? To put the question in its most profound sense, what is the appropriate relationship between the welfare of individuals and the social welfare? What follows is a brief review of alternative social welfare criteria. Since the main concern of this book is with measurement, the question of social welfare criteria—that is, how to use the measures—is off the main track. For a more extensive discussion of the problem, see Mishan (1960), especially section III, and Bowden and Bruce (1984).

In the literature on welfare economics there are basically four ways to approach this question. The first approach to the question is the so-called Pareto criterion. Only policy changes that make at least one person better off (that is, an individual experiences a positive welfare change) and make no individual worse off (that is, no individual experiences a negative welfare change) pass this criterion. This criterion deliberately rules out any attempts to add up, or otherwise make commensurable, the welfare measures of different individuals. Since virtually all actual public policy proposals impose net costs on at least some individuals, most policy actions by the state could not be accepted under this criterion. This would be particularly true in the environmental area, where environmental management costs are often channeled through the production sector while benefits accrue to households in the form of increased levels of environmental services. It is unlikely that this would result in a pattern of incidence of benefits and costs in which no one would lose. The restrictive features of the Pareto criterion have stimulated an ongoing search for a welfare criterion that would justify the state doing certain things that at least some people feel it should be able to do.

The second approach to the question was proposed in slightly different forms by Kaldor (1939) and Hicks (1939)—these are the two different forms of a potential compensation test discussed earlier. Let us review these tests in the present context of aggregation and social welfare.

As noted earlier, the Kaldor version of the test asks whether those who gain by the policy can fully compensate for the welfare losses of those who lose by the policy. The Kaldor version of the test would be satisfied if the sum of all individual CV and CS measures of welfare changes were greater than zero. The criterion is essentially one of potential Pareto improvement, since if the compensation were actually paid no one would lose from the policy.

The Hicks version of the potential compensation test asks whether those who lose from the policy could compensate the gainers for a decision not to proceed with the policy. If the answer is yes, the policy should be rejected according to the Hicks criterion. If the policy was rejected and compensation was actually paid, those who would have gained from the policy would be just as well off as if the policy had been adopted, and those who would have lost are at least as well off as they would have been with the policy. The Hicks version of the test takes acceptance of the project as its reference point. In effect, it is a decision to forgo the project that creates the gains and losses that are relevant to the Hicksian version of the potential Pareto improvement criterion.

Should compensation actually be paid in either the Kaldor or Hicks cases? If one thinks the answer should be yes, then the compensation test is transformed into a variation of the Pareto criterion in which the state serves to enforce the taxes and transfer payments that are necessary to ensure that no one actually experiences a welfare loss, assuming that such taxes and transfers would be costless. If one thinks that the answer should be no, this is equivalent to, in effect, assuming that all individual welfare changes are commensurate and can be summed together into an aggregate measure of welfare change. This is the efficiency criterion of the new welfare economics. According to the efficiency criterion, the objective of social policy is to maximize the aggregate value of all of the goods and services people receive, including environmental and resource services. One justification for the Hicks–Kaldor potential compensation test is that a large number of efficient projects will spread benefits sufficiently wide so that everyone is a net gainer from the set of projects taken as a whole, even though some might be losers on individual projects. See Polinsky (1972) for an interesting development of this line of reasoning.

Alternatively, one might believe that whether compensation should be paid depends upon who has to pay and who gets the benefits. This requires consideration of the equity (fairness) in the distribution of income as an element in the evaluation of social policy. The third approach to the question of social welfare criteria, proposed by Little (1957), makes explicit the concern for equality. He proposed a twofold test. First, does the policy pass the Kaldor test? Second, does the resulting change improve the distribution of income? The Little criterion legitimizes a concern with the distributional effects of changes in resource allocation, but it does not resolve the question of what constitutes an improvement.

The fourth approach to the question involves an attempt to make specific social judgments regarding equity, and to introduce equity considerations systematically into the evaluation of social policy. The most common proposal calls for the establishment of a social welfare function that gives different weights to individual welfare changes according to the relative deservingsness of the different individuals (Eckstein 1961; Haveman and Weisbrod 1975). Of course, the main problem with
the social welfare weight approach is the determination of the weighting function (Freeman 1971).

Nevertheless, willingness to make explicit value judgments about equity makes it possible to consider a wider range of policy choices. For example, if one opts for the Pareto criterion or the potential compensation version of the Hicks–Kaldor test, one rules out the possibility of accepting a project that has a sum of individual welfare changes that is less than zero, but would substantially improve the distribution of income. An example of such a policy would be one that imposes a welfare loss of $1,000 on a millionaire while bringing benefits of $99 to each of ten impoverished orphans. A welfare-weighting function could approve negative sum policies like this, provided that the weights given to the beneficiaries were sufficiently greater than the welfare weights of the losers. In addition, neither of these criteria would reject a project that imposes costs on no one, but distributes benefits only to the richest in our society. Some might make the value judgment that this, in itself, is undesirable. A social welfare function that included some measure of inequality of the aggregate distribution as an argument might reject inequality-creating projects like this, and it would also be likely to accept negative sum projects that reduced inequality.

The potential compensation test criterion is perhaps the most controversial feature of standard welfare economics. On the one hand, it has been criticized as being incompatible with the Pareto criterion since it allows for a ranking of projects that are Pareto noncomparable. On the other hand, many economists argue that lump sum transfers or other means of transferring wealth are a more appropriate way for addressing equity concerns. Thus, one should adopt projects that pass the potential compensation test and also take steps to efficiently address distributional concerns. In any case, these concerns have not deterred governments from using it for some kinds of policy choices, and economists from advocating greater use of it in a wider range of environmental and resource policy questions.

**Summary**

This chapter has provided a derivation and explanation of the compensating and equivalent measures of individual welfare change for changes in prices and quantities for both discrete and continuous goods. The compensating and equivalent measures answer different kinds of policy-relevant questions because they make different implicit assumptions about the relevant status quo. It is interesting to examine some hypothetical examples.

Suppose that the question is whether to locate a landfill in a particular neighborhood. The neighbors are likely to oppose this proposal, and suppose that it is accepted that the neighbors have a right to an undisturbed neighborhood. Then the relevant measure of the harm for locating the landfill in their neighborhood would be the sum of their compensating measures of loss (CV and CS). The appropriate measure of the gain to those who would use the landfill would be their willingness to pay to locate it in this neighborhood—also a compensating measure. Alternatively, it is argued that the larger society has a right to locate the landfill anywhere, then what is relevant is the neighbors’ willingness to pay to keep it out of their neighborhood. This is an equivalent measure of the potential loss (EV and ES). For the users of the landfill, the value of locating the landfill in this neighborhood is what its users would require to compensate them for locating it in a less desirable place—an equivalent measure of benefit.

Suppose, instead, that the offending facility is a polluting factory that has been in the neighborhood for a long time. If the neighbors are deemed to have a right to a clean neighborhood, then the appropriate reference point for welfare measurement is their utility levels after the factory has stopped polluting. This implies an equivalent measure of welfare change (EV and ES). Specifically, this is a measure of the compensation that the neighbors would require to forgo having the pollution stopped, and a measure of the factory owners’ willingness to pay to continue to pollute. Alternatively, if the factory has a right to pollute, compensating measures of the gain from stopping the pollution are appropriate (CV and CS).

In each case, the appropriate welfare measure can be found by examining the nature of the social transaction that is implied by the policy decision at hand, and by the implicit rights to the services of the environment presumed to be held by the various parties to the transaction. The results for the examples discussed here can be summarized in Table 3.2.

**Table 3.2 Implied property rights and associated welfare measures**

<table>
<thead>
<tr>
<th>Implicit &quot;rights&quot;</th>
<th>Policy question</th>
<th>Gainers</th>
<th>Losers</th>
</tr>
</thead>
<tbody>
<tr>
<td>To the present polluter</td>
<td>Require cleanup?</td>
<td>Neighbors, compensating measure (WTP)</td>
<td>Polluter, compensating measure (WTA)</td>
</tr>
<tr>
<td>To the potential polluter</td>
<td>Allow pollution?</td>
<td>Polluter, equivalent measure (WTA)</td>
<td>Neighbors, equivalent measure (WTP)</td>
</tr>
<tr>
<td>To the neighbors</td>
<td>Require cleanup?</td>
<td>Neighbors, equivalent measure (WTA)</td>
<td>Polluter, equivalent measure (WTP)</td>
</tr>
<tr>
<td>To the neighbors</td>
<td>Allow pollution?</td>
<td>Polluter, compensating measure (WTP)</td>
<td>Neighbors, compensating measure (WTA)</td>
</tr>
</tbody>
</table>

**References**


